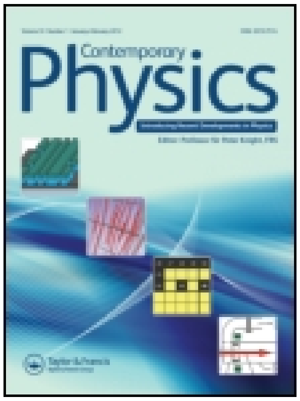

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Laser pulse analogues for gravity and analogue Hawking radiation

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Laser pulse analogues for gravity and analogue Hawking radiation

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Intense pulses of light may be used to create an effective flowing medium which mimics certain properties of black hole physics. It is possible to create the analogues of black and white hole horizons and a photon emission is predicted that is analogous to Hawking radiation. We give an overview of the current state of the art in the field of analogue gravity with laser pulses and of its implications and applications for optics.

Keywords: nonlinear optics; analogue gravity; Hawking radiation

1. Analogue gravity

'Analogue gravity' is the study of phenomena traditionally associated with gravitation and general relativity, by means of analogue models that can be realised in very different physical systems and that do not directly rely on gravity at all [1]. Following recent developments in the field, in this paper we attempt to give a brief overview of how laser-pulses may be used to create the analogue of an astrophysical event horizon. Analogues, as a tool to study the physics of one system in an often apparently and completely different system, are certainly not a novelty. Just to name one recent example, certain features of electron behaviour in solid state physics have been reproduced using light propagation in specifically engineered media, e.g. photonic crystals or optical waveguide arrays [2]. Some of these results have also led to significant technological developments in laser physics and are a clear example of why we should study, or attempt to study analogue models.

W.G. Unruh first proposed the possibility to reproduce the space–time geometry of a black hole by studying sound waves in a flowing medium [3]. If the flow is accelerated so as to pass from subsonic to supersonic, then sound waves propagating upstream in the subsonic region will be slowed down by the counterflow. As they approach the transition point from sub to supersonic counterflow they will be slowed down to zero-velocity, i.e. they will never be able to penetrate into the supersonic region. They will be blocked at the point at which the speed of sound and the speed of the flow become equal. This represents the analogue of an event horizon surrounding a black hole, i.e. the point in which the gravitationally induced flow of space becomes equal to the speed of light [4,5]. However, the analogy goes further than this. The underlying space–time geometry of a flowing fluid shows the same features as that of a black hole and even more impressively, quantum effects arising from the space–time curvature close to the horizon, such as Hawking radiation, are predicted to occur also in the analogue systems. Hawking radiation is the emission of light, or in general particles, from massive black holes that shed energy in the process and is often referred to as black hole evaporation. The existence of a temperature associated with a black hole was first anticipated by Bekenstein [6] on the basis of thermodynamic considerations and was later derived by Hawking in 1974 in terms of a thermal emission and explained on the basis of a quantum analysis of the vacuum state in the curved space–time close to the black hole horizon [7,8]. An intuitive picture of the process may be drawn by imagining that the vacuum fluctuation pairs close to the horizon are split so that the inner photon falls in, usually referred to as the 'negative mode' and the outside photon, or 'positive mode', escapes away from the black hole. Therefore, the outgoing photon cannot return to the vacuum and it becomes a real entity, gaining energy at the expense of the black hole. The particularity of Hawking radiation is that it is predicted to exhibit a blackbody spectrum with a temperature given by

$$T_{\mathrm{H}} = \frac{\hbar\kappa}{2\pi c k_{\mathrm{B}}}, \tag{1}$$

where $\hbar$ is the reduced Planck constant, c is the speed of light in vacuum and k_B is Boltzmann's constant. κ is

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the surface gravity, i.e. loosely speaking, the gravitational acceleration experienced at the horizon. For a black hole $k = c^4/4GM \sim 3 \times 10^{48}/M$ m s^{-2}, where G is the gravitational constant and M is the mass of the black hole. The very idea that such a process may occur has fascinated researchers for the last 40 years as can be evaluated by the huge number of published papers that study various aspects and problems of Hawking radiation. However, as shown above, T_{H} scales inversely to the black hole mass and a quick calculation tells us that the temperature of a typical black hole is of the order of 1 nK or less. This emission therefore has little hope of being directly observed given the surrounding temperature of the Cosmic Microwave Background which is nine orders of magnitude higher. The perspective of being able to possibly observe a similar process, under controlled conditions in an Earth-based laboratory, even if in an analogue setting, is truly exciting.

A large number of analogue models have been proposed in the last few years: acoustic analogues based on flowing fluids or even in the atmosphere [9], superfluids, Bose–Einstein condensates, gravity (not to be confused with gravitational) waves in water, ion rings and more recently optical analogues based on so-called luminous liquids (i.e. light propagation in a regime that mimics a hydrodynamical flow) and on laser pulses in nonlinear optical media. An overview of most of these is given in the excellent review paper by Barceló et al. [1]. Here we will focus attention on the last analogue model cited in the list, i.e. we will describe how intense laser pulses propagating in a dielectric medium may be used to recreate an effective non-uniform medium that flows at close to the speed of light and may thus be used to induce an effective horizon and observe analogue Hawking radiation.

Before moving on, a further comment is due in response to frequently recurring questions on the nature of the analogue model at play here. Indeed, it is often not clear where the applicability of an analogue model starts and where it ends. So to what extent may the results obtained in one system be applied to the other? And is the analogue model useful in both senses, i.e. in this specific case can optics really teach us something new about black hole physics? And are we learning anything new in optics?

The boundary of validity of analogue gravity is relatively well defined in the sense that it is clear that any analogue gravity model will only capture black hole kinematics, i.e. it will describe how waves move and propagate in the presence of an analogue horizon. If the metric that determines how the waves propagate can be identified with that of a black hole, then we do have a tool with which this propagation may be studied in the laboratory. However, the dynamics of a black hole will not in general be correctly modelled by an analogue model: the dynamics depend on the Einstein equations and are ultimately related to the fact that gravity is the driving force behind black hole evolution. There is no actual gravity in the analogue models and one must obviously refer to the relevant hydrodynamic equations (for fluids) or Maxwell equations (for electromagnetic waves) to model the analogue system evolution. The question of the importance of the analogue in one field or the other is a point that is continuously evolving and under discussion. At the current state of the art it is not clear if analogue gravity will be of use to astrophysicists or cosmologists. Even when considering what is expected to be analogue gravity's major breakthrough, i.e. the possibility to study in detail Hawking radiation, we must bear in mind that this is an *analogue* version of Hawking radiation and does not necessarily imply anything regarding astrophysical black hole evaporation. Nevertheless, we may still argue that such analogies are extremely important: they give us the means to transfer knowledge from one area of physics to another. In this specific case new discoveries can be made by comparing how light propagates in a moving medium with the propagation of light close to a gravitational event horizon. Even more interestingly, these concepts can be extended to virtually any kind of wave propagating in a moving medium such as acoustic oscillations in flowing fluids or in Bose–Einstein condensates, polaritons in semiconductor materials, gravity waves in flowing water etc. All of these are linked together by the common underlying analogy with flowing space in a gravitational field. So, whilst one may argue whether astrophysicists will learn anything or not from analogue gravity, there appears to be no doubt that the use of general relativity and the hunt for Hawking-like radiation in analogue systems is leading to significant and new discoveries. In the following we will discuss the new physics that emerge when considering how a travelling dielectric perturbation interacts with a laser pulse or with the vacuum state. The physics that emerge are indeed quite unlike any other known light–matter interactions.

2. Analogue gravity with optics in moving media

The study of light in moving media is certainly not a novelty and has a relatively long history. As far back as 1818, well before Einstein introduced the theory of special relativity, Fresnel discovered theoretically that the speed of light v_ϕ in a uniform and flowing medium depends on the flow velocity v:

$$v_\phi = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v. \tag{2}$$

Not many years passed and in 1851 Fizeau experimentally verified this prediction. Only after Einstein developed his theory of general relativity was it possible to make the first connection between electromagnetic waves propagating in dielectrics and in gravitational fields. The first work in this sense was developed by Gordon who derived the so-called Gordon metric that describes how light propagates in a non-stationary medium and to which we will refer below [10]. Pham Mau Quam in the late 1950s tackled the problem of ray optics in moving media [11] yet the problem of how light propagates in moving media that also exhibit a velocity gradient attracted little attention until very recently. Landau and Lifshitz give an overview of the general problem but explicitly note that they 'neglect slight effects due to the possibility of a velocity gradient' [12]. It is precisely these 'slight effects' that will be at the centre of the following discussion. A more recent review of optics in nonstationary media has been given by Shvartsburg [13] with particular emphasis on the interaction of light with a rapidly varying ionisation or plasma. Within a similar context, Rosanov has studied the 'parametric' Doppler effect in which light interacts with a medium that is at rest but has time-varying or moving parameters [14, 15]. A tightly correlated phenomenon is the so-called time refraction, first introduced by Mendonça by drawing a parallelism between the behaviour of light at a spatial boundary and a time-varying boundary [16, 17]: as is well known and summarised by Snell's relations, when light traverses a spatial (and time-stationary) boundary separating two media with different refractive indices, its wave-vector is modified. In a similar fashion, if the refractive index changes in time, i.e. there is a 'temporal boundary', then the frequency of light is modified. These ideas were then extended to account for quantum effects and, of particular interest in this context, included also the quantum vacuum and excitation of entangled photon pairs [18–20].

Over the last 10 years a number of papers have returned to the problem of light in moving media within the specific context of analogue gravity and with the explicit goal of evaluating the analogy between these systems and gravitational black holes [21–32]. However, it was only recently that Leonhardt proposed an idea by which an experimentally accessible layout may actually create the conditions required to observe the analogue of an horizon for light propagating in a dielectric medium [33].

2.1. *Recreating the flow of space*

The properties of space and time are conveniently described in terms of a metric. The metric defines the elementary length (infinitesimal interval) in terms of the space and time coordinates at that point and may be distorted by mass or, as in the case discussed below, by a flowing medium. The basic metric that describes a black hole, first derived by Schwarzchild, was later cast in different form by Painlevé and Gullstrand [34,35]. The Painlevè–Gullstrand metric is

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - (\mathrm{d}r - V\,\mathrm{d}t)^2, \tag{3}$$

where we are considering only radial trajectories (so as to eliminate an angular dependence) and

$$V = -\left(\frac{2GM}{r}\right)^{1/2}. \tag{4}$$

Based on these equations we may interpret space as if it were a fluid that is flowing with velocity V. This is the basis of the 'river model' [5] that allows an intuitively appealing yet mathematically correct understanding of how analogue models for gravity work. Figures 1(*a*) and (*b*) schematically show how space flows and falls into an astrophysical black hole or, under time reversal, emerges out of what is called a white hole. The flowing river of space moves in a Galilean fashion through a flat Galilean background space [5] and we may define a special point, the Schwarzschild radius, for which $r_S = 2GM/c^2$ and the flow velocity equals c: beyond this point the flow exceeds the speed of light. Therefore, because objects moving through the river must obey the laws of special relativity and their speed cannot exceed c, it is not possible to escape out of the black hole once inside r_S nor is it possible to penetrate inside the white hole beyond r_S.

On the basis of this reasoning, one may therefore attempt to construct a laboratory analogue using light in a flowing medium that recreates a flowing river of space. The original proposal by Unruh based on acoustic waves in a flowing fluid has a metric that can be reduced to a form similar to Equation (3) in which c represents the speed of sound [3]. However, it is extremely difficult to imagine a method by which we may actually induce a flow of matter at speeds that are close to the speed of light, as required in order to recreate an analogue optical horizon. One early proposal by Leonhardt attempted to bypass this obstacle by using so-called slow light [22, 36]: with electromagnetically induced transparency or metamaterials it is possible to slow light down to small fractions of c. However, the slowing down of light in these systems primarily affects only the *group* velocity whilst it was later realised that the important quantity for observing particle creation at a horizon is the *phase* velocity [36]. Leonhardt recently solved this issue by

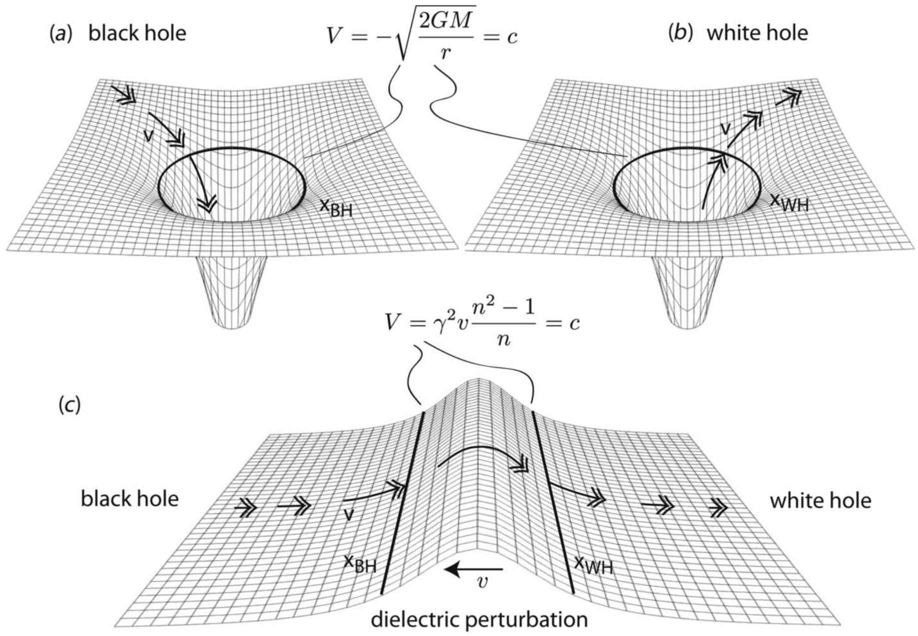

Figure 1. The flowing river of space in an astrophysical black (*a*) and white (*b*) hole. The black and white hole horizons are the points at which space flow becomes equal to c. Part (*c*) depicts the effective geometry induced by a one-dimensional dielectric perturbation moving from right to left. In the frame comoving with the perturbation, space flows as shown by the arrows. Longer arrows indicate larger velocities.

suggesting that the medium itself need not flow at all. All that is necessary is a localised perturbation of the refractive index that travels at speeds close to c. So the medium itself remains at rest in the laboratory frame and by using nonlinear optics (as described below) we may create an ultrafast perturbation. Just to fix ideas, we assume the perturbation to be gaussian-shaped with positive δ_n, so that the surrounding refractive index has some background value $n = n_0$ and then this gradually increases up to a maximum value $n = n_0 + \delta_n$. This perturbation of the dielectric medium is then made to move at a velocity v that is very close to the speed of light in the medium c/n_0. We may modify Fresnel's relation, Equation (2) to account for the fact that the perturbation is now localised: $v_\phi = c/n_0 + [1-1/(n_0+\delta n)^2]v$: we can see from this that an increase of δ_n in the local refractive index is indeed equally perceived by light as a local increase in the flow velocity, i.e. both lead to a slowing down of the light pulse. If $v \gtrsim c/n_0$, the perturbation will catch up with the light pulse or, in the comoving frame, the light pulse will be gradually sucked inwards. As the pulse is sucked in, the refractive index (or space flow velocity) increases and the speed at which the pulse falls in towards the perturbation increases. The light pulse will eventually pass the point x_{BH} of no return at which $\upsilon = c/(n_0 + \delta n_{\mathrm{BH}})$. This point is the analogue of a black hole horizon. In a similar fashion, one may consider the trailing edge of a perturbation with $v \lesssim c/n_0$: light approaching from behind will catch up with the perturbation. As it penetrates within the higher refractive index region it will be slowed down by the higher refractive index or, equivalently, by the faster space flow. The pulse will then be blocked at the point xWH for which $\upsilon = c/(n_0 + \delta n_{\mathrm{WH}})$. This is to all effects a time-reversed version of the black hole horizon, i.e. it is the analogue of a white hole horizon.

More precisely, we may consider the Gordon metric that describes space–time in the dielectric analogue [see below, Equation (7)]. Belgiorno et al. [37] has shown that by redefining the spatial coordinate, the Gordon metric in the comoving frame may be re-cast so that it is formally equivalent to the Painlevé–Gullstrand metric Equation (3):

$$ds^2 = c^2\,dt'^2 - (d\tilde{x}' - V\,dt')^2, \tag{5}$$

where an overall multiplicative (conformal) factor has been dropped and

$$V = \gamma^2 v \frac{n^2-1}{n}, \tag{6}$$

where $\gamma = 1/(1-\upsilon^2/c^2)^{1/2}$. This shows that the effect of the perturbation is indeed formally analogous to that of a mass and can be viewed in terms of space flowing with a velocity V that, for a fixed perturbation velocity, is determined by the local value of the refractive index

$n = n(\tilde{x}') = n_0 + \delta n(\tilde{x}')$. As in the Painlevé–Gullstrand coordinates, an horizon is formed when the space flow velocity equals the speed of light, $V = c$.

Figure 1(*c*) illustrates the geometry of a one-dimensional δn perturbation: the arrows indicate the equivalent flow of space. Interestingly, the flow of space is such that a single perturbation may recreate the analogue of both a black hole, on the leading edge, and a white hole on the trailing edge.

2.2. *White holes*

Black holes are relatively well-known objects whilst white holes, also solutions to the Einstein equations, are less studied due mostly to the fact that there is no obvious mechanism by which a gravitational white hole may form. A black hole will trap any incoming matter or light and forms as a natural consequence of the gravitational collapse of super-massive stars. A white hole may eject particles or light until it burns out and, most importantly it does not appear as the result of a gravitational collapse. Hawking specifically addressed this problem [38] and pointed out that the very nature of Hawking radiation implies that the black hole is at thermal equilibrium. Then, by the ergodic assumption, the system is equally likely to pass through all possible states if observed for a long enough time, including the time-reversal of any of those states. In other words, to an external observer a white hole and a black hole are completely indistinguishable and the emission of a white hole is the same as that of a black hole with the same mass [38, 39]. So we are justified in studying one or the other kind of hole (black or white), based on what is most convenient in a given context. Indeed, if we are unlikely to observe white holes in the cosmos, these objects actually turn out to be more the rule than the exception in the context of analogue gravity. In the original proposal by Leonhardt and co-workers [33], the travelling dielectric perturbation is generated by an intense laser pulse that propagates in an optical fibre. However, the same nonlinearities that generate the refractive index pertubation will also distort the laser pulse through a self-steepening effect that creates a very sharp shock front on the trailing edge [40–42]. However, as shown in Equation (10) below, only steep gradients in the perturbation lead to effective horizons implying that the trailing edge, i.e. white hole horizon will largely dominate over any effects from the leading-edge analogue black hole. In what follows we shall therefore restrict our attention only to the case analogue white hole horizons.

As a final comment, we note that analogue gravity, as all analogies, is at the level of a subset of the mathematical equations. By this we mean that analogue gravity does not reproduce all of the physics of gravity. Nor is the analogy expected to reproduce the exact physics in *all* aspects. If it did, then it would no longer be an analogy but rather it would be an 'identity'. In the specific case described in this paper, the analogy only goes as far as attempting to reproduce the kinematics of gravitational event horizons, e.g. features that are well described by the metric and resulting geodesics. It cannot reproduce the dynamics, e.g. the formation or evolution of gravitational black holes as these depend on the existence of mass and are described by the Einstein equations. And the Einstein equations are not reproduced in this analogy. The fact that gravitational white holes are unlikely objects is a result of the nature of gravitational dynamics. On the other hand, white holes are readily formed in optics as a result of the dynamics of intense light pulses in dielectric media. The analogy therefore is not connecting the horizon formation dynamics in the two settings but is only connecting the resulting kinematics, e.g. how a probe light pulse or how the electromagnetic vacuum behaves in the vicinity of a horizon once this is formed.

3. Dielectric white hole metrics and Hawking radiation

The relevant metric in the dielectric analogue context is the Gordon metric [10,21,30,37,43],

$$\mathrm{d}s^2 = \frac{c^2}{[n(x-vt)]^2}\,\mathrm{d}t^2 - \mathrm{d}x^2, \tag{7}$$

where the travelling dielectric perturbation is described by $n(x-vt) = n_0 + \delta n(x-vt)$. We may rewrite this in the perturbation comoving frame by means of a boost: $t' = \gamma[t-(v/c^2)x]$, $x' = \gamma(x-vt)$, so that

$$\begin{aligned}\mathrm{d}s^2 = {}&\gamma^2\frac{c^2}{n^2}\left[1-\left(\frac{nv}{c}\right)^2\right]\mathrm{d}t'^2\\ &+2\gamma^2\frac{v}{n^2}\left(1-n^2\right)\mathrm{d}t'\,\mathrm{d}x'-\gamma^2\left[1-\left(\frac{v}{nc}\right)^2\right]\mathrm{d}x'2.\end{aligned} \tag{8}$$

The primed coordinates, here and in the rest of this paper, indicate comoving coordinates. There is an ergosurface defined by putting the first term (g_{00}) equal to zero, i.e. for $1-nv/c = 0$. This ergosurface also corresponds to an horizon in the 1D + 1 (i.e. 1 spatial dimension + time) case [37] depicted in Figure 1(*c*) and exists when

$$\frac{c}{n_0+\delta n} < v < \frac{c}{n_0}. \tag{9}$$

This equation may be read in two different ways: for a given background index and perturbation amplitude, only perturbations with a certain velocity will give rise to an analogue horizon. Alternatively, for a given perturbation velocity and amplitude, only those frequencies that propagate with a refractive index that satisfies Equation (9) will experience the effect of the horizon. Indeed, in general n_0 varies with frequency ω due to material dispersion. We note that although relation (9) was not originally derived from a dispersive theory, recent models that account also for dispersion arrive at exactly the same equation (see e.g. [20]) where $n_0 = n_0(\omega)$ is the medium phase index. This equation therefore represents the fundamental relation against which one may compare measurements in order to verify if any observed radiation may be related to the presence of an horizon. For example, one may vary the velocity and/or the perturbation amplitude and search for consistency with Equation (9).

From the comoving-frame metric (8) we may deduce the equivalent of a surface gravity at the horizon which is found to be [37]

$$\kappa = \gamma^2 v^2 \frac{\mathrm{d}n}{\mathrm{d}x}\bigg|_H, \tag{10}$$

where the refractive index perturbation gradient is evaluated at the horizon H. This relation may then be substituted into Equation (1) to obtain the actual temperature predicted for Hawking radiation from the analogue horizon. Laser pulse induced perturbations may be extremely steep, with a rise from n_0 to $n_0 + \delta n$ over a distance of the order of 1 μm or even less. This leads to temperatures, measured in the comoving frame, that are easily of the order of 1–10 K, i.e. many orders of magnitude higher than in any other system proposed to date.

These formulas only show that if Hawking radiation is emitted by the analogue horizon, then it is expected to have a certain temperature. A full quantum electrodynamical model of the perturbation accounting for the interaction with quantum vacuum, such as that developed in [37] is required in order to show that Hawking radiation is actually emitted from the horizon. The model starts by considering the electromagnetic vacuum states in the absence of any perturbation and then compares these with those in the presence of the travelling perturbation. The so-called Bogoliubov coefficients that express the new states as a function of the old states can be used to directly evaluate the number of photons produced in such a scenario. The result clearly shows a logarithmic divergence of the phase of the electromagnetic field under conditions identical to Equation (9) [37]. It is this phase divergence, as originally pointed out also by Hawking, that leads to the creation of new modes, i.e. emission of radiation from the vacuum state. Moreover, this emission is found to follow the expected blackbody dependence with temperature

$$T = \frac{1}{\gamma}\frac{1}{1-(v/c)n_0\cos\theta}T_{\mathrm{H}}, \tag{11}$$

where T_H is evaluated from Equations (1) and (10), and θ is the angle of the direction of observation with respect to the propagation axis of the perturbation. The multiplicative factor in Equation (11) is simply the Doppler shift that transforms the temperature from the comoving frame to the laboratory frame. When viewed from the forward direction, $\theta = 0°$, the temperature measured in the laboratory frame is therefore predicted to be of the order 1000 K or more [33,37,44].

3.1. Dispersion

In order to create an effective flowing medium we must perform experiments in a dielectric material of some kind within which we generate the travelling perturbation. In general in any medium in which we decide to perform these experiments, the refractive index will vary as a function of frequency or, equivalently wavelength. This leads to a qualitative deviation from the ideal (dispersion-less) case analysed by Hawking. In Figure 2(*a*) we show the phase velocity of light with no dispersion (it is thus constant at all wavelengths) far away from the perturbation ($\delta n = 0$) and at the peak of the perturbation with $\delta n = 0.005$. If the perturbation velocity is tuned anywhere in between these two velocity values, for example to the value indicated by the dashed line, then the whole spectrum (shaded area) experiences an horizon and Hawking radiation will

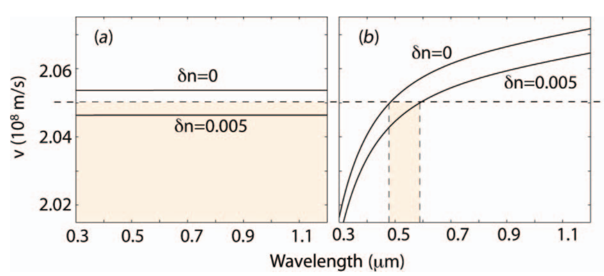

Figure 2. (*a*) Light phase velocity with no dispersion far away from the perturbation ($\delta n = 0$) and at the peak of the perturbation with $\delta n = 0.005$. The dashed line indicates a value of the perturbation v for which Hawking radiation will cover the full predicted blackbody spectrum. (*b*) Light phase velocity with dispersion: fused silica glass dispersion has been used in this example. For the same conditions as in (*a*), only a very restricted bandwidth of wavelengths will be emitted in the form of Hawking radiation.

cover the full predicted blackbody spectrum. In Figure 2(*b*) we show the same situation but now including dispersion: fused silica glass dispersion has been used in this example. For the same conditions as in (*a*), only a restricted bandwidth of wavelengths will be emitted in the form of Hawking radiation (shaded area) and the full blackbody spectrum will not be visible. On the one hand, dispersion appears therefore to 'ruin' the spectrum. But on the other it provides us with a remarkably effective method to test the presence of an horizon: the emitted spectrum depends critically on the velocity of the perturbation. If this can be controlled, then we have a very simple method by which we may compare precise spectral measurements against the straightforward prediction of Equation (9).

Dispersion has a further important consequence on Hawking radiation. In the absence of dispersion, radiation would accumulate at the white hole horizon for an infinite time. This would also lead to an infinite phase divergence and a consequently infinite blue-shift of the incoming frequency. Likewise, any emission observed far from a black hole horizon must originate from an infinitely blue-shifted vacuum fluctuation close to the horizon. Hawking radiation would therefore seem to originate from wavelengths that close to the horizon were smaller than the Planck scale, $\sim 10^{-35}$m. This in turn raises some doubts regarding the validity of the actual prediction of Hawking radiation as the laws of physics are expected to change radically at these length scales (the so-called transplanckian problem). Dispersion completely avoids this issue: as light accumulates on the white hole horizon it is blue shifted. However, in most dielectrics blue-shifted wavelengths will travel slower than red-shifted wavelengths implying that at a certain point the blue-shift will become such that the light will slow down and finally detach itself from the perturbation well before the Planck scale is reached. The study of analogue Hawking radiation therefore occurs in a regime in which transplanckian issues are of no concern or relevance.

In Figure 3(*a*) we show a typical dispersion relation that we will be dealing with. This is a simple quadratic dispersion curve, $n_0 = A + B\omega^2$, where $A = 1.44$ and $B = 10^{-31}$ s^2. Such a simplified dispersion relation can actually match the real dispersion of, for example, fused silica glass in the visible and near-infrared spectral region with sufficient precision to capture all of the necessary physics, at least at a qualitative level. The solid/dashed curves in Figure 3 indicate the dispersion branches that have positive/negative frequency in the laboratory reference frame. A common practice is to adopt the dispersion curves in the comoving frame rather than in the laboratory reference frame. We pass from one reference frame to the other using the Doppler relations

$$\omega' = \gamma(\omega - vk), \tag{12a}$$

$$k' = \gamma\left(k - \frac{v}{c^2}\omega\right), \tag{12b}$$

where $k = \omega[n(\omega) + \delta n]/c$. The laboratory frame dispersion curves in Figure 3(*a*) transform in the comoving frame (for $\upsilon = 0.99c/n$ and $\delta n = 0$) as shown in Figure 3(*b*). An alternative and useful representation with ω' as a function of ω is shown in Figure 3(*c*). We note that for increasing dn or v, the comoving frame dispersion curves will bend further downwards, as shown in Figure 3(*c*). The comoving frame dispersion curves are particularly convenient due to the fact that frequency ω' is a conserved quantity. This can be demonstrated on very simple grounds by deriving the hamiltonian for a space–time varying medium and by showing that ω' is a constant of motion [16]. An alternative route is based on a scattering model. The perturbation acts as a scattering defect and the scattered light will have a wave-vector $k_{\text{out}}(\omega)$ that is related to the input wave-vector $k_{\text{in}}(\omega_{\text{IN}})$ by momentum conservation [45]:

$$k_{\text{out}}(\omega) = k_{\text{IN}}(\omega_{\text{in}}) + \frac{\omega - \omega_{\text{in}}}{v}. \tag{13}$$

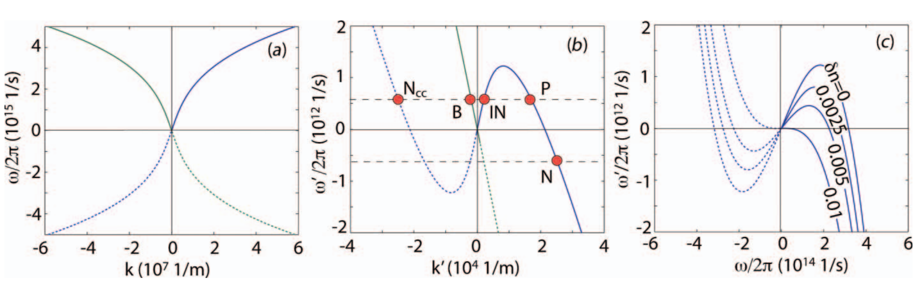

Figure 3. Dispersion curves for the numerical simulations in Figures 4(*a*) and 6(*b*). (*a*) Dispersion curves in the laboratory reference frame. (*b*) Dispersion curves in the comoving frame. The full-circles indicate the position of the IN, P and N mode frequencies and the horizontal dashed lines correspond to $\pm\omega'_{\text{IN}}$. In (*c*) we plot the comoving frequency, $\omega'/2\pi$, as a function of the laboratory frame frequency, $\omega/2\pi$ for various values of δn.

This momentum conservation relation may be transformed into the comoving frame using Equation (12) and leads to

$$\omega' = \omega'_{\mathrm{IN}}. \tag{14}$$

In other words, momentum conservation in the laboratory reference frame is equivalent to frequency conservation in the comoving frame. We may therefore use this to predict how an input probe wave is transformed during the interaction with the perturbation by simply looking for the intersections of the comoving dispersion relation with a horizontal line that passes through ω'_{IN}. We call these intersection points 'modes', in the sense that they identify specific modes of the electromagnetic field and are described by well-defined ω' and k' values (or ω' and k' in the comoving reference frame). In particular, in the following we will continuously refer to the input mode as the 'IN' mode and the positive or negative frequency Hawking modes generated at the horizon as the 'P' and 'N' modes. In Figure 3(*b*) the horizontal dashed line intersects the dispersion curve at two points for positive frequencies. The IN mode has positive group velocity $v'_g = \mathrm{d}\omega'/\mathrm{d}k'$ and thus approaches the perturbation, i.e. it is moving forward on the comoving frame. The scattered mode, indicated with 'P', occurs with a negative gradient and is thus reflected backwards from the perturbation (note that in the laboratory frame, both IN and P modes will be moving in the forward direction). A third mode is possible and is indicated with 'B': this is just the IN mode that is propagating backwards and is usually not considered as only the forward propagating IN mode is excited. Finally, a fourth mode is available, namely the intersection with $-\omega'_{\mathrm{IN}}$: this gives what we will call the negative mode, indicated with 'N'. This mode is allowed as it is the complex coniugate of the 'N *cc*' mode that lies at $+\omega'_{\mathrm{IN}}$. However, we prefer to consider the N mode rather than the N *cc* mode as the former has positive frequency in the laboratory reference frame and will correspond to the mode actually measured in experiments.

We note that mode conversion from the IN mode to the P mode has been observed in a wide variety of settings (although this may not be immediately apparent due to the different terminology with respect to that used here):

(1) In optical fibres it is possible to excite a soliton pulse as the result of a balance between nonlinear (intensity induced) frequency broadening and dispersion. However, at high input intensities high order solitons are generated that then breakup as the result of an instability and shed blue-shifted light that is usually called a dispersive wave or Cherenkov radiation (not to be confused with the Cherenkov radiation generated by superluminal charged particles) [46]. The dispersive wave emission obeys a momentum conservation law which, neglecting a nonlinear phase correction term, is identical to Equation (13) [46–49] and thus falls under the same general explanation presented here.

(2) In higher dimensions, e.g. in 2D waveguides or in bulk media, self-focusing and self-induced spatiotemporal reshaping of the input pulse lead to the formation of the so-called X-waves. X-waves are characterised by two hyperbolic branches in the spectrum when viewed in angle-frequency coordinates: one branch passes through ω_{IN} and the other passes through the P mode frequency at zero angle. A more general description may be given in which the whole X-wave is actually expressed using only Equation (13) [45,50–53].

The P mode therefore emerges as an ubiquitous feature in nonlinear optics and it owes this ubiquity to the fact that it is simply a restatement of moment conservation.

The same reasoning may of course be applied to the negative N mode: this mode too is a result of momentum conservation and should therefore be expected. However, to date no (optical) measurements have ever been performed that report the existence of this mode. The dispersion relations simply tell us which are the allowed modes, but do not tell us if the modes will actually be created.

The P and N modes together form the classical analogues of the Hawking pairs emitted from an horizon. The existence of these modes is a very general feature that is related only to the form of the dispersion relation and to the existence of a natural comoving reference frame in which frequency conservation leads to the excitation of negative frequencies. Recently this same reasoning has been applied to surface waves travelling in flowing water. By creating a gradient in the water flow, negative frequencies were observed, generated at the horizon [54]. These first measurements were then further developed and led to the first demonstration of *stimulated* Hawking emission [55]. The horizon is stimulated by a probe wave and the amplitudes of the emitted P and N waves are measured. According to the theory, Hawking radiation in both the gravitational and analogue context will be characterised by precise relations that link the norms of the emitted waves (normalised with respect to the norm of the input wave $|\mathrm{IN}|^2$) [7,8,55,56]:

$$|P|^2 - |N|^2 = 1, \tag{15}$$

$$\frac{|N|^2}{|P|^2} = \exp(-\alpha\omega). \tag{16}$$

Equation (15) implies that $|\mathrm{P}|^2 + |\mathrm{N}|^2 > 1$ and the Hawking effect will lead to amplification. Equation (16) imposes a strict relation between the two modes: it can be easily verified that indeed (15) and (16) imply a thermal emission for the N mode, $|\mathrm{N}|^2 = 1/[\exp(\alpha\omega) - 1]$, where α may be linked to a blackbody temperature $T_\mathrm{H} = \hbar/\alpha k_\mathrm{B}$.

Gerlach gave a description of the black hole horizon in terms of a parametric amplifier [57]. In the absence of a probe pulse the horizon will amplify vacuum fluctuations but it will of course likewise convert and amplify any classical probe pulse that is sent onto it [39]. Parametric amplification is a well-studied phenomenon in wave physics, in particular in the context of nonlinear optics. Very efficient excitation and amplification of vacuum states is achieved for example using crystals with a second order, or so-called $\chi^{(2)}$ nonlinearity [58], i.e. crystals that exhibit a nonlinear response that scales with the square of the input electric field. This same mechanism is actually the most widely used and robust method for generating quantum correlated photon pairs that have then in turn been used to test quantum theories and develop quantum information transmission and manipulation. The photon distribution of the radiation excited by these optical methods is also thermal. However there is a fundamental difference with respect to Hawking radiation: the thermal emission obtained by standard nonlinear optical parametric processes is thermal with a *different* temperature characterising each mode (i.e. frequency in the monochromatic limit) [59,60]. Conversely, Hawking emission is composed of radiation that has exactly the same temperature T_H over the whole spectrum [57,59]. Parametric amplification at a horizon in the form of Hawking radiation is therefore a very specific and peculiar effect that is quite unlike usual nonlinear optical parametric amplification.

4. Numerical simulations of one-dimensional dielectric white holes

An interesting question raised by these findings and predictions is 'what does Hawking radiation correspond to in the framework of the Maxwell equations?'. As should be expected, analogue Hawking radiation does emerge from Maxwell's equations yet it is a new and unexpected effect that has not been predicted before.

We performed numerical simulations using the finite-difference time-domain technique applied to the discretised Maxwell equations [61]. The equations solved are

$$\frac{\partial E_y}{\partial x} = -\mu\frac{\partial H_z}{\partial t},$$

$$\frac{\partial H_z}{\partial x} = -\frac{\partial(\varepsilon E_y)}{\partial t}, \tag{17}$$

where $\varepsilon = \varepsilon\,(x - vt) = [n_0 + \delta n(x - vt)]^{1/2}$ is the medium permittivity. We underline that there are no nonlinearities involved in these equations: only linear propagation is simulated and the travelling refractive index perturbation is included in ε. In our simulations we took a super-gaussian form for the perturbation: $\delta n = \delta n_\mathrm{max}\exp[-((x\text{-}vt)/\sigma)^\mathrm{m}]$, where m is an even integer.

In these studies we do not directly verify the emission of radiation from the vacuum state. Emission from the horizon is stimulated by an incoming classical probe pulse and we study how this pulse evolves and thus gather information on the underlying Hawking radiation mechanism. As mentioned above, we will only focus attention on the white hole, i.e. on how the probe pulse interacts with the steep, trailing edge of the perturbation.

Figure 4 shows an example of such a simulation (an animation, WH-1.mp4, of the same simulation is also available for download with this paper). Dispersive effects are introduced through numerical dispersion that depends on the grid resolution and may thus be controlled [62]. Figure 5(a) shows the dispersion in the comoving frame relative to the simulations in Figure 4. The perturbation has $v = 0.99c/n_0$, maximum amplitude $\delta n_\mathrm{max} = 0.01$ and supergaussian order $m = 26$ so that δn raises from 0 to $\sim \delta n_\mathrm{max}$ over a distance $\sim 1\mu$m. The input probe pulse is taken with initial wavelength 4 μm and is placed behind the perturbation. This is equivalent to studying the evolution of a mode that attempts to enter a white hole. Figures 4(a)–(d) show the electric field profile at various propagation distances within a window centred on the perturbation: the input probe pulse catches up with the perturbation where it is blocked at the horizon and frequency shifted until it finally starts to lag behind (due to dispersion that decreases the pulse velocity with decreasing wavelength). Figures 4(e)–(h) show the spectra relative to each electric field profile: the input spectral peak (IN) is transformed into two distinct peaks (P and N). We underline once more that this is a purely linear simulation, i.e. the observed frequency conversion is not the result of an optical interaction involving e.g. $\chi^{(2)}$ or $\chi^{(8)}$ nonlinearities. Rather, this frequency conversion finds a simple explanation in terms of the generation of positive and negative Hawking modes: in Figure 5(*a*) we show the modes on the dispersion curve at the horizon. As can be seen, both the P and N modes lie on the curve and both conserve ω'_{IN}. Moreover, by repeating such a simulation for many different input frequencies and taking the ratio of the photon numbers in the two output modes, $|N|^2/|P|^2$, it is possible to verify that these satisfy relation (16), i.e. the emission is thermal with the same temperature over the whole spectrum of input frequencies [63].

Figure 6 shows the result of a simulation with the same input parameters as in Figure 4 with the only difference that the input wavelength is now 2 μm and the probe pulse is placed inside the perturbation which is now moving faster than the pulse (see the animation, WH-2.mp4 of this simulation). This is equivalent to studying the evolution of a mode that exits a white hole. As the pulse exits the perturbation, it is frequency converted, as before, to a P and N mode. The dispersion curves corresponding to this simulation are shown in Figure 5(*b*): note that because the input probe pulse has a phase velocity that is lower than the perturbation velocity, in the comoving frame it now has initial negative (comoving) frequency.

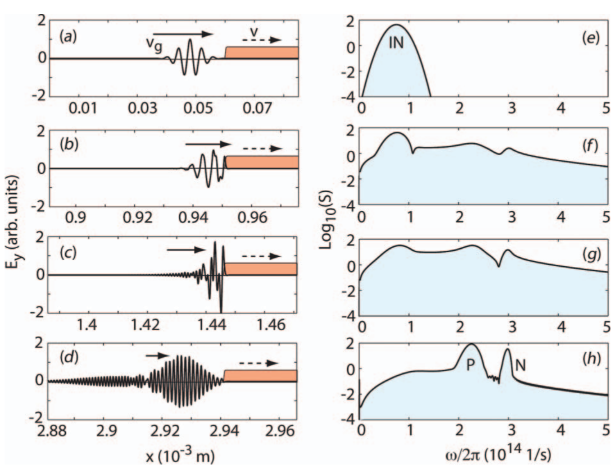

Figure 4. Numerical simulations of stimulated Hawking radiation. (*a*)–(*d*) Evolution of the electric field of an input few-cycle laser pulse with an initial wavelength of 4 μm. The shaded area indicates the refractive index perturbation. The arrows indicate the qualitative amplitude of the velocities (solid arrow for the probe pulse, dashed arrow for the perturbation). The spectra (S) relative to each of these graphs are shown in (*e*)–(*h*) in logarithmic scale: the input spectral peak is indicated with 'IN'. The ouput spectrum clearly exhibits two distinct blue shifted peaks, the positive and negative Hawking modes indicated with 'P' and 'N' in (*h*).

5. Creating an effective moving medium with a laser pulse

So far the discussion has referred to a generic refractive index perturbation without actually mentioning how this may be generated. The idea originally proposed by Leonhardt and co-workers [33] is based on the Kerr effect [58]: a sufficiently intense laser pulse propagating in any isotropic medium such as a gas, liquid or amorphous solid will excite a nonlinear polarisation response

$$P = \varepsilon_0(\chi^{(1)} + \chi^{(3)}E^2)E, \tag{18}$$

where $\chi^{(1)}$ is the linear susceptibility that is related to the linear refractive index $n_0 = (1 + \chi^{(1)})^{1/2}$, $\chi^{(3)}$, is the

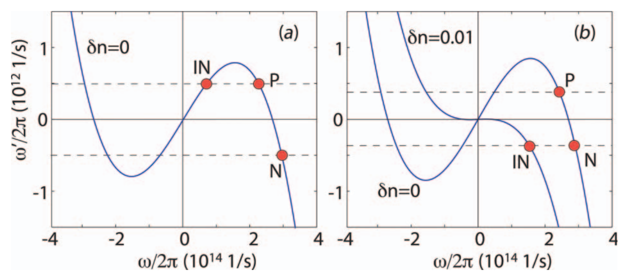

Figure 5. Dispersion curves for the numerical simulations in Figures 4(*a*) and 6(*b*). We plot the comoving frequency, $\omega'/2\pi$, as a function of the laboratory frame frequency, $\omega/2\pi$. The full-circles indicate the position of the IN, P and N mode frequencies.

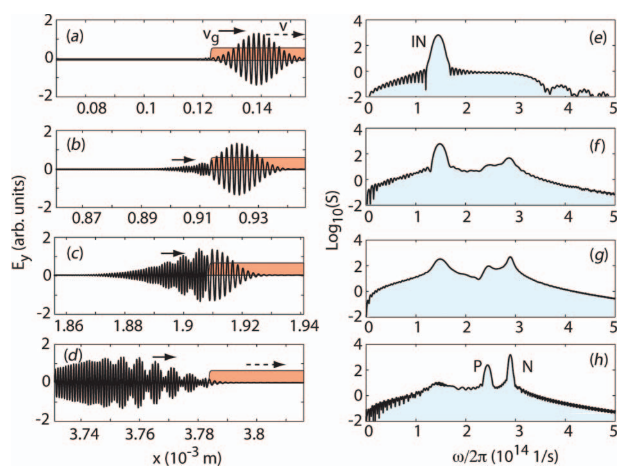

Figure 6. Numerical simulations of stimulated Hawking radiation. (*a*)–(*d*) Evolution of the electric field of an input few-cycle laser pulse with an initial wavelength of 2 μm. The input pulse now starts from inside the perturbation with *lower* initial group velocity $v_g < v$ and thus exits the perturbation, passing through the white hole horizon. The spectra (S) relative to each of these graphs are shown in (*e*)–(*h*) in logarithmic scale.

third-order nonlinear susceptibility (also know as the Kerr nonlinearity) and $E = |E\cos(\omega t)$ is the electric field of the intense laser pulse that excites the medium. The third-order polarization term may thus be re-written as $P_{NL} = 1/4\chi^{(3)}|E|^3 \cos(3\omega t) + 3/4\chi^{(3)}|E|^3\cos(\omega t)$. The first term oscillates at 3ω, i.e. it acts as a source for third harmonic frequency generation and may be neglected (unless the beams or the medium are specifically engineered so as to enhance this process). The second term oscillates at the input frequency ω and from Equation (18) we define an effective refractive index $n = (1 + \chi^{(1)} + 3/4\chi^{(3)}E^2)^{1/2} \simeq n_0 + n_2 I$, where the nonlinear index is $n_2 = (3/8\chi^{(3)}n_0$ and $I = |E|^2$. The intensity profile of a laser pulse usually has a gaussian-like form, i.e. $I = I(x\text{-}vt) = \exp[-((x\text{-}vt)/\sigma)^2]$, where the pulse speed will be given by the group velocity of light in the medium, $v = v_g = d\omega/dk$. In other words, an intense laser pulse propagating in a nonlinear Kerr medium will create a refractive index perturbation $\delta n = n_2 I$ that travels close to the speed of light. We note that the reasoning above may be generalised without loss of generality to the case in which the Kerr medium is excited by an intense laser pulse and the resulting perturbation acts upon a second, weak probe pulse [33, 58,63].

There are various methods by which the Kerr medium may be excited to induce a refractive index perturbation. However, successful measurements of Hawking radiation do impose some additional constraints:

(1) The intensity profile should be stationary during propagation in order to recreate stationary excitation conditions. This is not a trivial requirement due to the fact that the same Kerr effects that generate the perturbation also lead to back-reaction on the pump laser pulse and detrimental effects such as pulse splitting, self-focusing and white light generation.
(2) The existence and frequency range of an horizon is completely determined by the perturbation velocity as seen in Equation (9). So ideally we would like to control the speed v.

Bearing this in mind, a few experimental setups have been proposed and are summarised in Figure 7. Leonhardt's original proposal was based on the use of optical solitons propagating in highly nonlinear photonic crystal fibres. These are fibres with micro-structured cores that on the one hand allow one to tightly confine light within the core region, so as to increase the pulse intensity and amplitude of the perturbation, and on the other allow one to engineer the dispersion relation and thus the group velocity of the stationary soliton. The $\delta n_{\max}$ generated by these solitons is usually of the order of 10^{-4}. Experiments using fibre solitons therefore satisfy our list of requirements and indeed the first evidence of horizon-related frequency conversion was experimentally observed using such fibres in 2008 [33].

Another option is to attempt to harness the nonlinear propagation of laser pulses in bulk media. One possibility that has been proposed [44,63,64] is to use so-called filaments. The term filament, or light filament, denotes the formation of a dynamical structure with an intense core that is able to propagate over extended distances much larger than the typical diffraction length while keeping a narrow beam size without the help of any external guiding mechanism [65]. These sub-diffractive 'light-bullets' may be either generated spontaneously of by pre-shaping the laser beam. Spontaneous filaments are born as a consequence of a spontaneous spatio-temporal reshaping of an input Gaussian-shaped laser pulse when it is loosely focused into a nonlinear Kerr medium. If the input power is larger than a certain critical threshold power, the pulse will self-focus as a result of the spatially varying refractive index perturbation that the pulse generates and that acts in a similar fashion to a focusing lens. At the same time the spectrum is broadened and both the transverse and longitudinal profiles of the pulse are simultaneously reshaped until a quasi-stationary (or 'dynamical') state is formed.

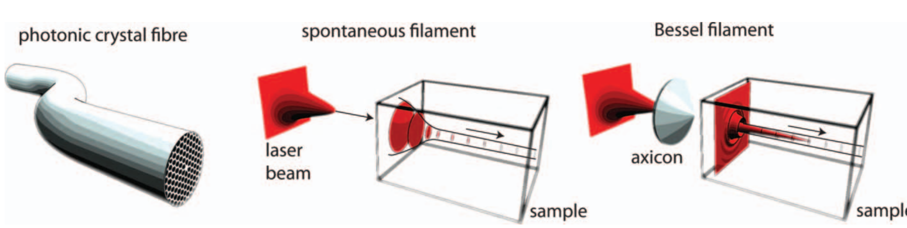

Figure 7. Sketch of three different methods employed to generate intense laser pulses with quasi-stationary propagation over long distances. Photonic crystal fibres have micro-structured cores that allow one to both tightly confine light and control the material dispersion. Spontaneous filaments are obtained by loosely focusing an intense laser pulse in a bulk Kerr medium, e.g. fused silica glass.

Spontaneous filaments exhibit a number of features that are attractive for the generation of optical horizons: (i) they are extremely simple to obtain and are characterised by a very intense peak, e.g. $I \sim 10^{12}$–10^{8} W cm^{-2} which propagates over distances of the order of 2–10 diffraction lengths. In fused silica $n_2 \sim 3 \times 10^{16}$ cm^2 W^{-1} so we may have $\delta n_{\text{max}} \sim 10^{-3}$. (ii) Along the longitudinal coordinate the pulse may split in two and the trailing pulse will be drastically shortened and exhibit an extremely sharp shock front on the trailing edge. Moreover, the trailing peak propagates with a velocity that is significantly slower than the input pulse group velocity. By controlling the input conditions, e.g. wavelength, focusing, power, it is possible to control to a certain extent both the steepening effects and the peak velocity. Figure 8(*a*) shows an example of the numerically simulated evolution of the longitudinal profile of a 1.055 μm, 100 fs long laser pulse that undergoes filamentation in fused silica. The pulse splits into two daughter pulses: the rear pulse has significantly higher intensity and exhibits a self-steepened shock front on its trailing edge with a nearly single optical duration. Figure 8(*b*) shows the velocity of this peak over a longer propagation distance. As can be seen v varies and gradually accelerates. This feature has been used in experiments: by selecting the emission at different x it is possible to study the behaviour at different v [63,64].

A further possibility that has been proposed is the use of Bessel filaments. In this case the transverse profile of the input laser beam is re-shaped into a Bessel-like pattern using a conical lens (also called an axicon). The axicon transforms the laser beam by redirecting light along a cone at an azimuthally symmetric angle θ towards the optical axis. Light propagating towards the axis will therefore interfere and the resulting interference pattern will be a non-diffracting Bessel pattern. Moreover, simple geometric considerations show that the central Bessel peak propagates along the x-axis with velocity $v = v_g/\cos\theta$. Therefore, by simply changing the angle of the axicon, it is possible to control the propagation velocity of the Bessel pulse. We note that the spontaneous filament leads to a trailing intense peak that is slowed down with respect to the input pulse group velocity, v_g whilst the Bessel pulse travels faster.

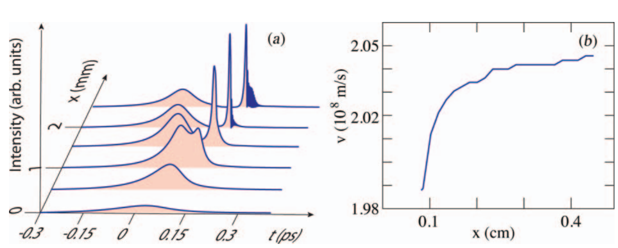

Figure 8. (*a*) Example of the numerically simulated longitudinal profile of a spontaneous filament at various propagation distances. (*b*) Evolution of the intense trailing peak velocity.

6. Experiments

Laser-pulse induced analogues are, to date, the only analogue systems in which experiments are currently being carried out to investigate and probe the quantum properties of analogue Hawking emission. These experiments were initiated in 2008 by the work of Philbin et al. [33]: a soliton was created inside a photonic crystal fibre and blue-shifting of part of the soliton radiation was observed as a consequence of the interaction with the self-generated white hole horizon. This was a purely classical effect but clearly demonstrated the possibility to generate horizons in dielectric media using intense laser pulses.

This idea was later extended to filament induced perturbations. The spectral transformations of a filament pulse are significantly richer than in a fibre due to the additional transverse degree of freedom. Very specific hyperbolic-shaped spectra are observed in angle-wavelength coordinates. These features were shown to be reproduced and predicted very precisely within the framework of a model based on the metric (8), thus confirming that by using the basic mathematical tools of general relativity it is possible to capture relevant details of optical pulse propagation in the presence of a travelling perturbation [44].

These first experiments were then adapted so as to search for signatures of Hawking radiation. The experimental layout is depicted in Figure 9: the input laser pulse is sent on to a 2 cm long sample of pure fused silica. Filaments were formed by either loosely focusing the input pulse with a 20 cm focal length lens (spontaneous filaments) or by replacing the lens with an axicon so as to generate a $\theta \sim 7^{\circ}$ Bessel filament. Light emitted in the forward direction is extremely intense with an average photon number of the order 10^{15} photons/pulse. Considering that Hawking radiation is unlikely to provide more that 1 photon/pulse, a huge and extremely challenging suppression of the laser pump pulse radiation is required. The photons emitted from the sample were therefore collected at 90° rather than in the forward direction: by using horizontally polarised pump pulses, Rayleigh scattering is effectively suppressed so that scattered photons at wavelengths in the predicted Hawking emission range are below 0.001 photons/pulse. By employing both spontaneous and Bessel filaments in separate measurements it was possible to measure radiation emitted from perturbation with various velocities. The results, adapted from [63,64], are summarised in Figure

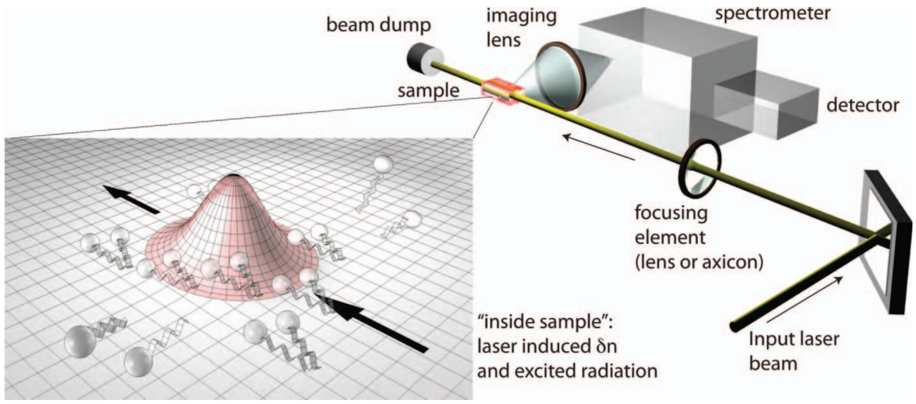

Figure 9. Sketch of the experimental layout. The input laser beam is focused into the Kerr sample (fused silica glass) using either a 20 cm focal length lens (to induce spontaneous filamentation) or an axicon with a 20° cone base angle (to create a Bessel filament). The inset shows a schematic representation of the Kerr sample: the perturbation propagates following the arrows and photon pairs, corresponding to the negative and positive Hawking pairs, are excited.

10(*a*): the solid curves represent the predicted Hawking emission wavelengths based on Equation (9), for increasing δ_n. The solid bars summarise the results from four measurements: the vertical extension indicates the measured spectral bandwidth (at half maximum) and the horizontal bars indicate the velocity variation of the perturbation in the case of spontaneous filaments (indicated with S). The measurements follow the expected dependence and indicate a tight correlation between the wavelength or colour of the emitted radiation and the perturbation velocity thus supporting the claim that this radiation is indeed emitted from the analogue horizon. The measurement indicated with B refers to the Bessel filament and is shown in more detail in Figure 10(*b*). In this case there is no spread of the perturbation velocity and it was possible to probe the emission for increasing peak intensities, i.e. increasing δ_n. The bandwidth of the emitted radiation increases with intensity and increases predominantly to longer wavelengths. This is precisely in agreement, also at a quantitative level [63,64], with Equation (9).

On the basis of these results, these measurements have been proposed as the first experimental evidence of Hawking-like emission from an analogue horizon [63,64]. However, this is not generally considered to be conclusive evidence and further measurements are called for in order to verify some open issues: the measured photon numbers, of the order of 0.1–0.01 photons/pulse, appear to be too high to be accounted for on the basis of a blackbody emission. Yet other models [20] appear to give predictions that confirm the measurements. Moreover, the theory predicts that photons will be generated in correlated pairs. The photons collected at 90° are likely to have suffered strong scattering after emission in the forward direction and it will therefore be very unlikely to observe any kind of pair-correlation with such a setup. Other experimental layouts, e.g. based on fibres which will confine the photon pairs along the same direction, are therefore required.

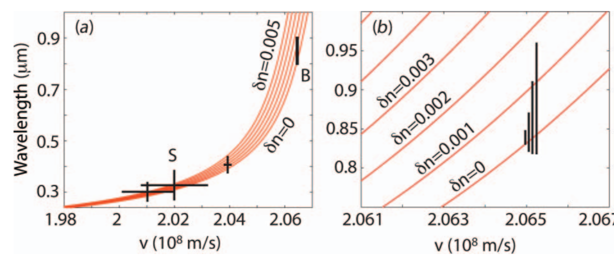

Figure 10. (*a*) Measured radiation wavelength for various perturbation velocities. The red curves show the predicted velocity dependence of the radiation wavelengths. The Bessel measurement is expanded in (*b*): the four different vertical bars indicated four different spectrum bandwidths obtained by increasing the input laser pulse intensity. The bars are slightly displaced horizontally for clarity but they actually all have the same $v = 2.065 \times 10^8$ cm s^{-1}.

7. Conclusions and perspectives

Laser pulses have been demonstrated in a variety of settings to generate analogue white holes and horizons that transform light according to the predictions of models that are derived within the context of general relativity. There is an intrinsic beauty in space–time geometries, that same beauty that pushed Einstein to develop, and others to accept, the theory of general relativity [66]. The extension to the study of the 'geometry of light' [67], is no exception in this sense. It is possibly not clear to date how far-reaching this extension will be and if it will allow one to gain further insight into quantum gravity theories or black hole physics. However, analogies between black hole kinematics and flowing media are certainly extending the limits of our understanding across various disciplines, e.g. waves in fluids, acoustics and optics. And thanks to the common underlying tools derived from general relativity, discoveries developed in one field apply also to the others and a deeper insight is achieved by directly comparing similar or different behaviours across the various disciplines.

The optical analogue is still in its infancy and we expect a strong development in the next few years. A number of recent proposals are based either directly on the presence of an horizon or on the same technology required to build an horizon. For example Demircan et al. have studied an optical transistor that acts through an optical event horizon [68], McCall et al. have proposed a temporal cloaking device that uses pulses that split and then recombine thus cancelling out portions of history [69] and Ginis et al. have proposed a frequency converter based on a metamaterial analogue of cosmological expansion [70]. Stimulated Hawking mode conversion as seen in the numerical simulations presented here and in the literature [63] represents a novel kind of optical amplifier and is awaiting experimental demonstration. Such an amplification mechanism could then in turn lead to the first 'black hole laser' whereby a wave is trapped between two horizons that form a cavity: at each reflection from the white hole horizon light is amplified through a stimulated Hawking process with a resulting laser-like behaviour [71].

These are just a few ideas and possibilities. There are certainly many more that await to be investigated and brought to light through the blending of general relativity with flowing media.

Notes on contributor

The author obtained a degree in Physics at the University of Milano, Italy (1998) and an Mphil in Physics at the Optoelectronics Research Centre, Southampton, UK (2000). He then carried out research for a number of years in the telecommunications industry and obtained a Ph.D. from the Université de Nice-Sopha Antipolis, France (2007). After a number of years at the University of Insubria, Italy, he is currently Reader in Physics at Heriot-Watt University, Edinburgh, UK. His research has been mostly devoted to laser physics and nonlinear optics and more recently to the application of laser pulses for studying light in moving media.

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