EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Neutron Stars:

Rotational & Magnetic Field Instabilities

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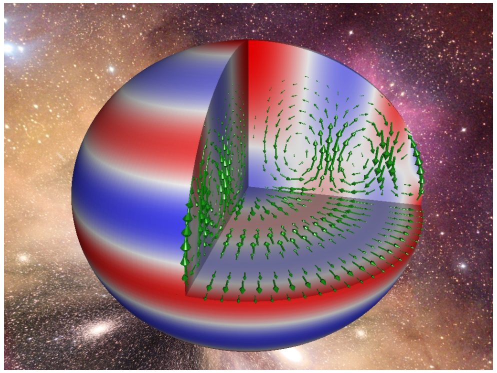
List of Issues

- Nils (physics)
  - Modes : Instabilities , r-modes
  - Glitches
  - Cooling
  - Superfluidity

- Kostas (dynamics)
  - Instabilities f-modes
  - Magnetized NS (magnetars)
    - QPOs
    - Non-linear evolutions of magnetic fields
    - Emission of GWs

Neutron Star "ringing"



p-modes: main restoring force is the **pressure (f-mode)** ***(>1.5 kHz)***

$$\sigma \approx \sqrt{\frac{M}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the **Coriolis force**

$$\sigma \approx \Omega$$

w-modes: pure **space-time modes** (only in GR) ***(>5kHz)***

$$\sigma \approx \frac{1}{R}\left(\frac{M}{R}\right)$$

Torsional modes (t-modes) ***(>20 Hz)*** shear deformations. **Restoring force, the weak Coulomb force of the crystal ions.**

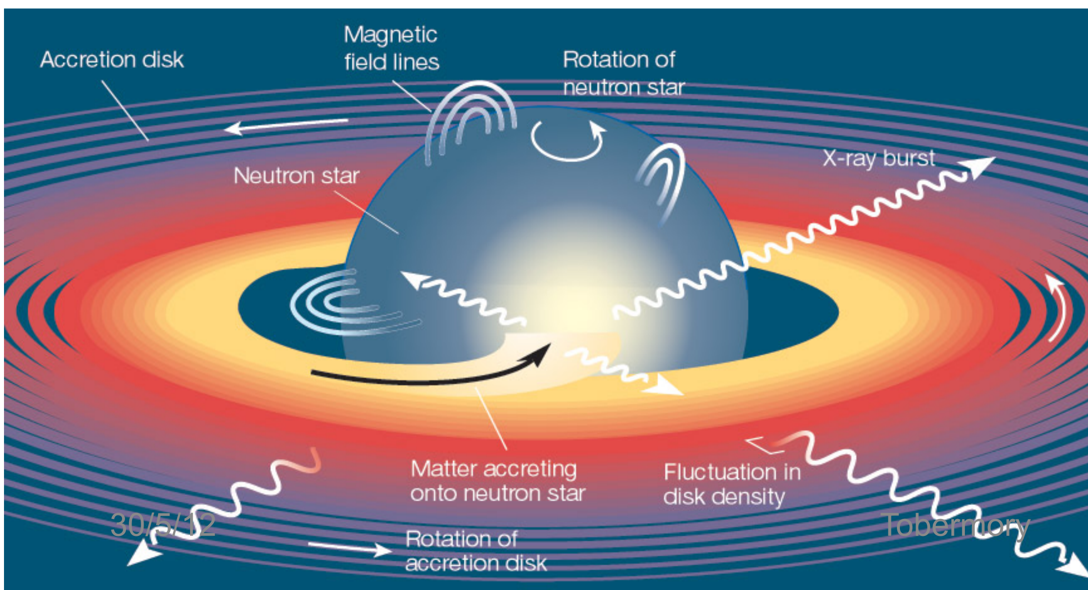
$$\sigma \approx \sqrt{\frac{v_S}{R}}$$

… and many more

Effect of Rotation & Magnetic Fields

ROTATION

- **Frame dragging**
- **Quadrupole deformation**
- **Rotational instabilities**



MAGNETIC FIELD

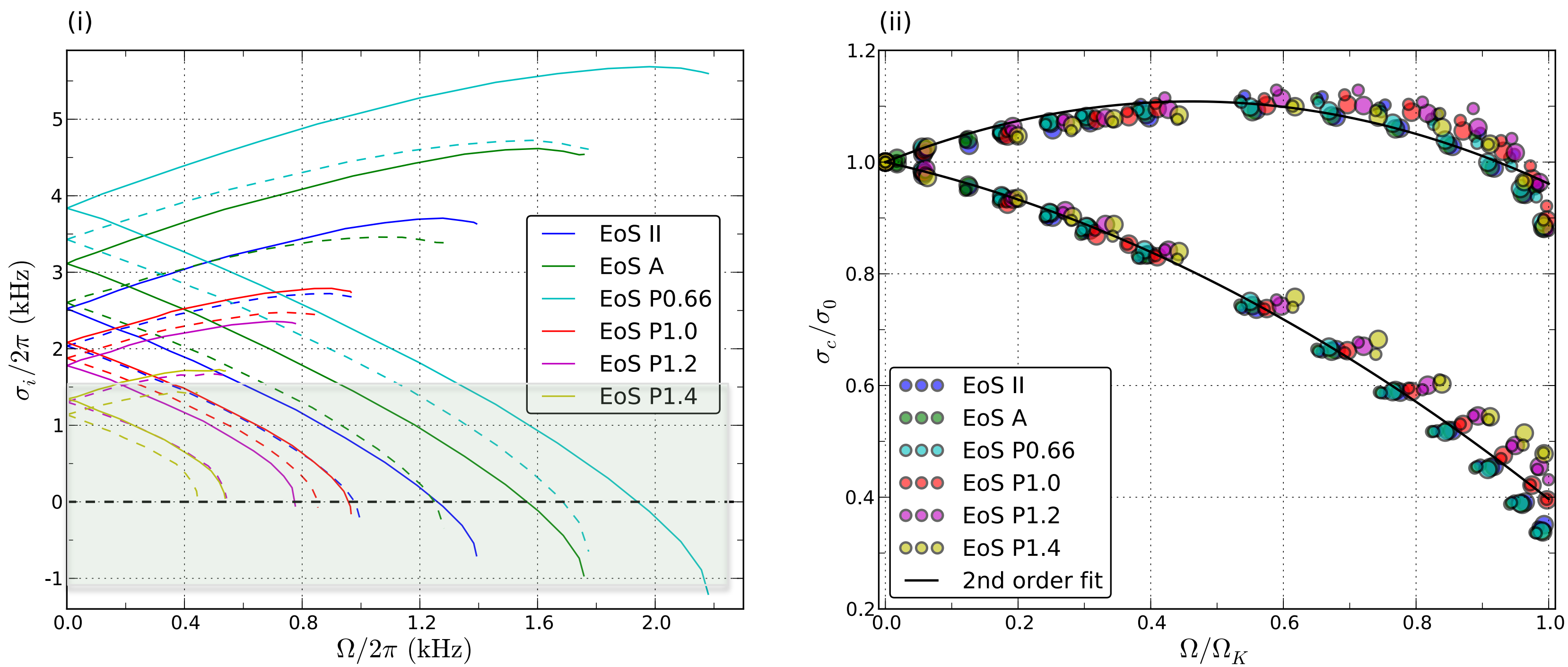
- **No significant effect** in the fluid frequencies and damping/growth times

$$\frac{\text{magnetic energy}}{\text{gravitational energy}} \sim \frac{B^2 R^3}{GM^2/R} \sim 10^{-4}\left(\frac{B}{10^{16} G}\right)^2$$

- For **magnetars** we may observe **Alfvén oscillations**

f-modes: Asteroseismology I

We can produce empirical relation relating the parameters of the neutron stars to the observed frequencies.



Frequency

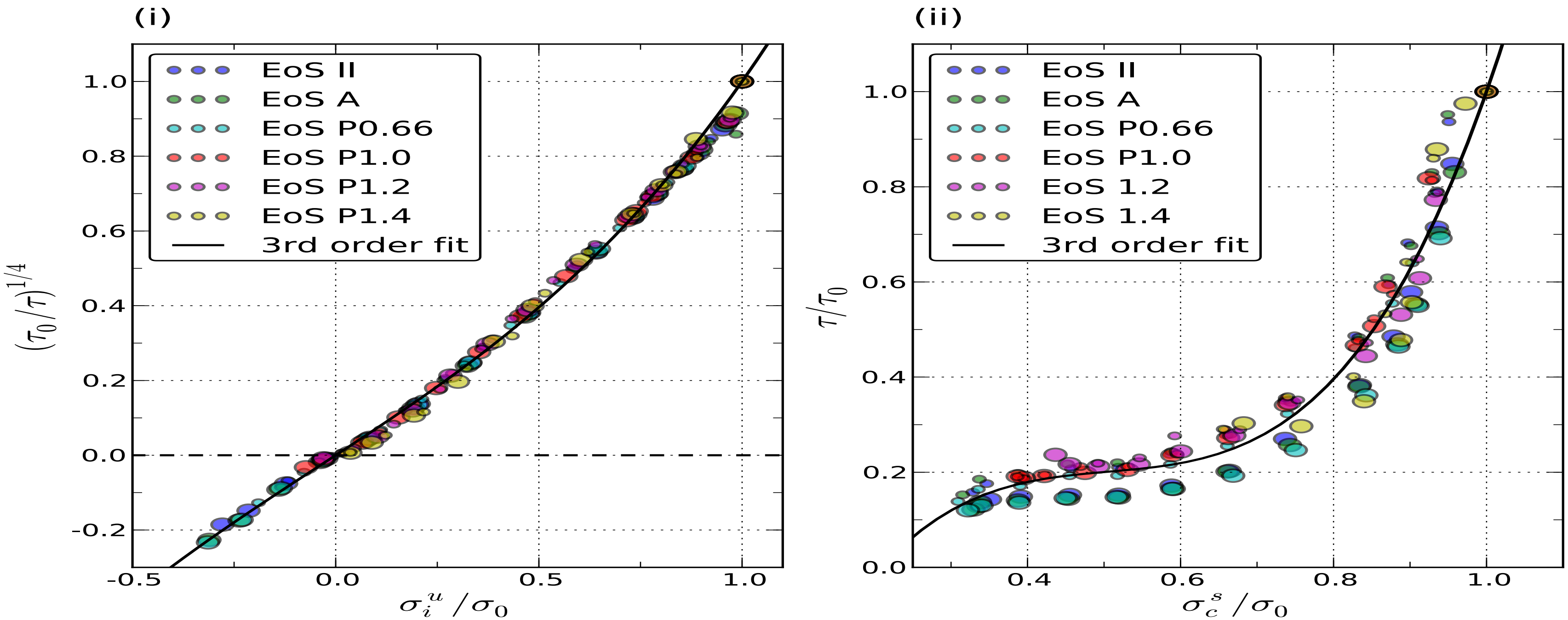
$$\frac{\sigma}{\sigma_0} \approx 1 + 0.63\left(\frac{\Omega}{\Omega_K}\right) - 0.32\left(\frac{\Omega}{\Omega_K}\right)^2 + \dots \quad (m=2)$$

$$\frac{\sigma}{\sigma_0} \approx 1 - 0.41\left(\frac{\Omega}{\Omega_K}\right) - 0.53\left(\frac{\Omega}{\Omega_K}\right)^2 + \dots \quad (m=-2)$$

Gaertig-Kokkotas 2008, 2010

f-modes: Asteroseismology II

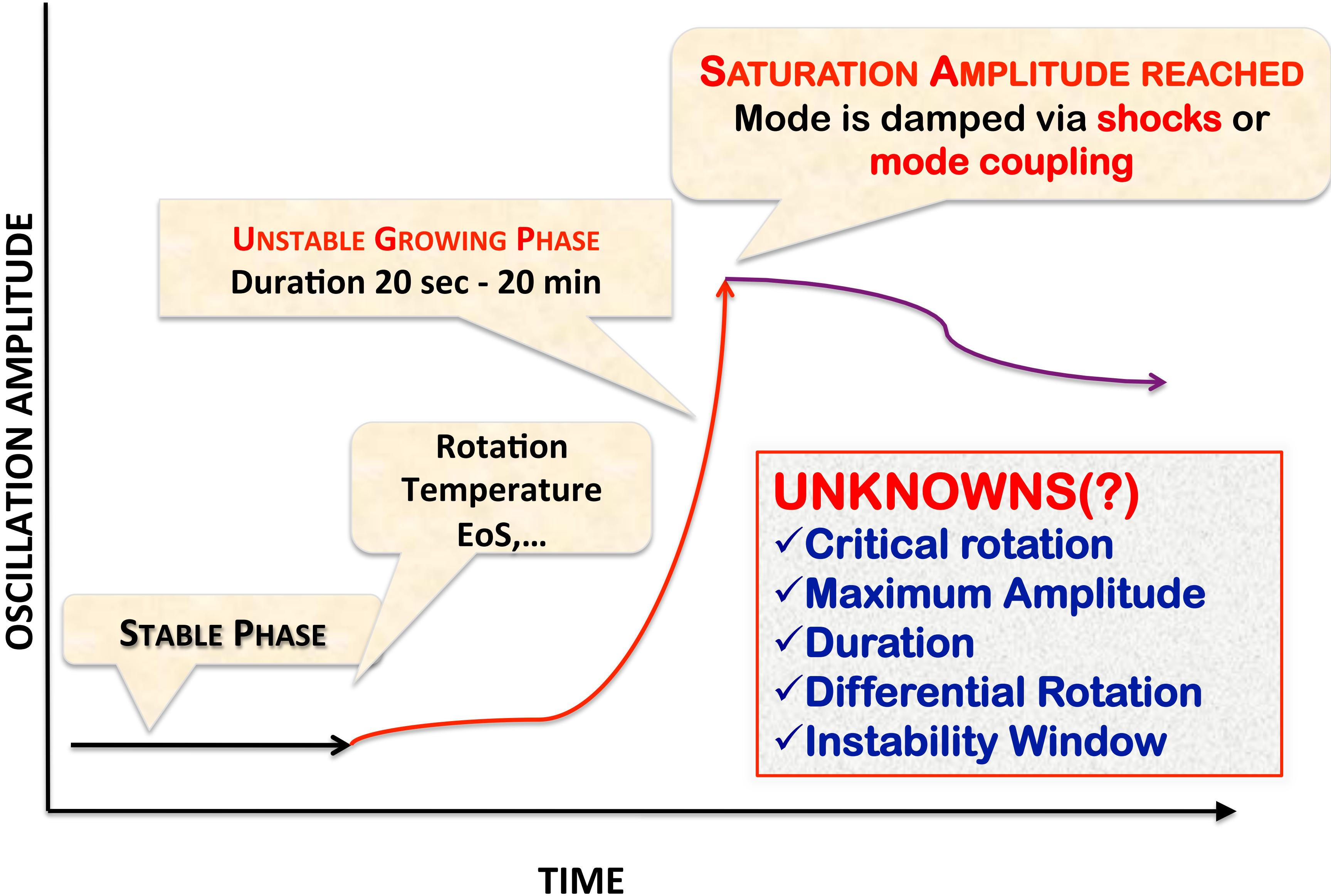
Damping or Growth Time



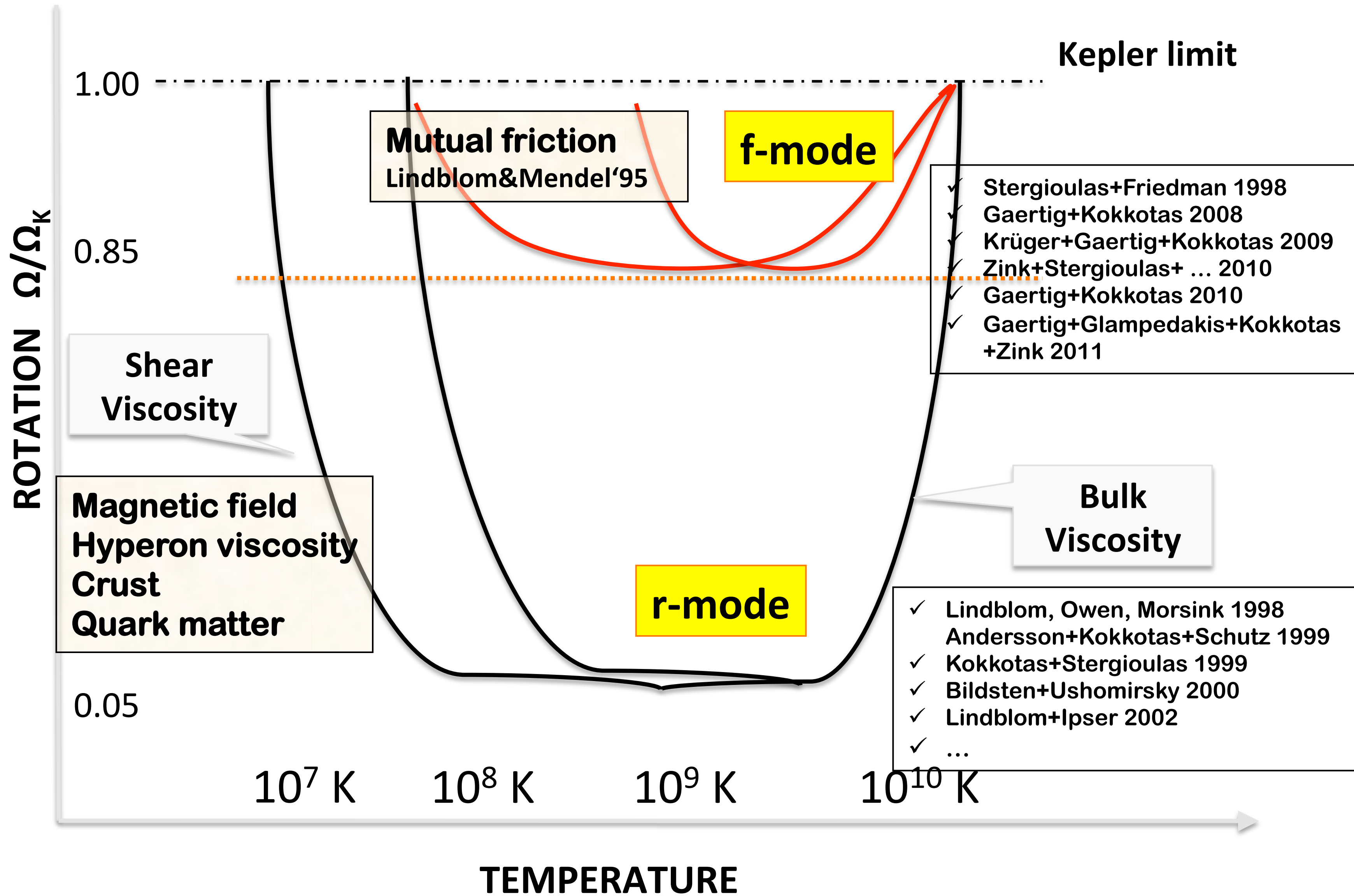
$$\left[\frac{\tau_0}{\tau}\right] \approx -0.66\left[1 - 7.33\left(\frac{\sigma_c}{\sigma_0}\right) + 15.06\left(\frac{\sigma_c}{\sigma_0}\right)^2 - 9.26\left(\frac{\sigma_c}{\sigma_0}\right)^3\right]$$

$$\left[\frac{\tau_0}{\tau}\right]^{1/4} \approx \mathrm{sgn}(\sigma_i)0.71\left(\frac{\sigma_i}{\sigma_0}\right)\left[1 + 0.048\left(\frac{\sigma_i}{\sigma_0}\right) + 0.35\left(\frac{\sigma_i}{\sigma_0}\right)^2\right]$$

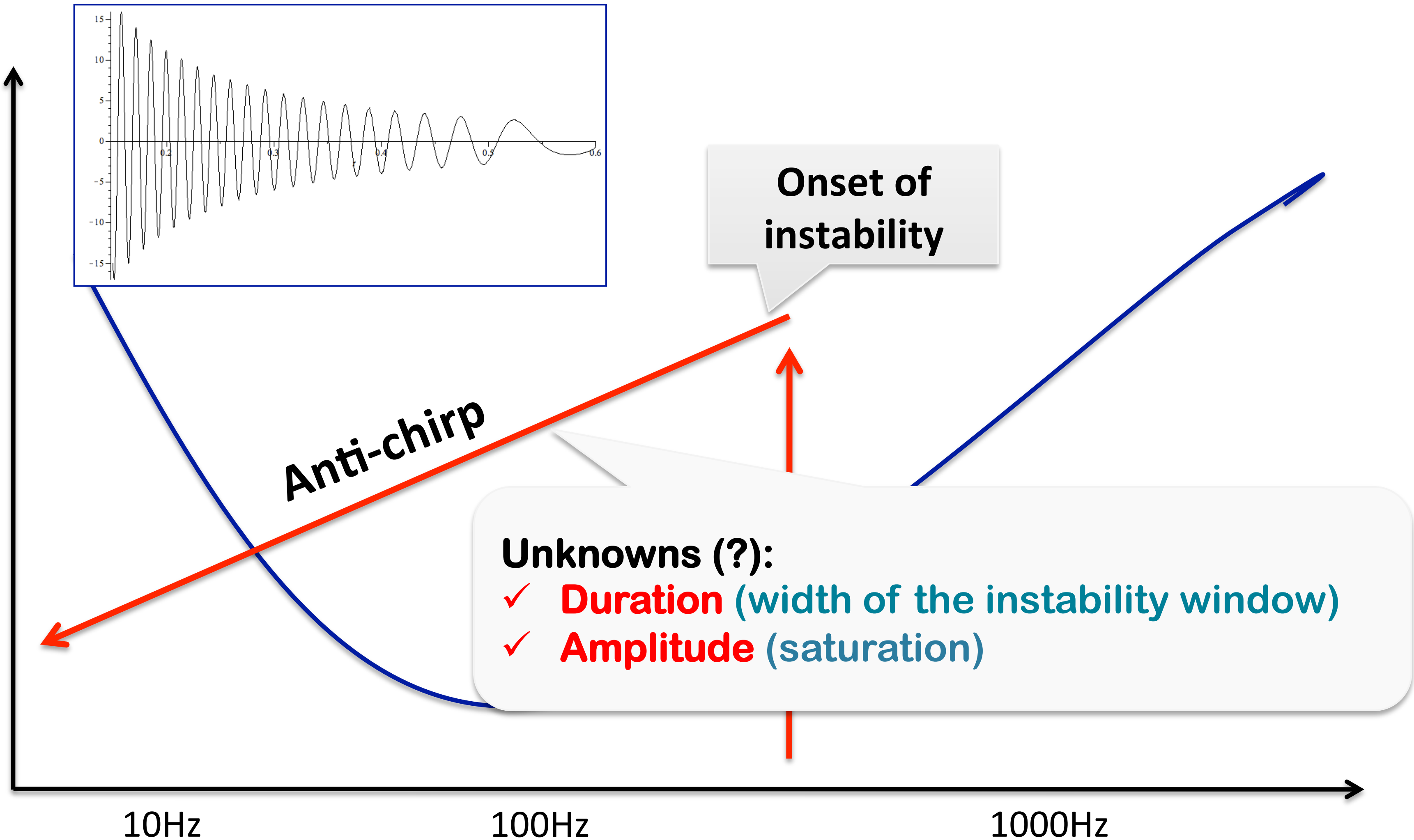
The Excitation of Secular Instabilities



INSTABILITY WINDOW



f-mode Instability



f-mode: Instability window

$$E = \frac{1}{2}\int\left[\rho\delta u^a \delta u_a^* + \frac{\delta p}{\rho}\delta\rho^*\right]d^3x \quad \Rightarrow \quad E \approx \sigma^2$$

$$\frac{dE}{dt} = -\sigma_i\left(\sigma_i + m\Omega\right)N_\ell\left|\delta D_{\ell m}\right|\sigma_i^4 \quad \Rightarrow \quad \frac{dE}{dt} \approx \sigma_i^6$$

$$\frac{1}{\tau_{GR}} = -\frac{1}{2E}\left(\frac{dE}{dt}\right) \quad \approx \quad \sigma_i^3\left(\sigma_i + m\Omega\right)$$

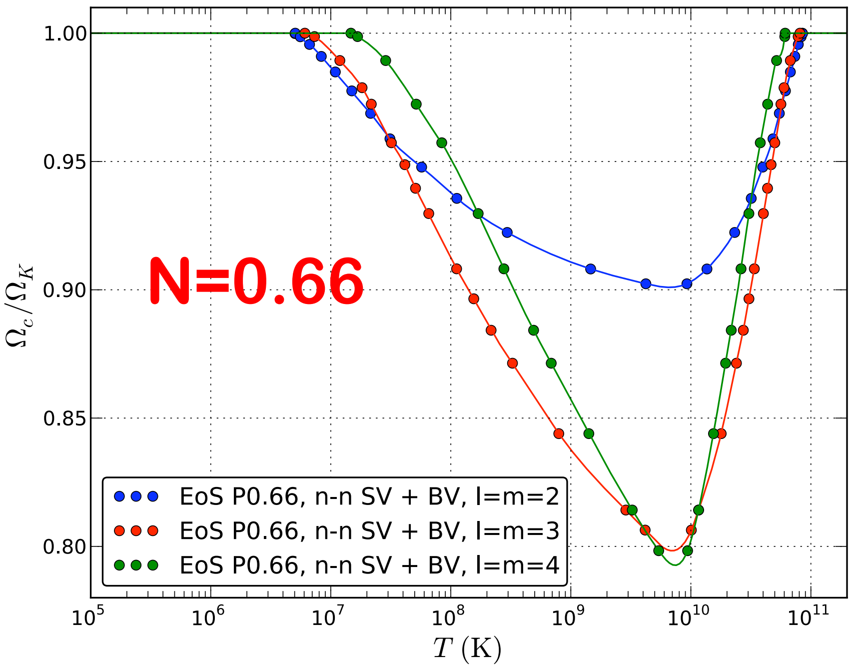
$$\frac{1}{\tau_{BV}} = \frac{1}{2E}\int \zeta\,\delta\sigma\,\delta\sigma^* d^3x$$

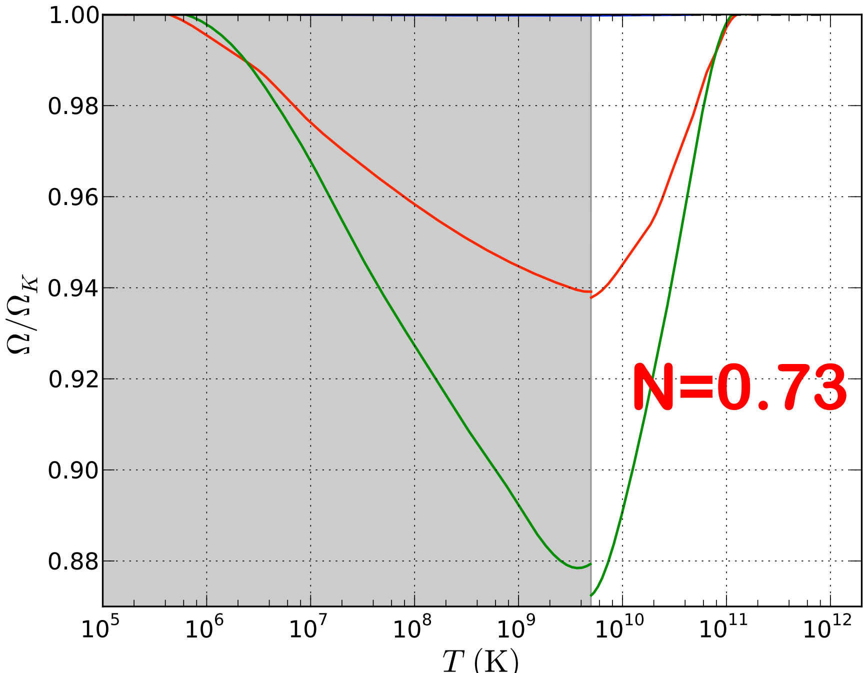
$$\frac{1}{\tau_{SV}} = \frac{1}{2E}\int \eta\,\delta\sigma^{ab}\,\delta\sigma^*_{ab} d^3x$$

$$\frac{1}{\tau_{GR}} = \frac{1}{\tau_{SV}} + \frac{1}{\tau_{BV}}$$

Ipser-Lindblom 1991

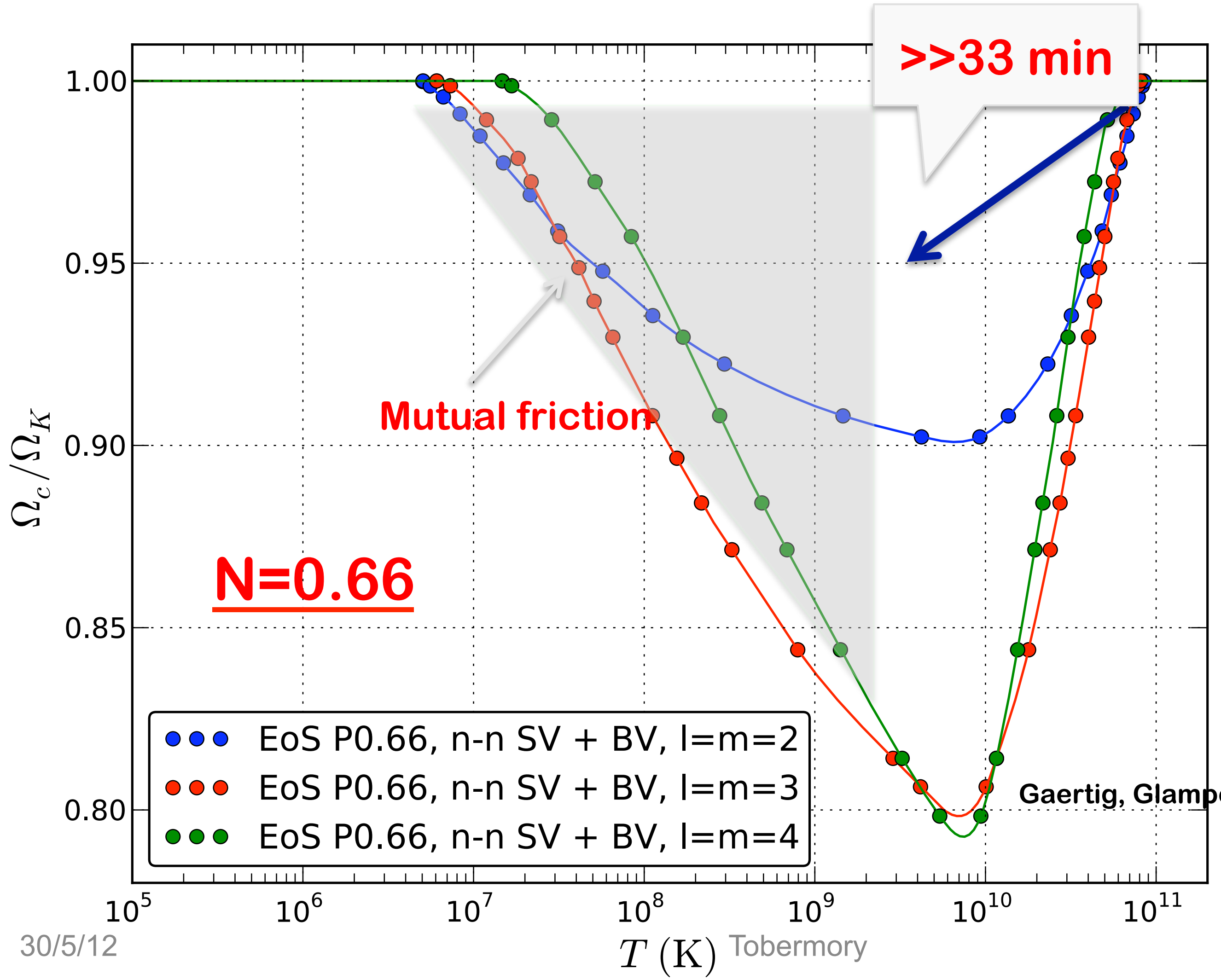
Gaertig-Glampedakis-Kokkotas-Zink 2011





Instability Window

- ✓ For the **first time** we have the window of f-mode instability in **GR**
- ✓ **Newtonian: (l=m=4)** Ipser-Lindblom (1991)



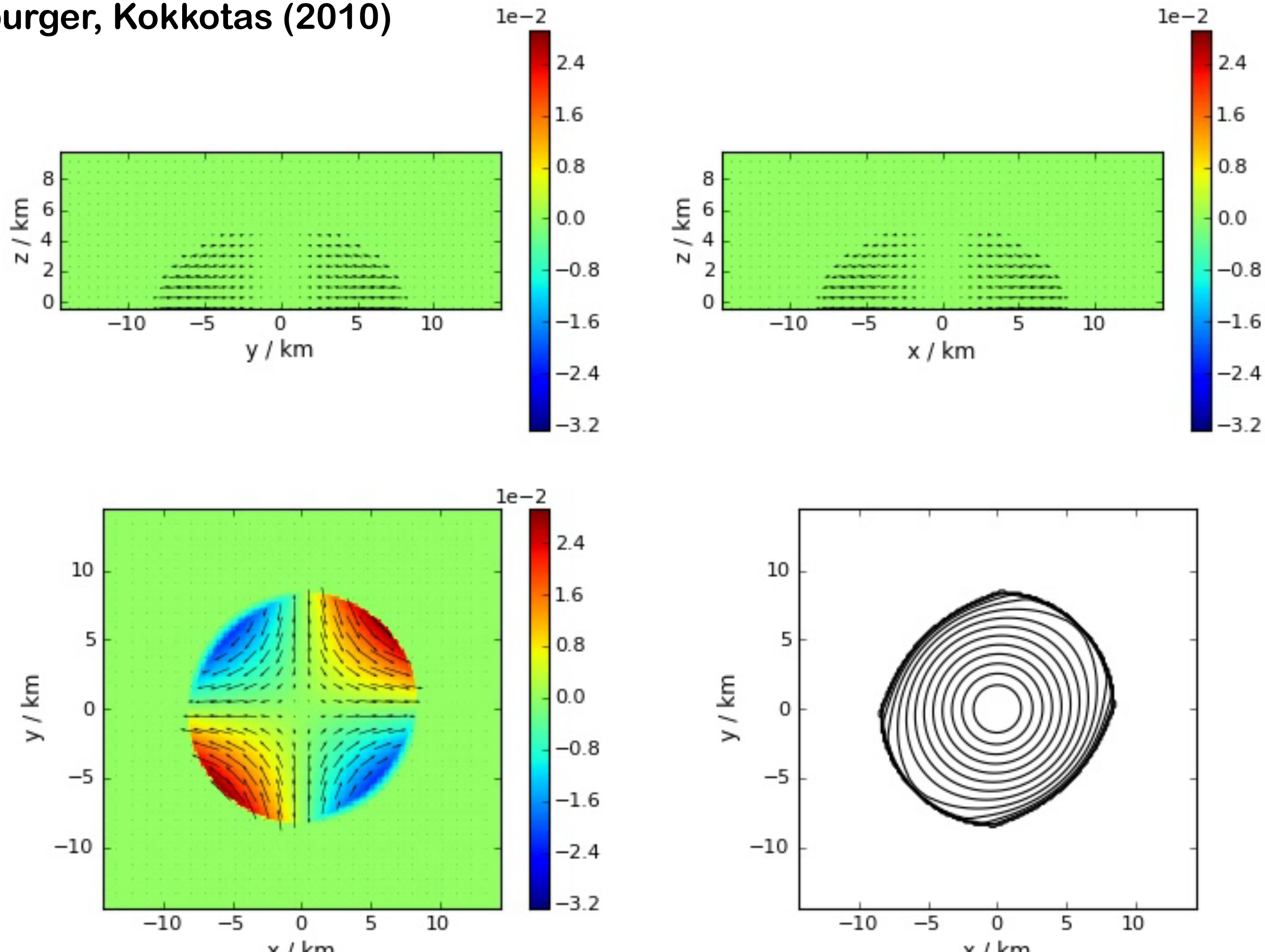
...typical Growth Time

~38 sec	l=2
~56 sec	l=3
~264 sec	l=4

Gaertig, Glampedakis, Kokkotas, Zink (2011)

Animation of the l=m=2 f-mode

Kastaun, Willburger, Kokkotas (2010)



- ✓ **Quasi-Radial &Axisymmetric: damped due to shock formation**
- ✓ **Non-axisymmetric: damped due to wave breaking on the surface**

Detectability of an unstable f-mode at 20Mpc

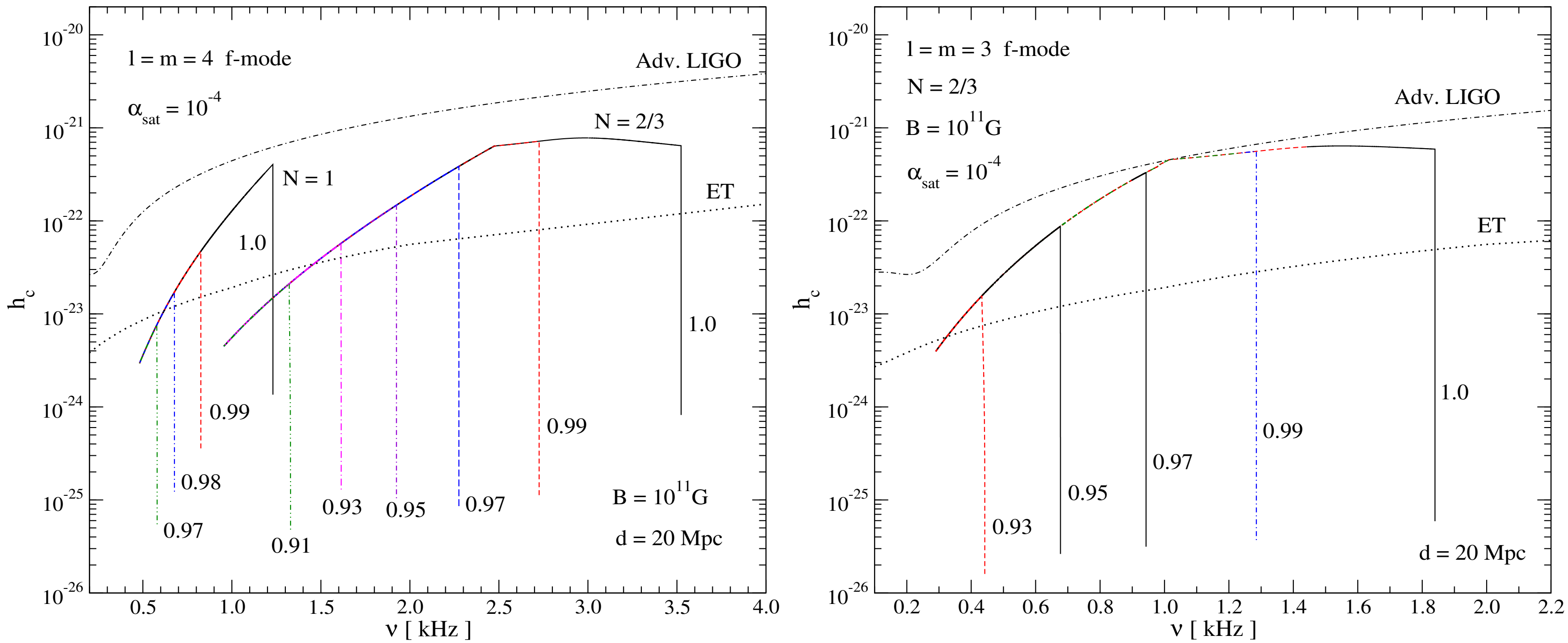
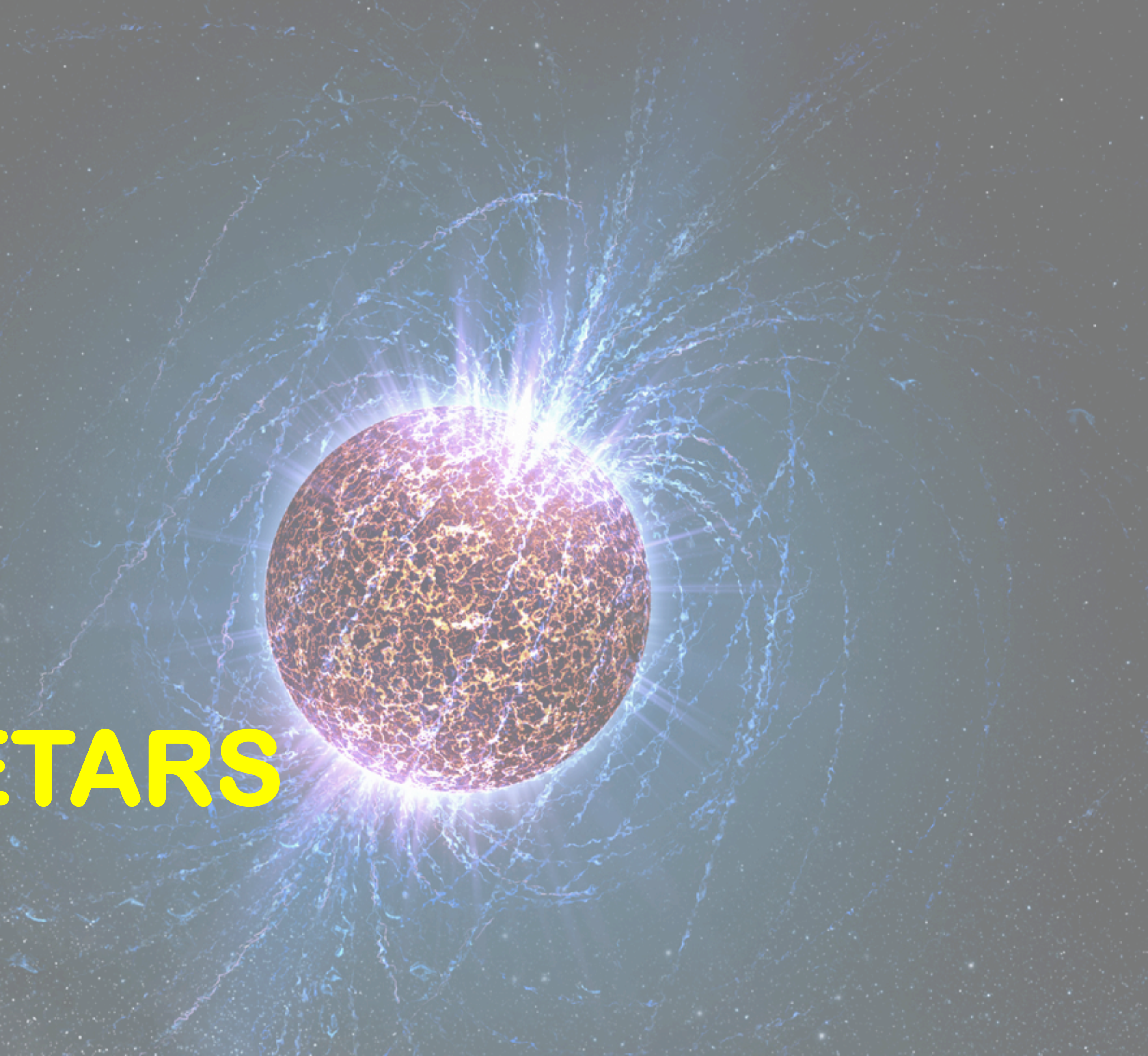


FIG. 1: N=1 polytrope with $B = 10^{11}$G.

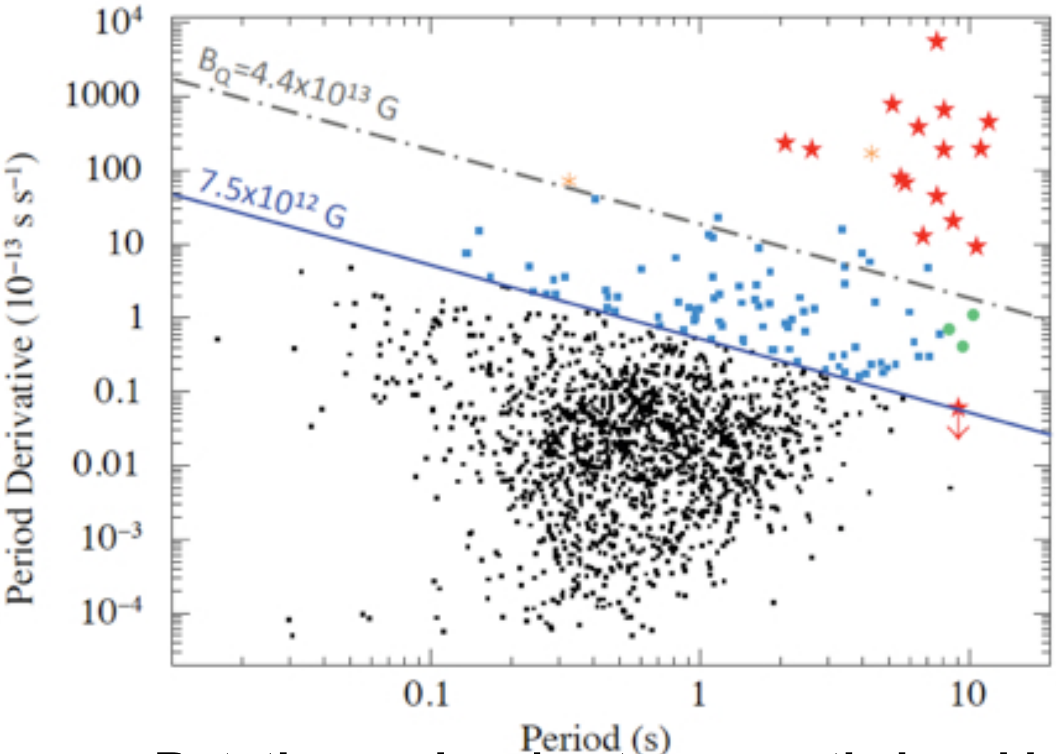
Here we assume that the saturation amplitude is **$\alpha=10^{-4}$** → **$10^{-6}\, M_{\odot}\, c^2$**

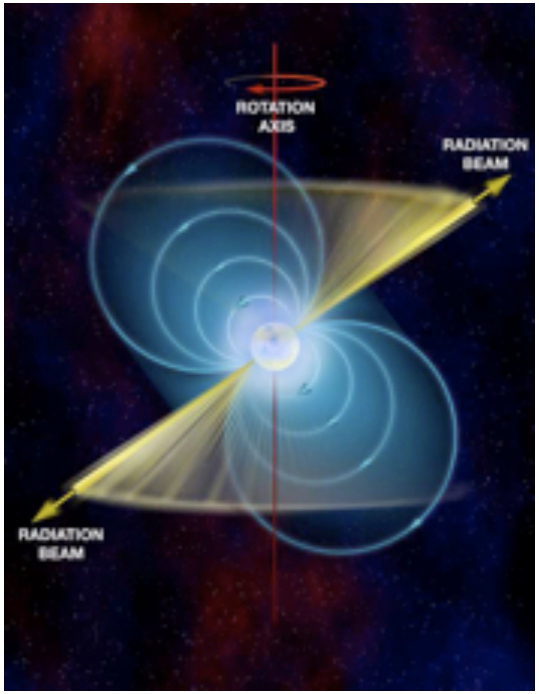
If the initial magnetic field is **> 10^{12}** Gauss the instability can be supresed significantly

MAGNETARS



Strong Magnetic Fields in Neutron Stars





- **Rotation varies due to magnetic breaking**

$$B_d \sim 3.2 \times 10^{19} \left(P\dot{P}\right)^{1/2} G$$

- **Only infer exterior dipole component**

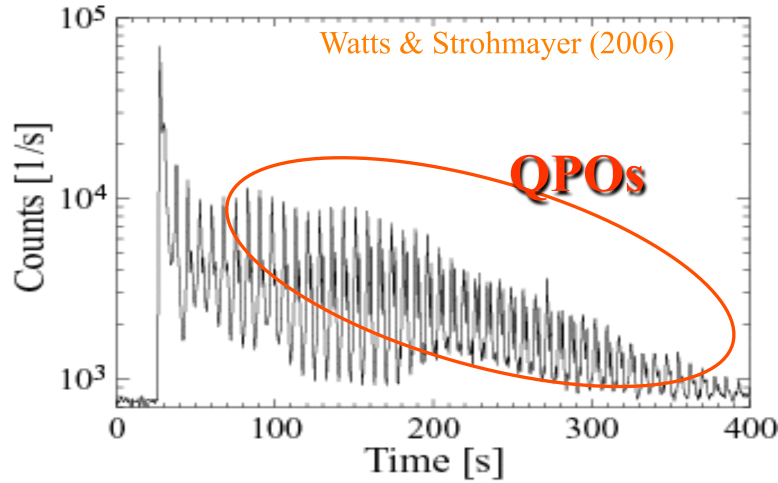
- **Magnetars**

$$B_d \geq 10^{14} - 10^{15} G$$

Magnetars

- **Young, slowly spinning (P~10s) systems (about 21)**
- **Exhibit regular γ-ray flares**
  - Believed to be powered by magnetic field
  - Either trigger or are preceded by starquakes
  - Some linked to glitches
- **Three giant flares observed with peak luminosities ~10^{47} erg/s**
  - March 5, 1979 : SGR 0526-66
  - August 27, 1998 : SGR 1900+14
  - December 27, 2008: SGR 1806-20
- **Giant flares**
  - QPOs – 10's -100's of Hz
  - Magnetic field reconstruction
  - Possible f-mode excitation

Magnetars:
Quasi-Periodic Oscillations



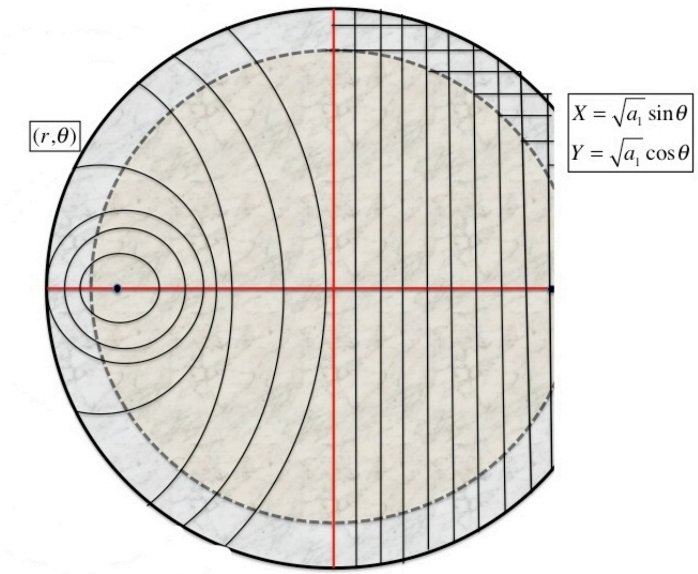
✓ **Giant flares in SGRs**

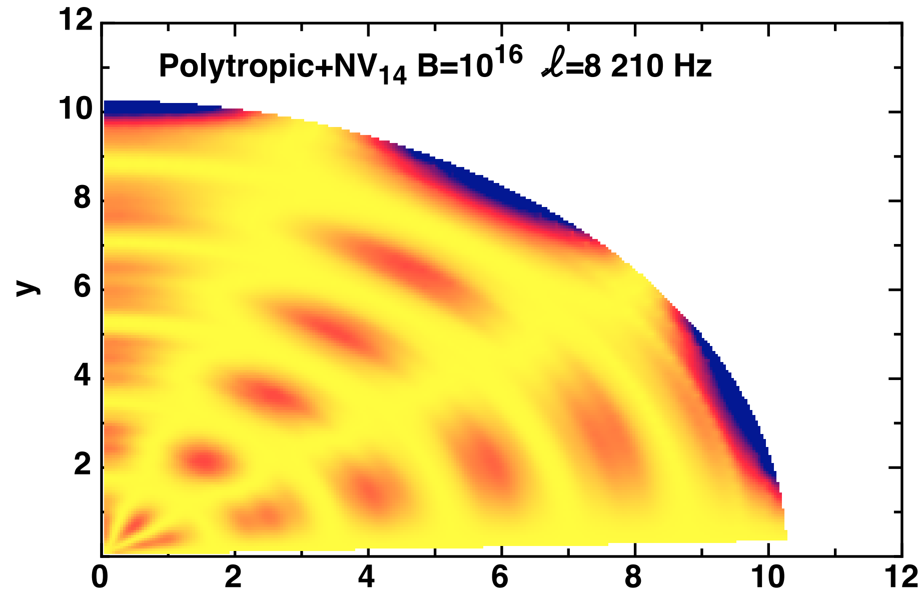
- Up to now, **three giant flares** have been detected.
  - *SGR 0526-66 in 1979,*
  - *SGR 1900+14 in 1998,*
  - *SGR 1806-20 in 2004*
- **Peak luminosities** : $10^{44} - 10^{46}$ erg/s
- A decaying tail for several hundred seconds follows the flare.

✓ **QPOs in decaying tail** (Israel *et al.* 2005; Watts & Strohmayer 2005, 2006)

- **SGR 1900+14** : 28, 54, 84, and 155 Hz
- **SGR 1806-20** : 18, 26, 29, 92.5, 150, 626.5, & 1837 Hz
  (possible additional frequencies : 720 & 2384 Hz)

Alfven Continuum + Discrete oscillations





Only Crust Oscillations

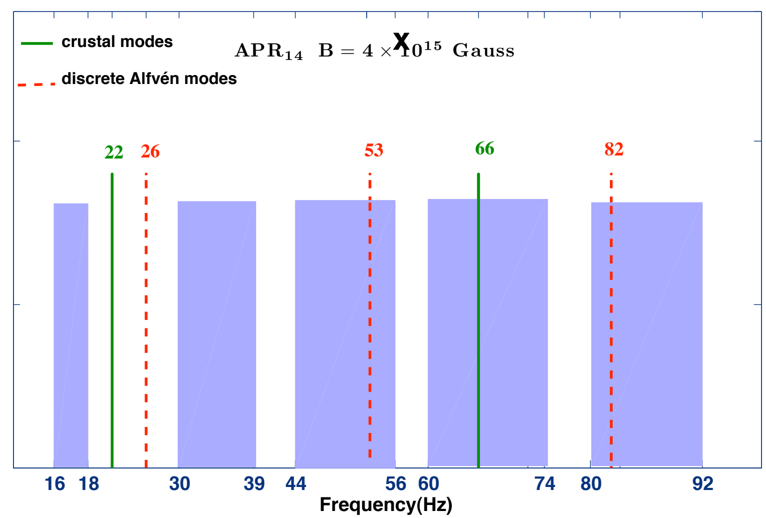
- Sotani,Kokkotas, Stergioulas 2007
- Samuelsson, Andersson 2007
- Steiner, Watts 2009

Without Crust

- Levin 2007
- Sotani,Kokkotas, Stergioulas 2008
- Colaiuda, Beyer, Kokkotas 2009
- Cerda-Duran, Stergioulas, Font 2009

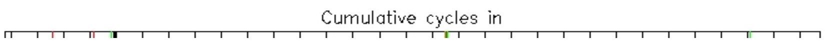
With Crust

- Van Hoven, Levin 2011, 2012
- Cerda-Duran, Stergioulas, Font 2011
- Gabler et al 2011
- Colaiuda, Kokkotas 2011, 2012



ONLY AXISYMMETRIC AXIAL OSCILLATIONS

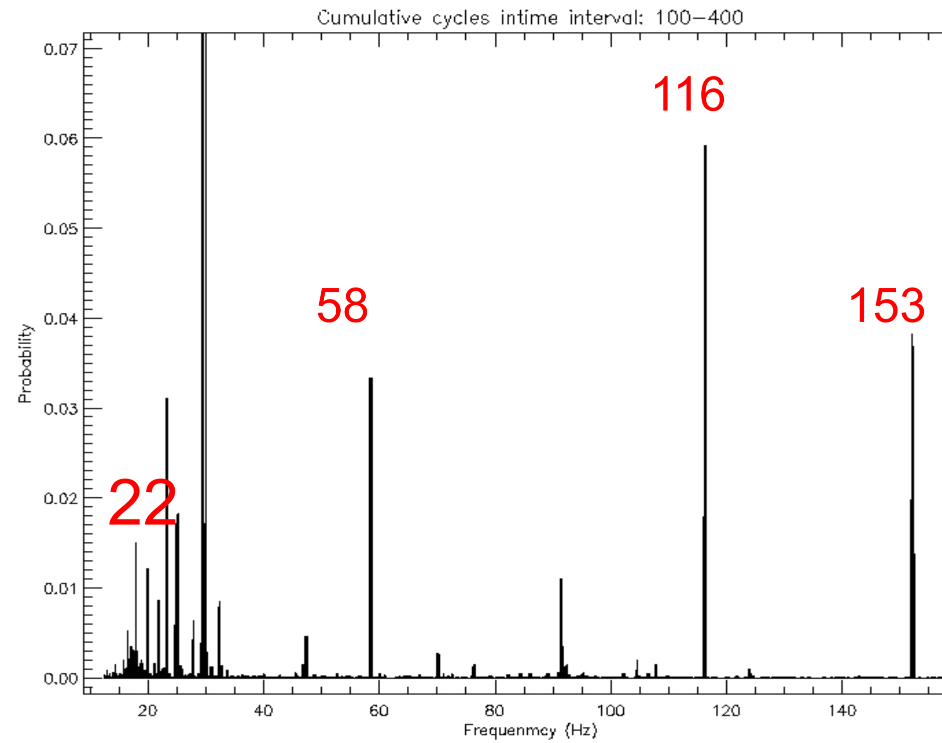
Gregory & Loredo method application to the SGR flare



QPO frequencies as expected by Colaiuda, Beyer, Kokkotas (2009)

Table 2. Frequencies of the Alfvén QPOs and their ratios for different stellar model with a toroidal component of the magnetic field and a surface magnetic field $B = 4 \times 10^{15}$ Gauss. For open magnetic 'string' the frequencies are taken for points near the critical point (L_n) and near the y-axis (U_n), both for even and odd parity. We also give the value of the frequencies for the lower closed magnetic 'string' (C_n).

Model	ζ (km^{-1})	n	$f_{L_n^{\mathrm{even}}}$ (Hz)	$f_{L_n^{\mathrm{odd}}}$ (Hz)	$f_{U_n^{\mathrm{even}}}$ (Hz)	$f_{U_n^{\mathrm{odd}}}$ (Hz)	f_{C_n} (Hz)
		0	15.15	30.30	16.24	33.94	15.76
WFF_{14}	0.24	1	46.06	61.20	50.91	66.66	31.15
		2	75.75	90.90	81.20	99.99	79.38
		0	18.95	38.46	22.85	49.6	19.51
APR_{14}	0.18	1	56.83	76.91	69.67	98.65	39.01
		2	95.30	114.8	117.0	147.7	57.96
		0	16.21	32.42	19.35	41.84	16.73
APR_{14}	0.20	1	48.11	64.84	59.05	83.67	33.47
		2	64.84	97.79	99.88	125.5	51.25
		0	11.47	22.93	13.92	29.90	11.88
L_{14}	0.20	1	34.40	45.46	42.18	58.57	23.75
		2	56.52	68.39	70.05	88.46	35.63

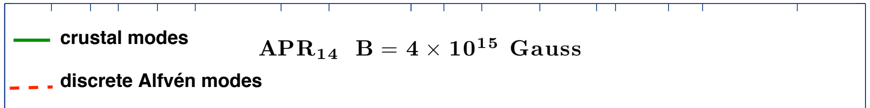


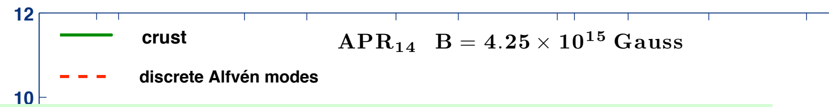
At least three more frequencies detected by our method …

Hambaryan, Neuhaeuser, Kokkotas 2011

SGR 1806-20 SGR 1900+14

Colaiuda, Kokkotas 2011a





Latest News (Colaiuda, Kokkotas 2011b)

- ✓ The combination of poloidal+toroidal magnetic fields leads to PURE discrete spectrum
- ✓ The results of the magnetar seismology remain unchanged !

Explain all observed QPOs (!) (?)

EoS : APR (NV)
Mass: $M = 1.4M_{\odot}$
Radius: 11.57
B-field : $2\mathrm{x}10^{15}$ Gauss
Crust : 0.099R

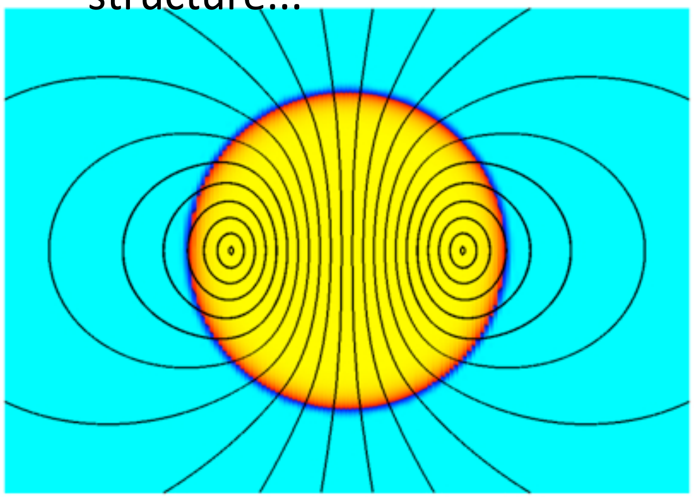
EoS : APR or WFF
Mass: $M=1.4M_{\odot}$
Radius: 11.57 or 10.51 km
B-field : $4.25\mathrm{x}10^{15}$ or $4\mathrm{x}10^{15}$ Gauss
Crust : 0.099R or 0.085R

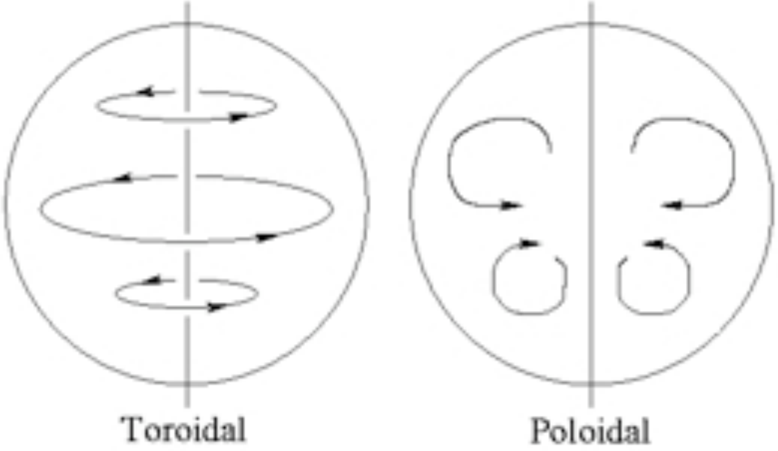
Magnetars & Grav. Waves

- **3D - GRMHD simulations of known and arbitrary initial magnetic field configurations**

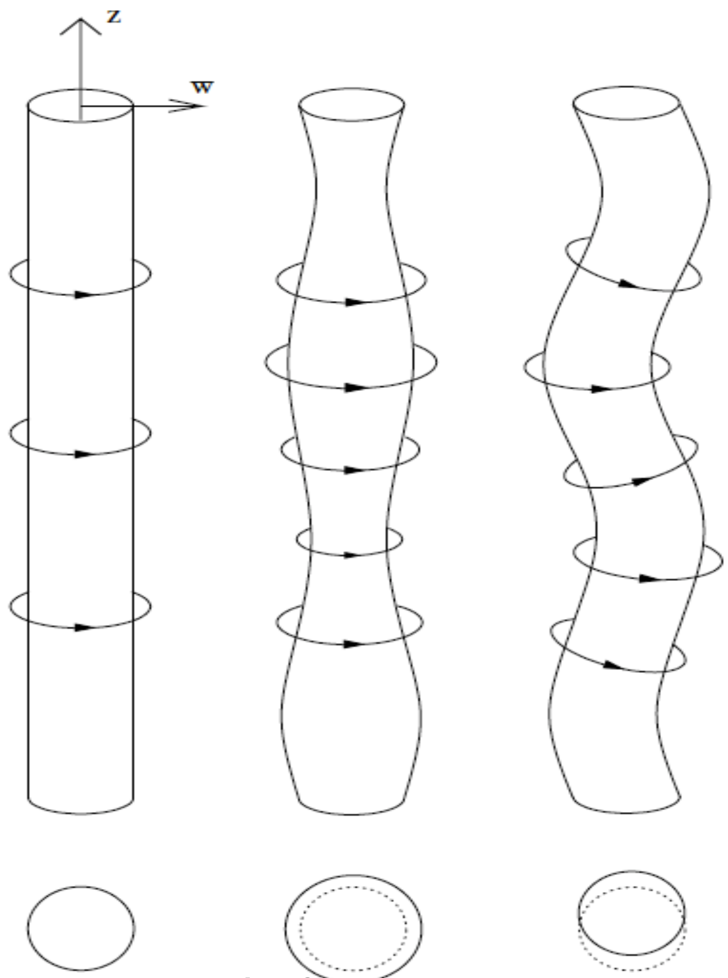
- **Magnetic field instabilities relevant for flare generation**
  instability mechanisms, relevant timescales, phenomenology (GR)

- **Understanding stable magnetic field configurations** mixed poloidal - toroidal configurations, relevant strengths of components, multipolar structure...



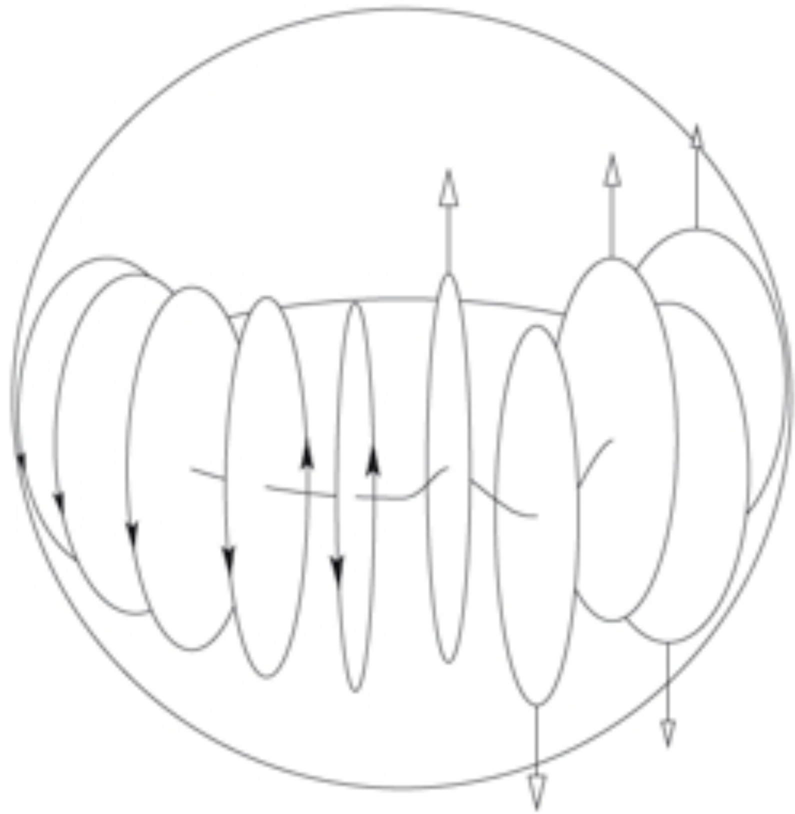


Poloidal Field Instabilities



k=0
“Varicose/
Sausage”

k=1
kink



Markey & Tayler 1973
Braithwaite& Spruit 2006

Our GR-MHD-Hydro Code

THOR

- ✓ 3 Dimensional
- ✓ Fully Non-Linear
- ✓ General Relativistic
- ✓ GR-MHD
- ✓ Cactus

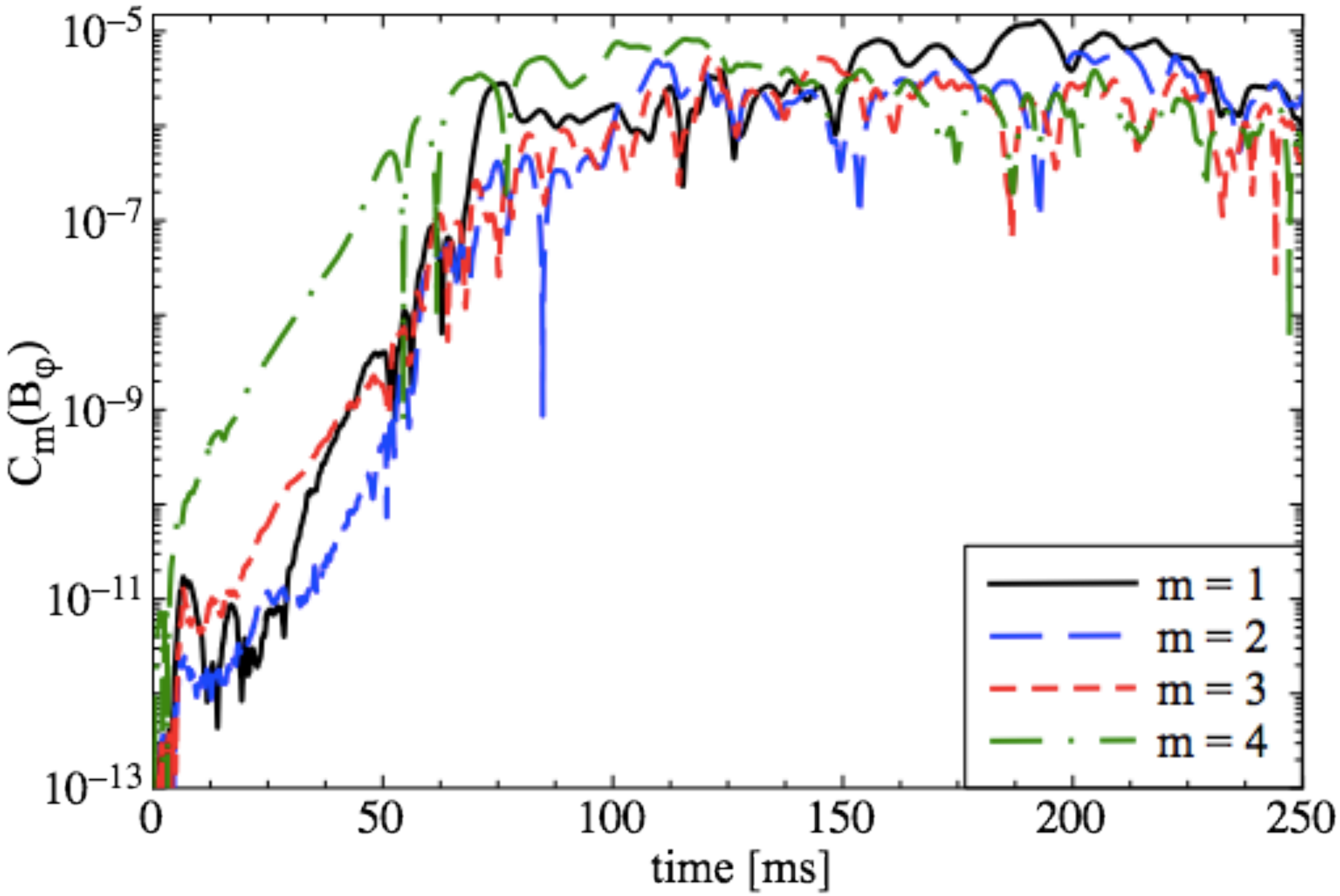
↓

Horizon

GPU version of THOR
(maybe the fastest code in the market!)

Poloidal Field Instabilities

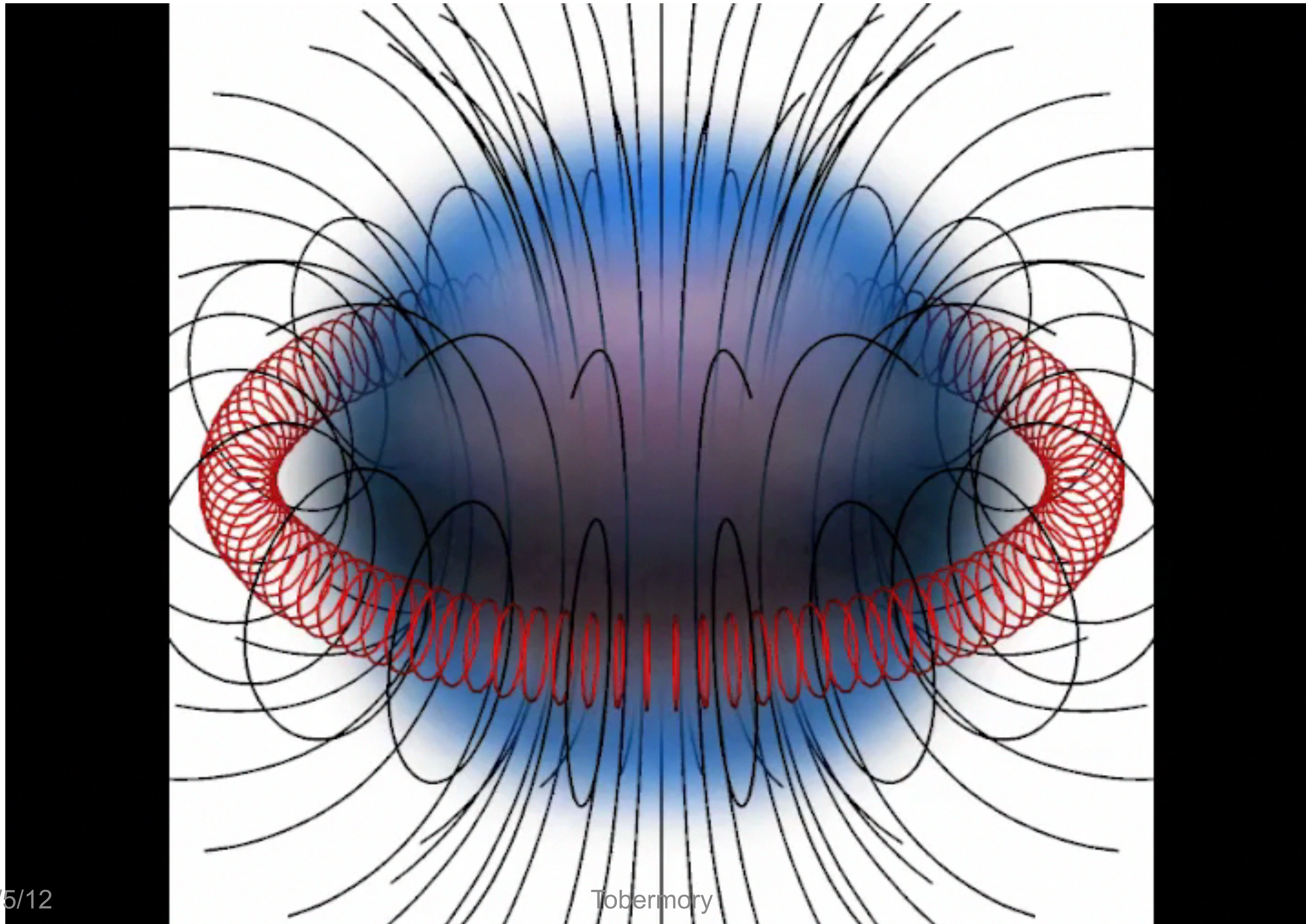
Lasky, Zink, KK, Glampedakis ApJL (2011)



Similar simulations by AEI group but for much shorter time

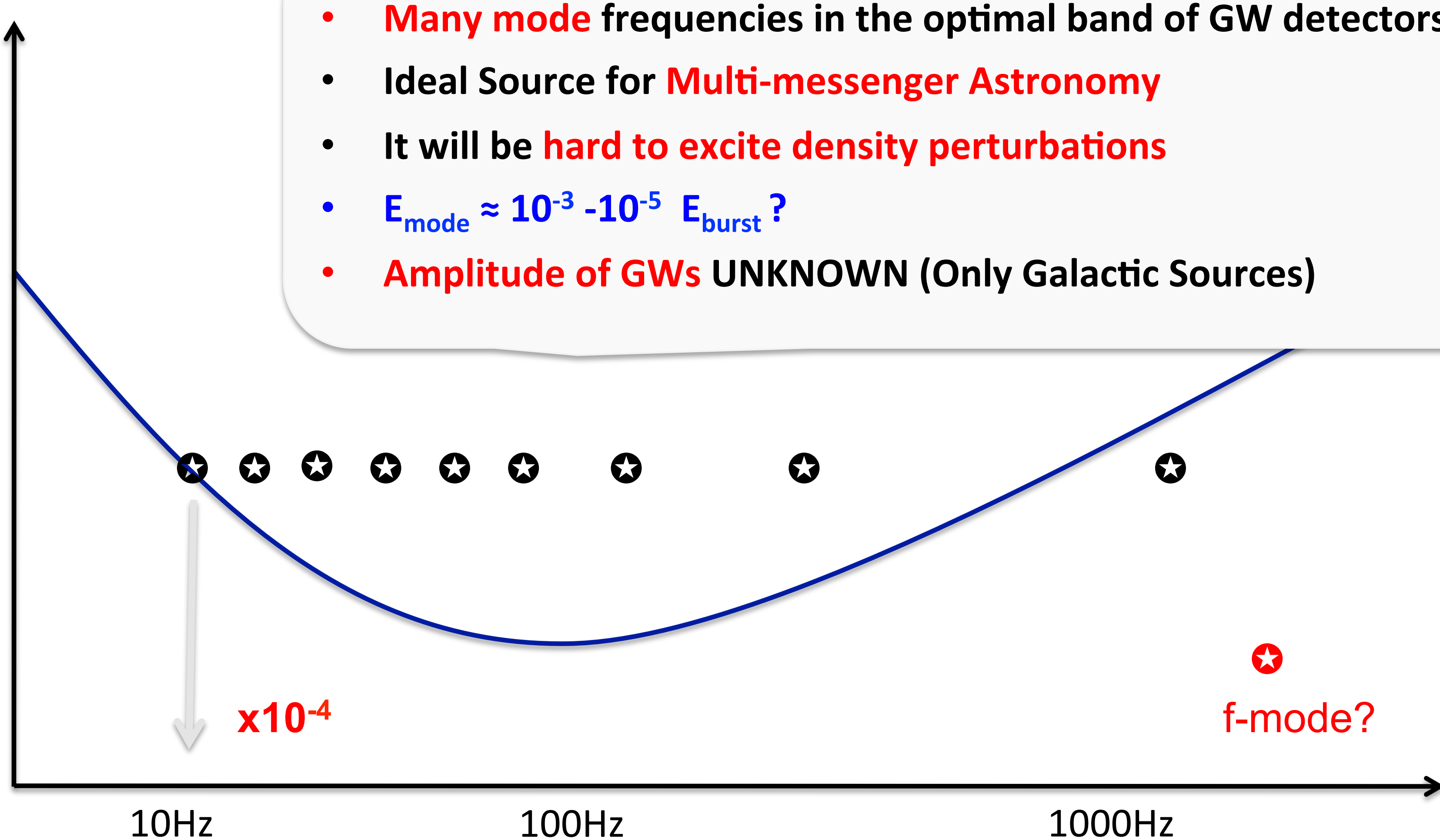
Simulation of Magnetic Field Instability

Lasky, Zink, KK, Glampedakis ApJL (2011)



Magnetars & GWs

- Many mode frequencies in the optimal band of GW detectors
- Ideal Source for Multi-messenger Astronomy
- It will be hard to excite density perturbations
- $E_{mode} \approx 10^{-3} - 10^{-5}\ E_{burst}$?
- Amplitude of GWs UNKNOWN (Only Galactic Sources)



GW from Magnetars

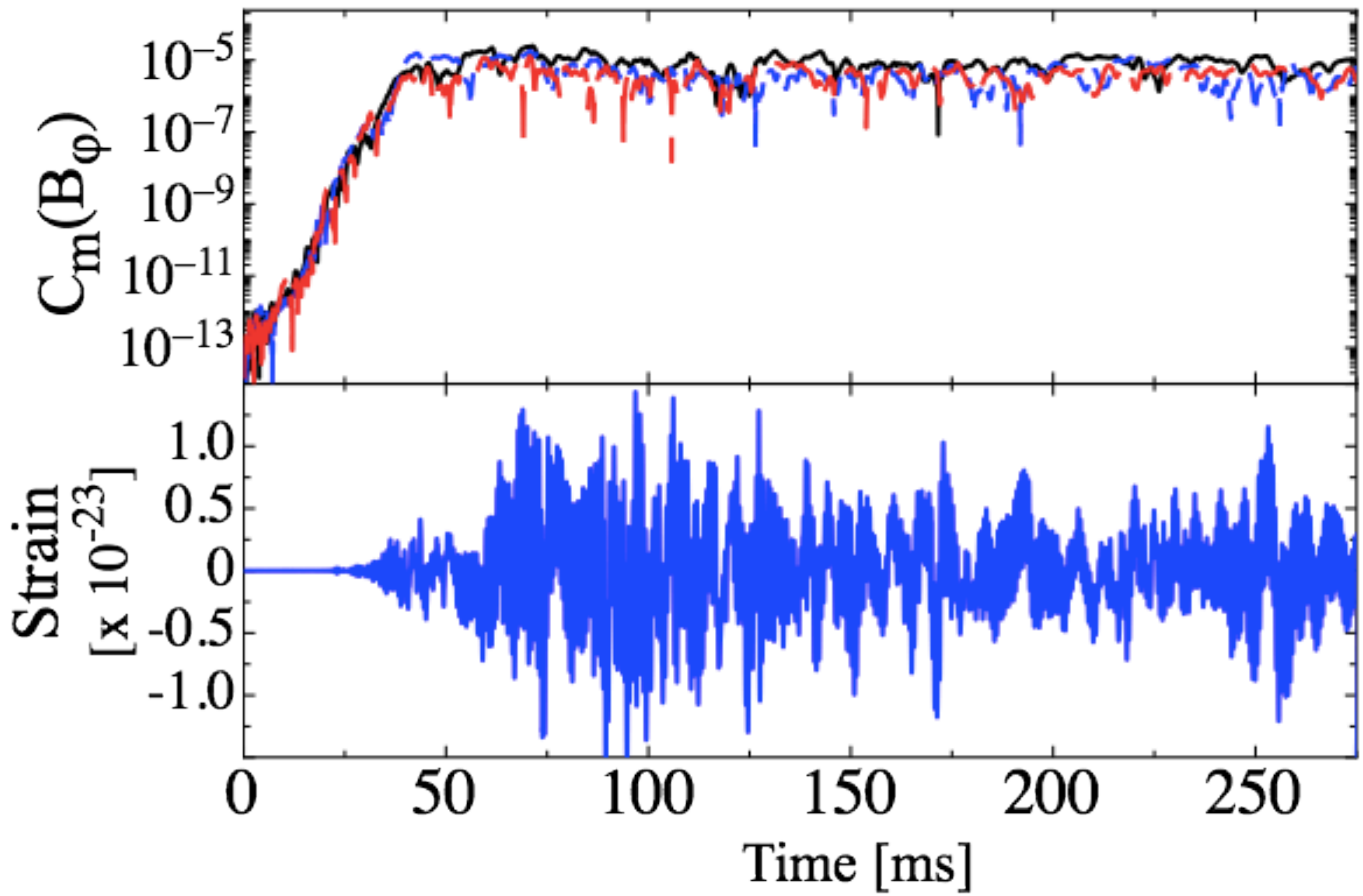
- **EM energies**: SGR 1806-20, 2004 …… $\sim 5\text{x}10^{46}$ erg
- GW energy upper limits (Abadie etal 2011)
  - White noise: $3\text{x}10^{44}$ erg
  - F-mode: $2\text{x}10^{47}$ erg

Theoretical Work

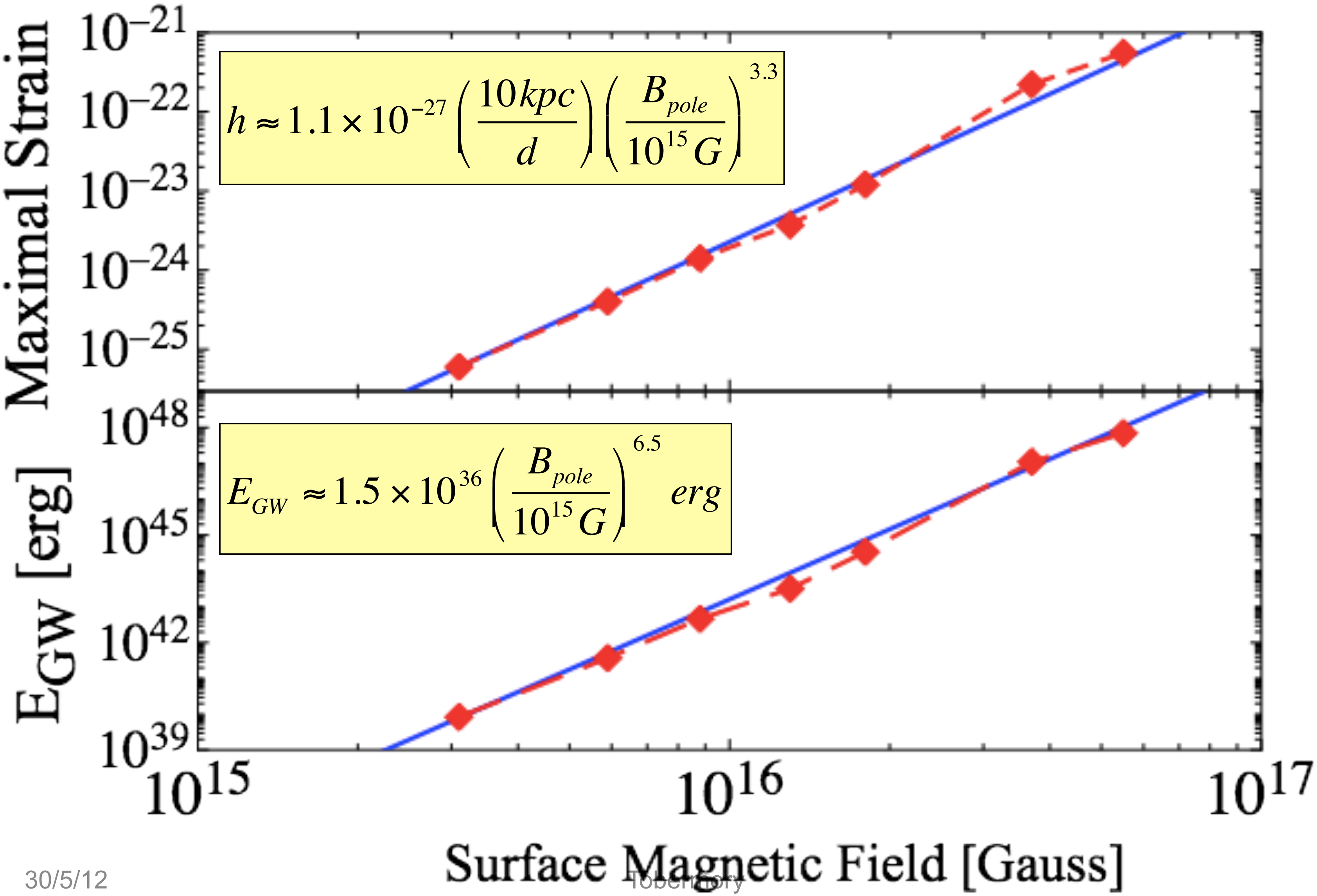
- **Ioka (2001)** – change in moment of inertia from optimal B-field reconfigure ………………………………….. 10^{49} erg
- **Corsi & Owen (2011)** – as above ……... 10^{49} erg
- **Levin & van Hoven (2011)** – excitation of f-mode from external field excitation ………………………………… $\leq 10^{41}$ erg
- **Ciolfi etal (2011)** - NR excitation of the f-mode from inertial field rearrangements ($B \sim 10^{17}$) **(S/N ~2-5)**

Gravitational Waves from Magnetars

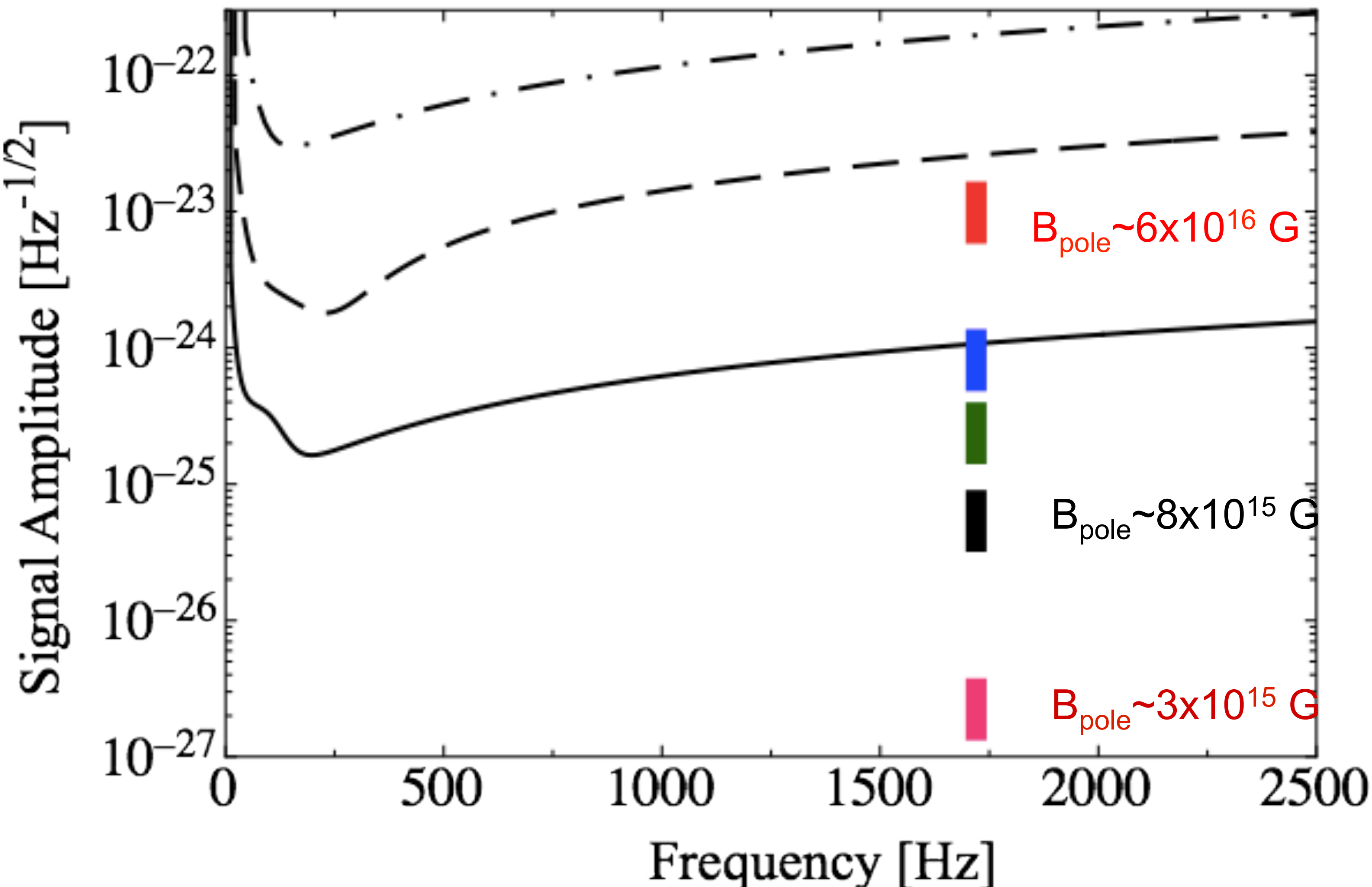
Zink, Lasky, Kokkotas (2011)



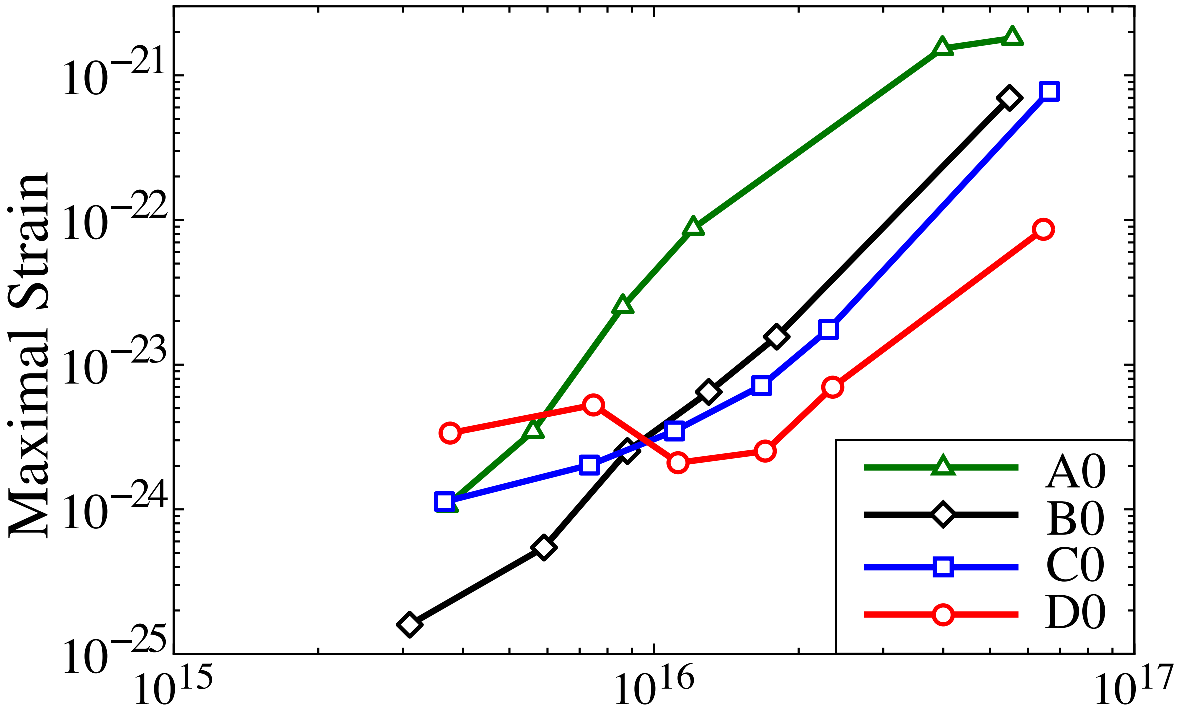
Gravitational Waves



Detectability I

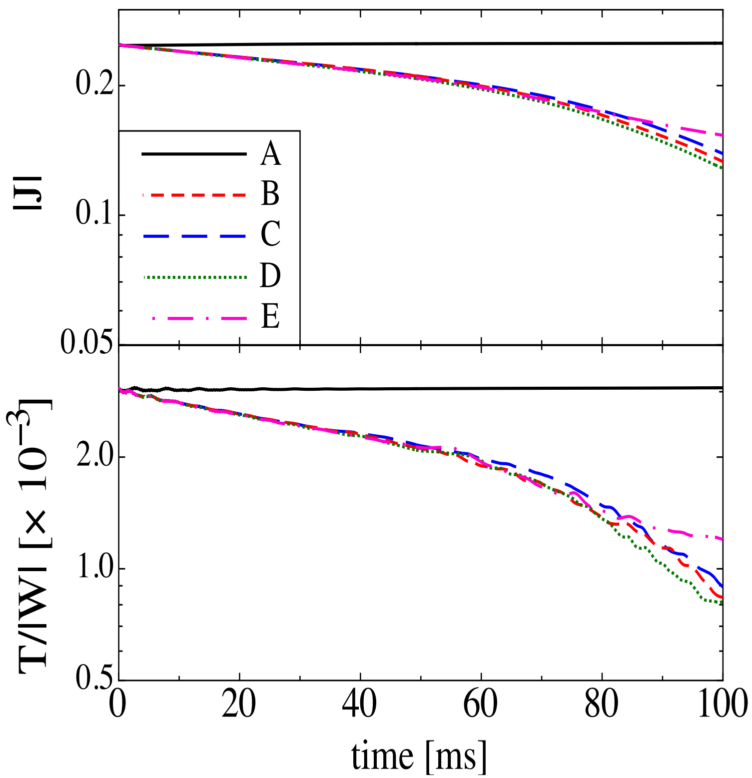


More models and EoS



$$E_{\mathrm{GW}} = 1.7 \times 10^{36} \left(\frac{R}{10\,\mathrm{km}}\right)^{9.6} \left(\frac{M}{M_\odot}\right)^{3.6} \left(\frac{B_{\mathrm{pole}}}{10^{15\,\mathrm{G}}}\right)^{5.8} \mathrm{erg.}$$

Fast Rotating Magnetars

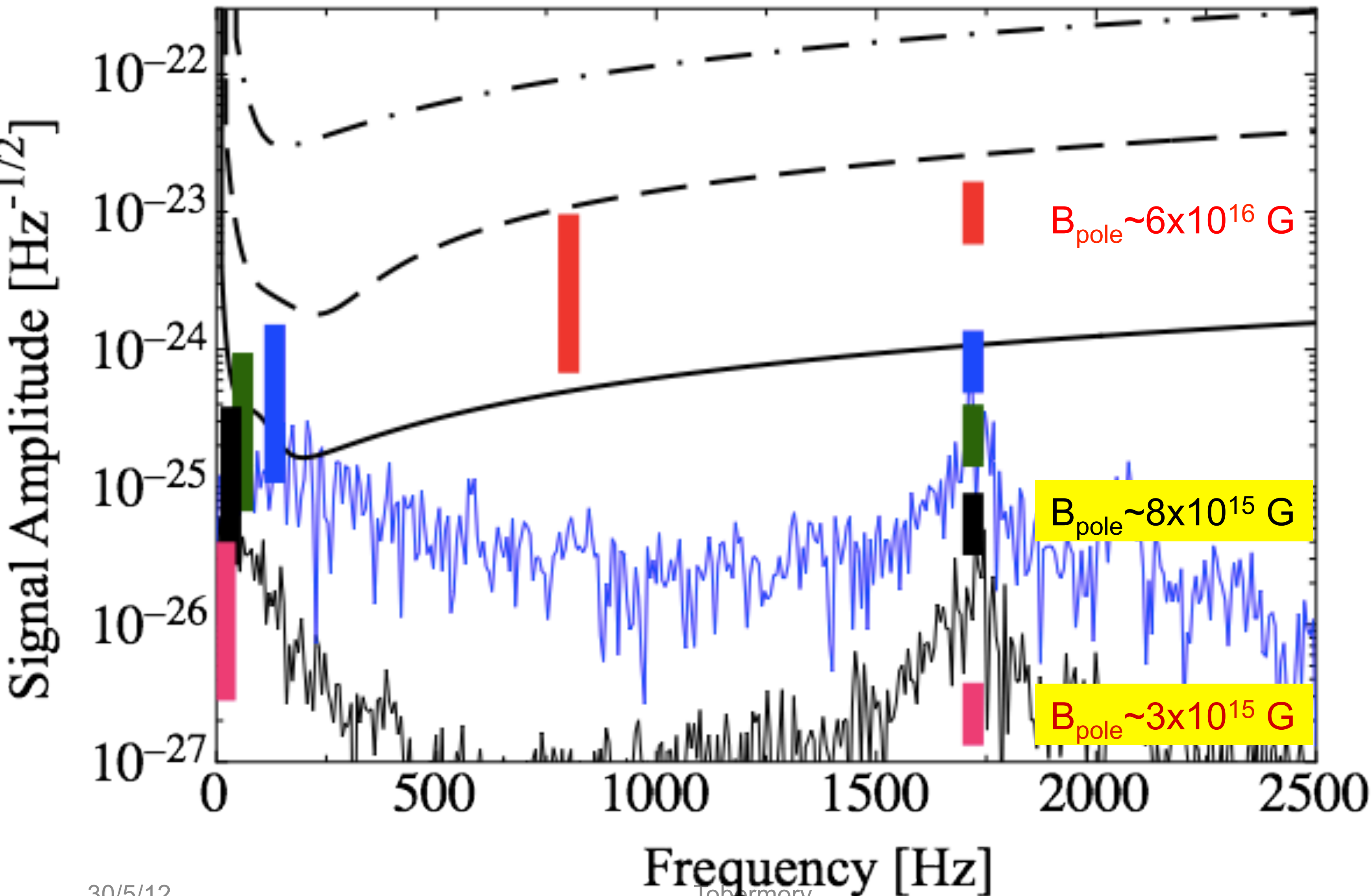


$$J = \int_\Sigma n_a T^a{}_\phi \sqrt{\gamma} d^3x,$$

Significant loss of angular momentum

- Most probably numerical reason
- As the magnetic field becomes weaker the loss becomes smaller

Detectability II



Conclusions

- **Rotational Instabilities of Neutron Stars**
  - ✓ Are potential sources for GW **beyond** our galaxy
  - ✓ **Many open issues** (growth time, EoS, non-linear coupling,…) **have already been resolved**.

- **Dynamics of magnetars**
  - ✓ Offers the possibility to **understand their structure**
  - ✓ Most probably a **weak source** for GW with the present generation detectors