gravitational-wave bursts
with memory

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Objectives:

- Provide a general overview of the memory effect applicable to ***all*** GW sources (***not*** just compact-object binaries).

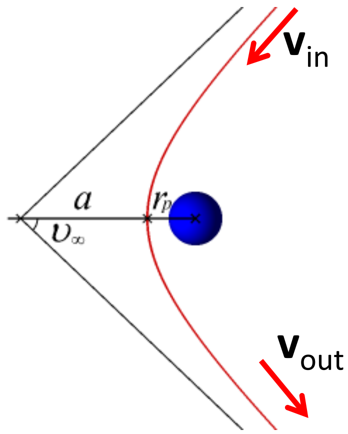
- Understand how to describe memory signals and make rough estimates of their sizes.

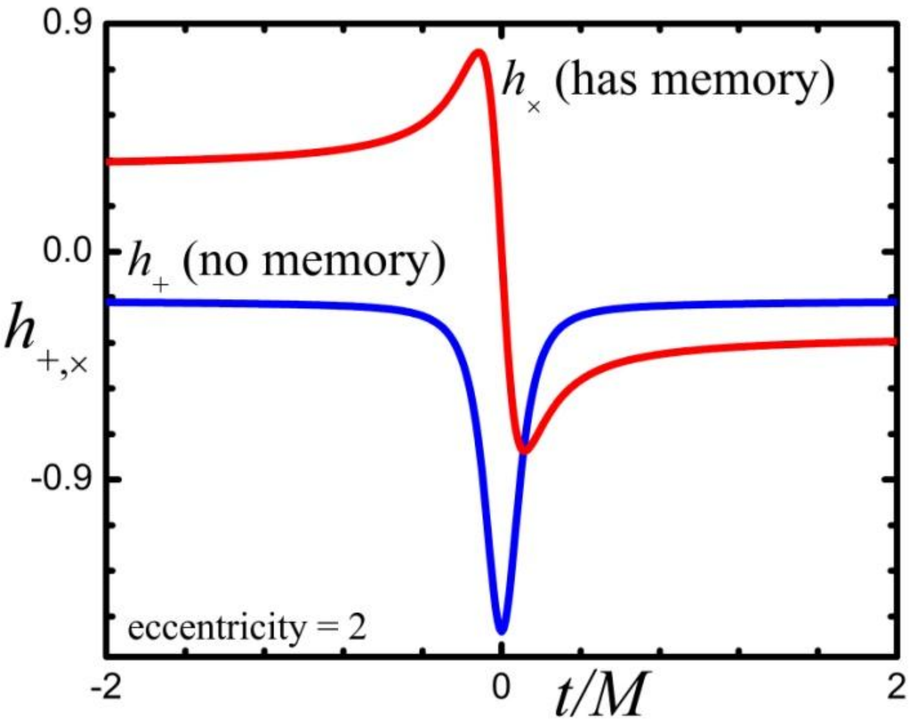
- Discuss specific sources of memory:
  - Core-collapse supernovae
  - GRBs
  - Two-body scattering
  - Compact-object binaries

- Concluding remarks.

Examples of memory:

Two-body scattering/hyperbolic orbits

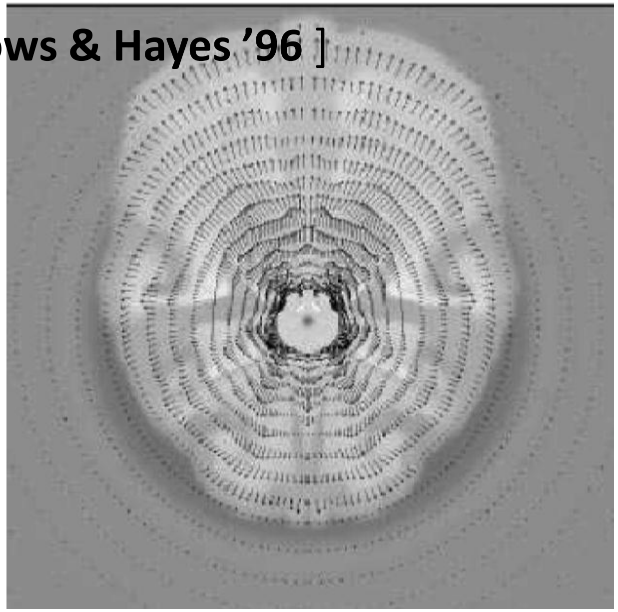


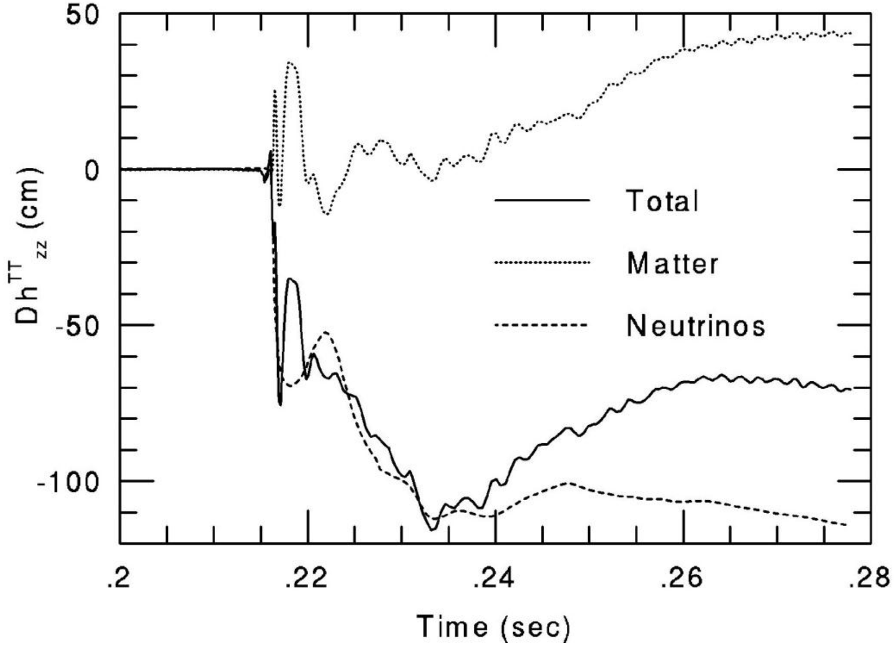


[Turner ‘77, Turner & Will ’78, MF ‘11]

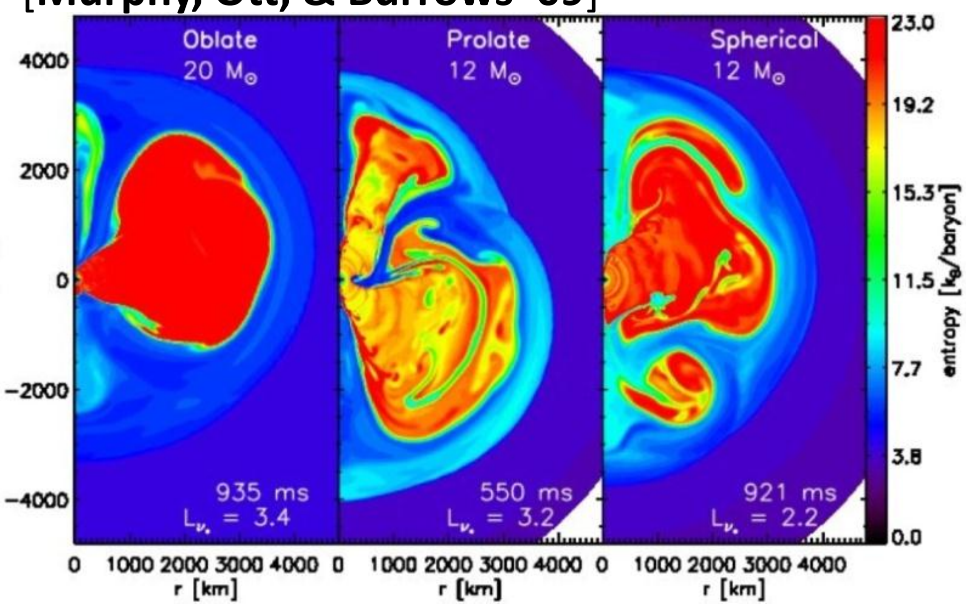
Examples of memory: Core-collapse supernovae

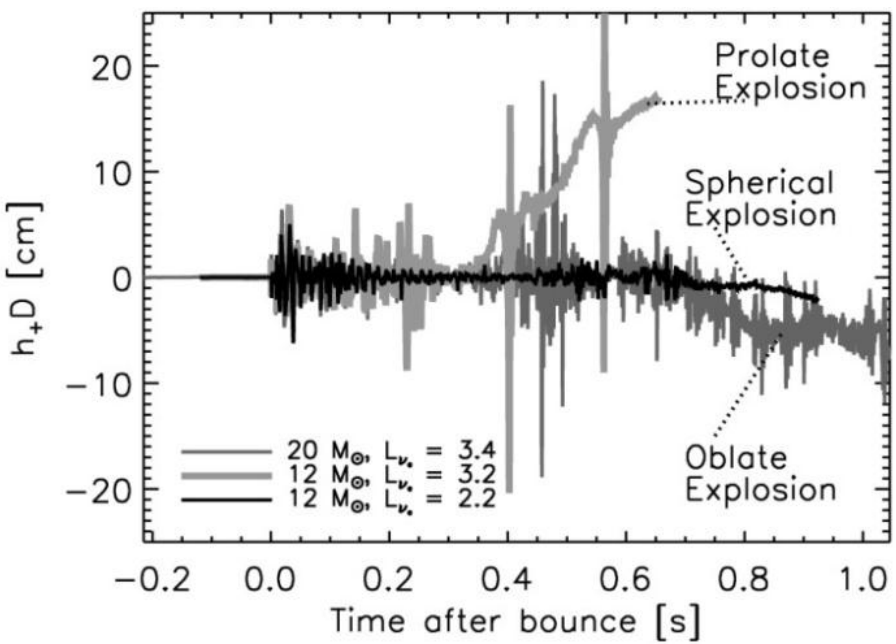
[Burrows & Hayes ’96]





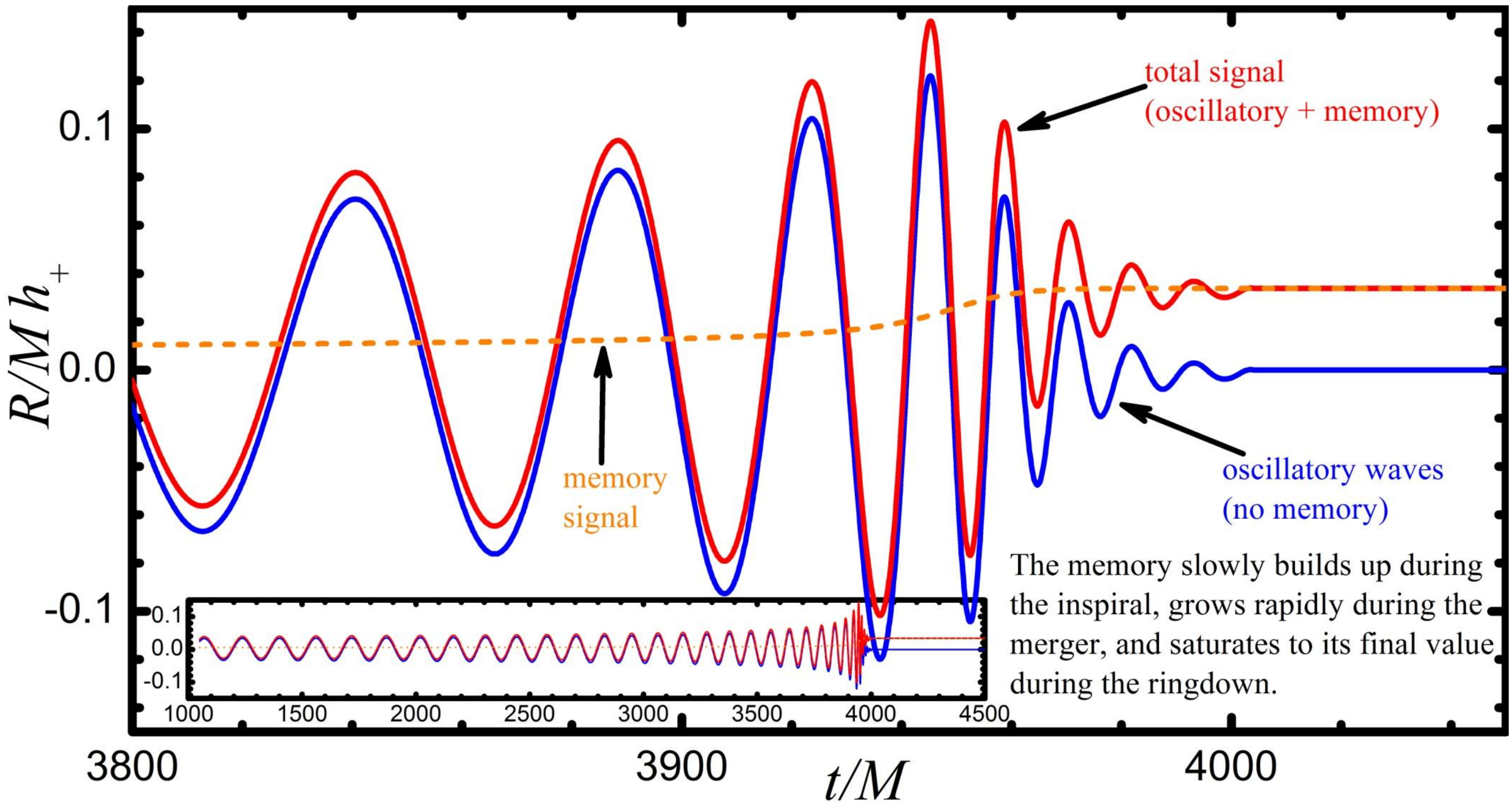
[Murphy, Ott, & Burrows ’09]



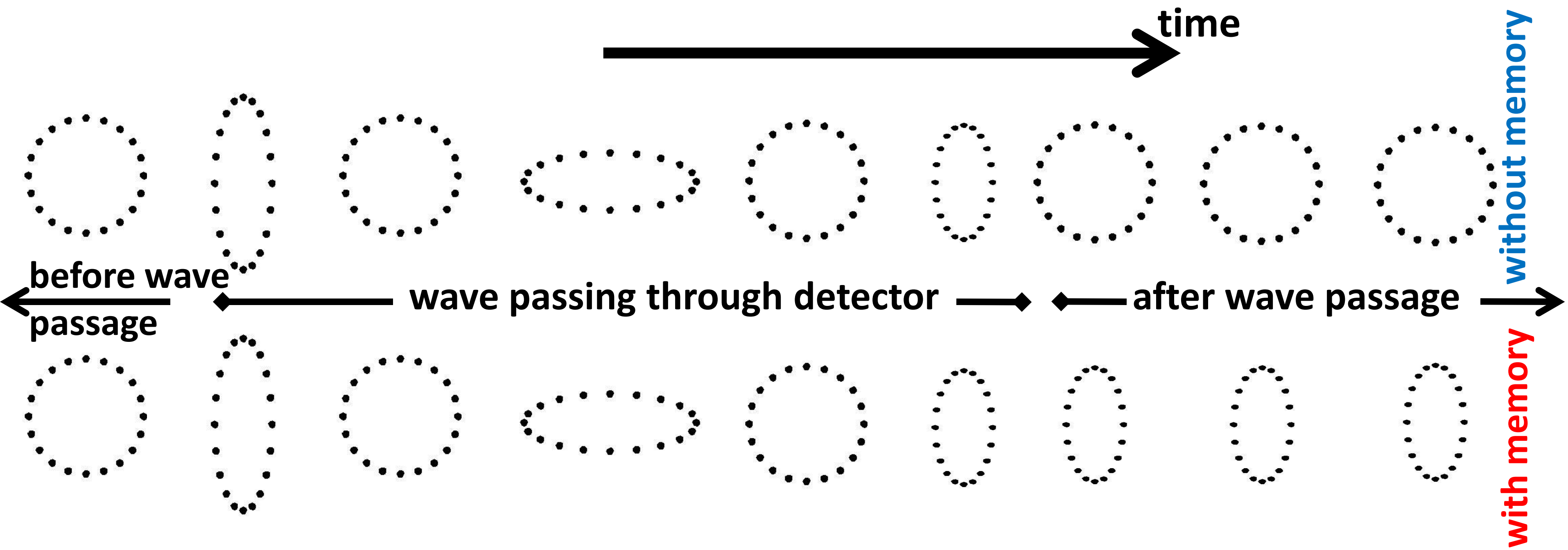


Examples of memory:

Binary black-hole mergers



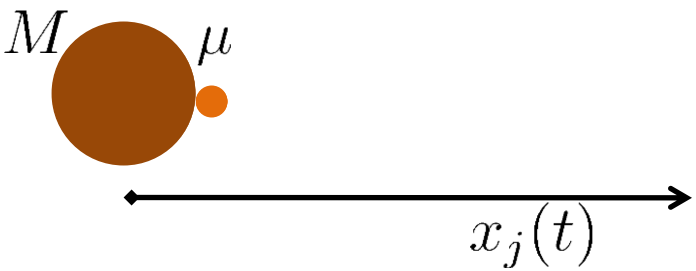
Why is this called “memory”?



[GW propagating perpendicular to the screen]

Understanding the memory effect:

[Zel'Dovich & Polnarev '74;
Braginsky & Grishchuk '85;
Braginsky & Thorne '87]



$$\dot{x}_j(t) \to v^j_\infty$$

$$h^{\mathrm{TT}}_{jk} \approx \frac{2}{R}\ddot{\mathcal{I}}^{\mathrm{TT}}_{jk} \qquad \mathcal{I}^{\mathrm{TT}}_{jk} = \mu[x_j x_k]^{\mathrm{TT}}$$

$$\ddot{x}_j = -\frac{M}{r^3}x_j$$

$$\begin{aligned}\ddot{\mathcal{I}}^{\mathrm{TT}}_{jk} &= \mu[x_j\ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j\dot{x}_k]^{\mathrm{TT}} \\ &= 2\mu\left[\dot{x}_j\dot{x}_k - \frac{M}{r^3}x_j x_k\right]^{\mathrm{TT}}\end{aligned} \longrightarrow \quad \Delta h^{\mathrm{TT}}_{jk} = \frac{4\mu}{R}\Delta[v^j v^k]^{\mathrm{TT}}$$

Understanding the memory effect:

General formula for the memory jump in a system w/ N components [Braginsky & Thorne '87, Thorne '92]

$$\Box \bar{h}_{ij} = -16\pi \sum_{A=1}^{N} T_{ij}^{\mathrm{pp},A}$$

$$\Delta h_{ij}^{\mathrm{TT}} = \Delta \sum_{A=1}^{N} \frac{4M_A}{R\sqrt{1-v_A^2}} \left[\frac{v_A^j v_A^k}{1-\boldsymbol{N}\cdot\boldsymbol{v}_A} \right]^{\mathrm{TT}}$$

$$\Delta h_{ij}^{\mathrm{TT}} = \lim_{t\to+\infty} h_{ij}^{\mathrm{TT}}(t) - \lim_{t\to-\infty} h_{ij}^{\mathrm{TT}}(t)$$

For neutrinos or GWs, the above eq. reduces to [Thorne '92, Epstein '78]:

$$\Delta h_{ij}^{\mathrm{TT}} = \frac{4}{R}\int_{-\infty}^{T_R} dt \int d\Omega \frac{dL}{d\Omega}\left[\frac{n^j n^k}{1-\boldsymbol{N}\cdot\boldsymbol{n}}\right]^{\mathrm{TT}}$$

Understanding the *nonlinear* memory effect:

- Arises from the GW stress-energy tensor (GWs produced by GWs)
  [Blanchet & Damour '92, Christodoulou '91]

- For inspiralling binaries the nonlinear memory modifies the waveform at ***leading*** (Newtonian) order:

$$h_+ = -2\frac{\mu}{R}v_{\rm orb}^2\left[(1+\cos^2\Theta)\cos[2\varphi(t)-2\Phi] + \frac{1}{96}\sin^2\Theta(17+\cos^2\Theta) + O(v_{\rm orb}^{1/2})\right]$$

[Wiseman & Will '91]

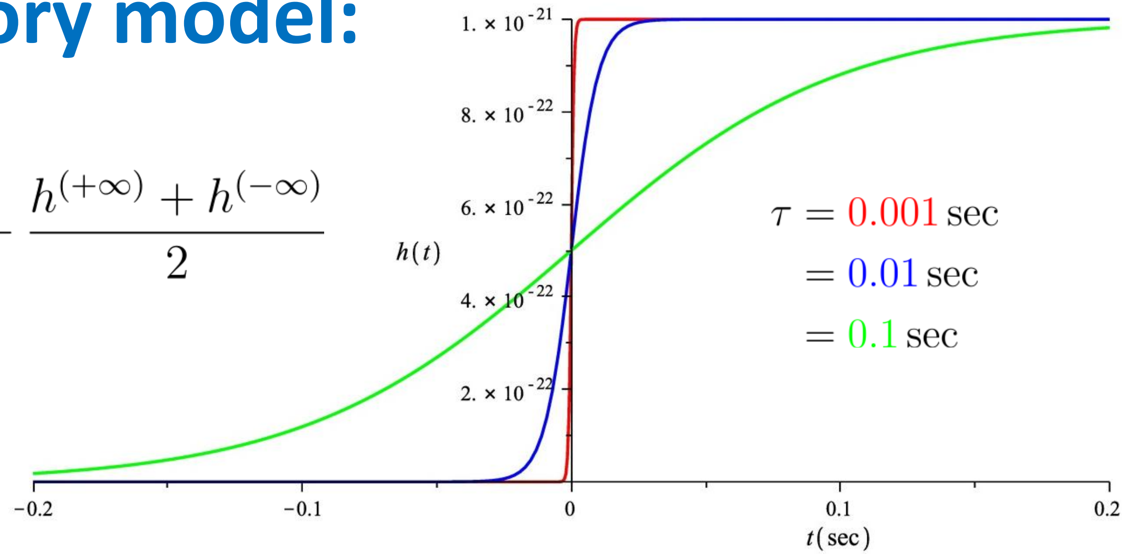
Why?

$$\Delta h^{jk}_{\rm mem} \sim \frac{\Delta E_{\rm GW}}{R} \qquad \Delta E_{\rm GW} \sim \Delta E_{\rm binding} \sim \frac{\mu M}{r} \sim \mu v_{\rm orb}^2$$

$$h^{ij}_{\rm oscil.} \propto \frac{1}{R}\ddot{\mathcal{I}}_{ij} \sim \frac{\mu}{R}v_{\rm orb}^2$$

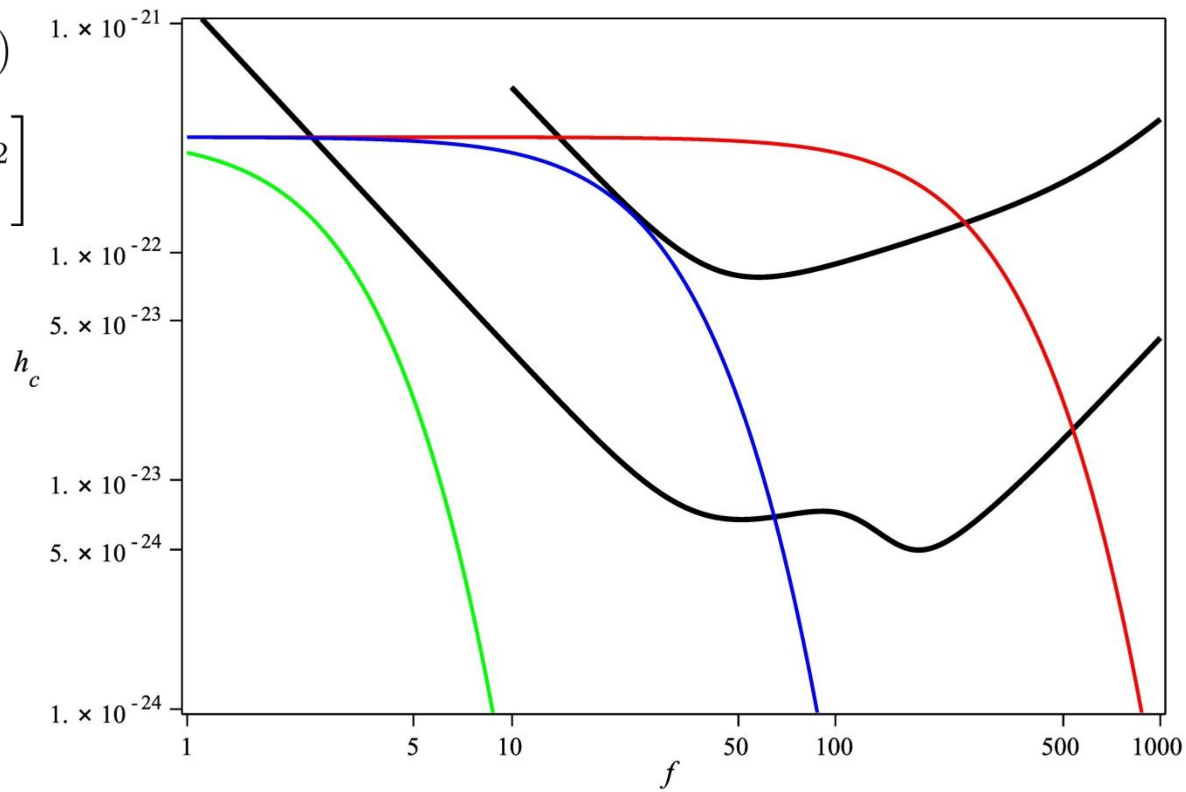
Simple analytic memory model:

$$h^{(\mathrm{mem})} = \frac{h^{(+\infty)} - h^{(-\infty)}}{2} \tanh\left(\frac{t}{\tau}\right) + \frac{h^{(+\infty)} + h^{(-\infty)}}{2}$$



$$\tilde{h}^{(\mathrm{mem})}(f) = \frac{\Delta h^{(\mathrm{mem})}}{2} i\pi\tau\, \mathrm{csch}(\pi^2 \tau f)$$
$$\approx i\frac{\Delta h^{(\mathrm{mem})}}{2\pi f}\left[1 - \frac{\pi^2}{6}(\tau f)^2\right]$$

$$\Delta h^{(\mathrm{mem})} \equiv h^{(+\infty)} - h^{(-\infty)}$$

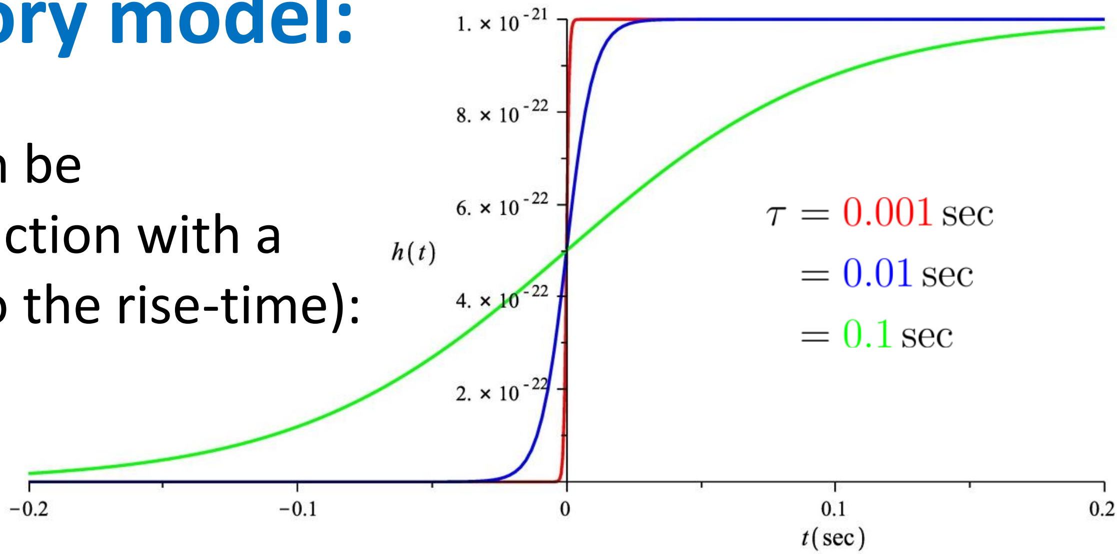


$$h_c(f) = 2f|\tilde{h}(f)|$$
$$h_n(f) = \sqrt{5 f S_n(f)}$$

[MF, PRD ’11]

Simple analytic memory model:

Generically, a memory signal can be approximated as a Heaviside function with a high-frequency cutoff (related to the rise-time):



$$h_{+,\times}^{(\mathrm{mem})(\mathrm{t})} = \Delta h_{+,\times}^{(\mathrm{mem})} \Theta(t)$$

$$\tilde{h}_{+,\times}^{(\mathrm{mem})}(f) = \frac{i \Delta h_{+,\times}^{(\mathrm{mem})}}{2\pi f},\ 0 < f < f_c,\ f_c \sim \frac{1}{\tau_{\mathrm{rise}}}$$

$$\begin{aligned} \tilde{h}^{(\mathrm{mem})}(f) &= \frac{\Delta h^{(\mathrm{mem})}}{2} i\pi\tau \,\mathrm{csch}(\pi^2 \tau f) \\ &\approx i \frac{\Delta h^{(\mathrm{mem})}}{2\pi f} \left[1 - \frac{\pi^2}{6}(\tau f)^2\right] \end{aligned}$$

[MF, PRD ’11]

Simple analytic memory model:

Estimate SNR:

$$\langle \rho^2 \rangle = \int_0^\infty \frac{h_c^2(f)}{h_n^2(f)} \frac{df}{f} \qquad h_c(f) = 2f \langle |\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2 \rangle^{1/2}$$

$$h_n(f) = \sqrt{5 f S_n(f)}$$

$$|\tilde{h}_{+,\times}^{(\mathrm{mem})}(f)| = \frac{\Delta h_{+,\times}^{(\mathrm{mem})}}{2\pi f}, \quad 0 < f < f_c, \quad f_c \sim \frac{1}{\tau_{\mathrm{rise}}}$$

$$\langle \rho^2 \rangle^{1/2} = \frac{\langle |\tilde{h}_+^{(\mathrm{mem})}|^2 + |\tilde{h}_\times^{(\mathrm{mem})}|^2 \rangle^{1/2}}{\hat{N}} = \frac{\Delta h^{(\mathrm{mem})}}{\hat{N}} \qquad \hat{N} = \pi \left[\int_0^{f_c} \frac{df}{f h_n^2(f)} \right]^{-1/2}$$

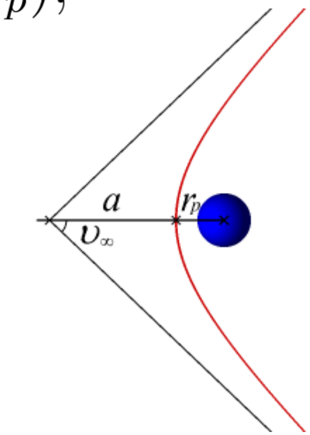
For $f_c \in [f_{\mathrm{min}}, f_{\mathrm{high}}]$

Detector	$\hat{N}_{\mathrm{min}}$	$\hat{N}_{\mathrm{high}}$
Initial LIGO	3.8×10^{-21}	2.3×10^{-21}
Advanced LIGO	3.9×10^{-22}	1.8×10^{-22}
Advanced Virgo	5.1×10^{-22}	2.2×10^{-22}
ET-b	2.9×10^{-23}	1.4×10^{-23}
ET-c	3.8×10^{-23}	1.4×10^{-23}
ET-d	5.4×10^{-23}	1.6×10^{-23}
LISA	5.3×10^{-21}	2.5×10^{-21}
PTA		6.1×10^{-16}
SKA		1.9×10^{-17}

[MF, PRD ’11]

Memory sources: gravitational scattering

For high velocity scatterings ($e \gg 1$ or $v_\infty^2 \gg M/r_p$),



$$\Delta h^{(\mathrm{mem})}_{e\gg 1} = 8\sqrt{\frac{2}{5}}\eta \frac{M}{R}\frac{M}{r_p}$$

$$\Delta h^{(\mathrm{mem})}_{e\gg 1} = 3\times 10^{-18}\left(\frac{\eta}{0.25}\right)\left(\frac{M/10M_\odot}{R/10\mathrm{kpc}}\right)\left(\frac{20M}{r_p}\right),$$

$$= 6\times 10^{-21}\left(\frac{\eta}{10^{-5}}\right)\left(\frac{M/10^6M_\odot}{R/20\mathrm{Mpc}}\right)\left(\frac{20M}{r_p}\right),$$

$$= 3\times 10^{-18}\left(\frac{\eta}{0.25}\right)\left(\frac{M/10^6M_\odot}{R/1\mathrm{Gpc}}\right)\left(\frac{20M}{r_p}\right).$$

$$\text{for } \; f_{\mathrm{rise}} \equiv \frac{1}{\tau_{\mathrm{rise}}} > 170\mathrm{Hz}\left(\frac{10M_\odot}{M}\right), \text{or}$$

$$> 1.7\mathrm{mHz}\left(\frac{10^6M_\odot}{M}\right)$$

Note that the nonlinear memory scales like:

$$\Delta h^{(\mathrm{nonlin.\,mem})} \sim \eta^2\frac{M}{R}\left(\frac{M}{r_p}\right)^{7/2}$$

and is suppressed by several orders of magnitude in hyperbolic binaries.

[MF, PRD ’11]

Memory sources: supernovae

Simulations from multiple groups show a memory effect due to anisotropic matter or neutrino emission:
[Burrows & Hayes ‘94, Murphy, Ott, Burrows ’09, Kotake et al ‘09, Muller & Janka ’97, Yakunin et al ‘10]

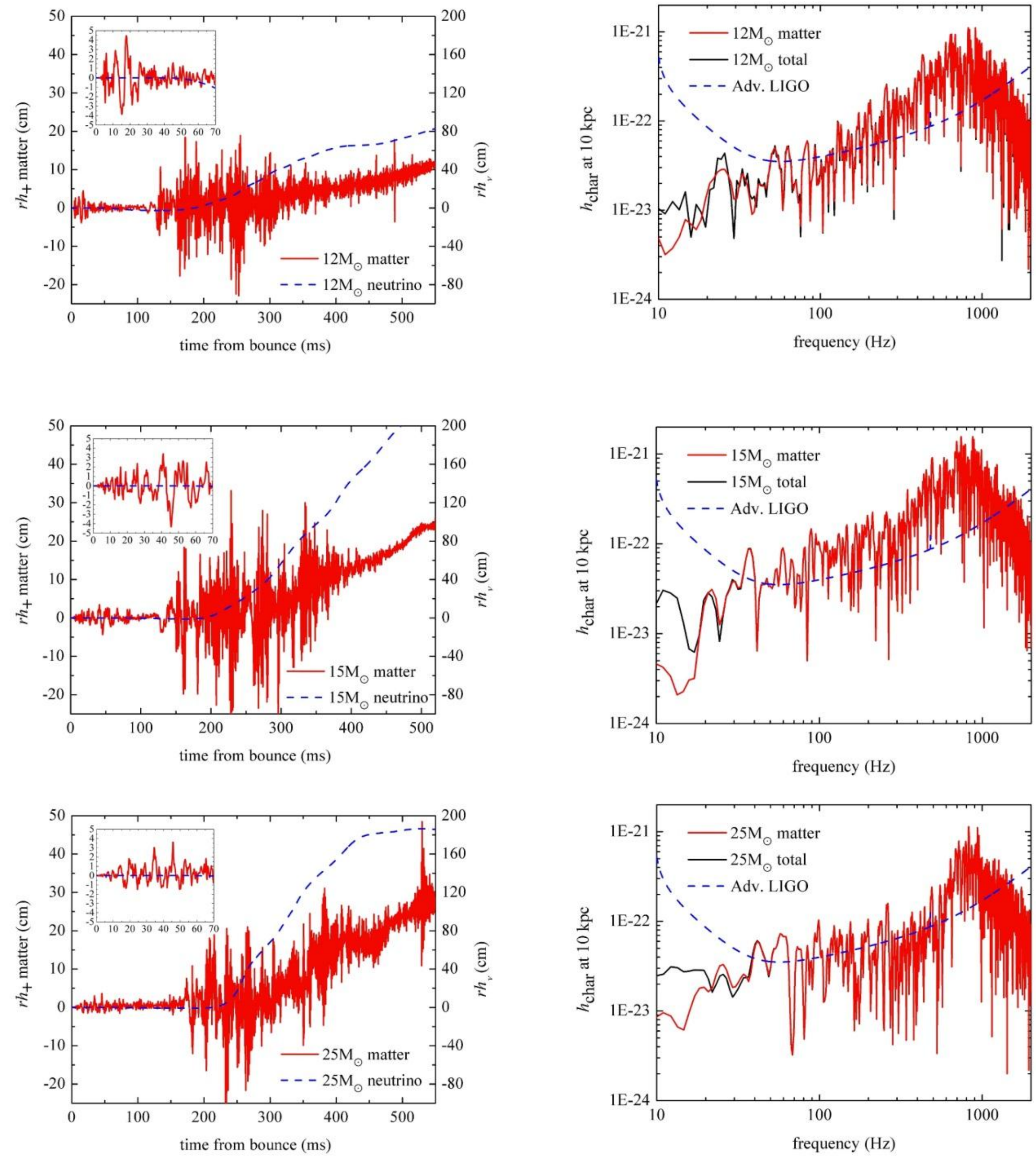
$$\Delta h^{(\mathrm{mem})}_{\mathrm{matter}} \sim 10^{-21} \left(\frac{10\,\mathrm{kpc}}{R} \right)$$

$$\Delta h^{(\mathrm{mem})}_{\nu} \sim 7 \times 10^{-21} \left(\frac{10\,\mathrm{kpc}}{R} \right)$$

$$\text{but} \quad f_c \lesssim 10\,\mathrm{Hz}$$

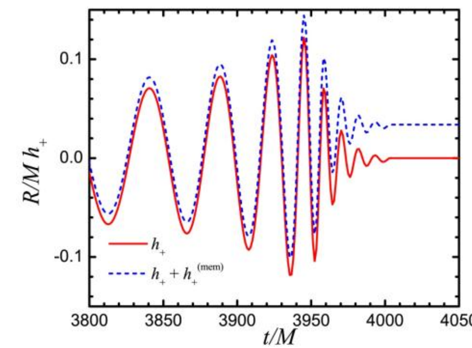
Size of memory varies among simulations depending on input physics.
[reviews by Ott’09 & Kotake ‘11]

[Yakunin et al ’10]

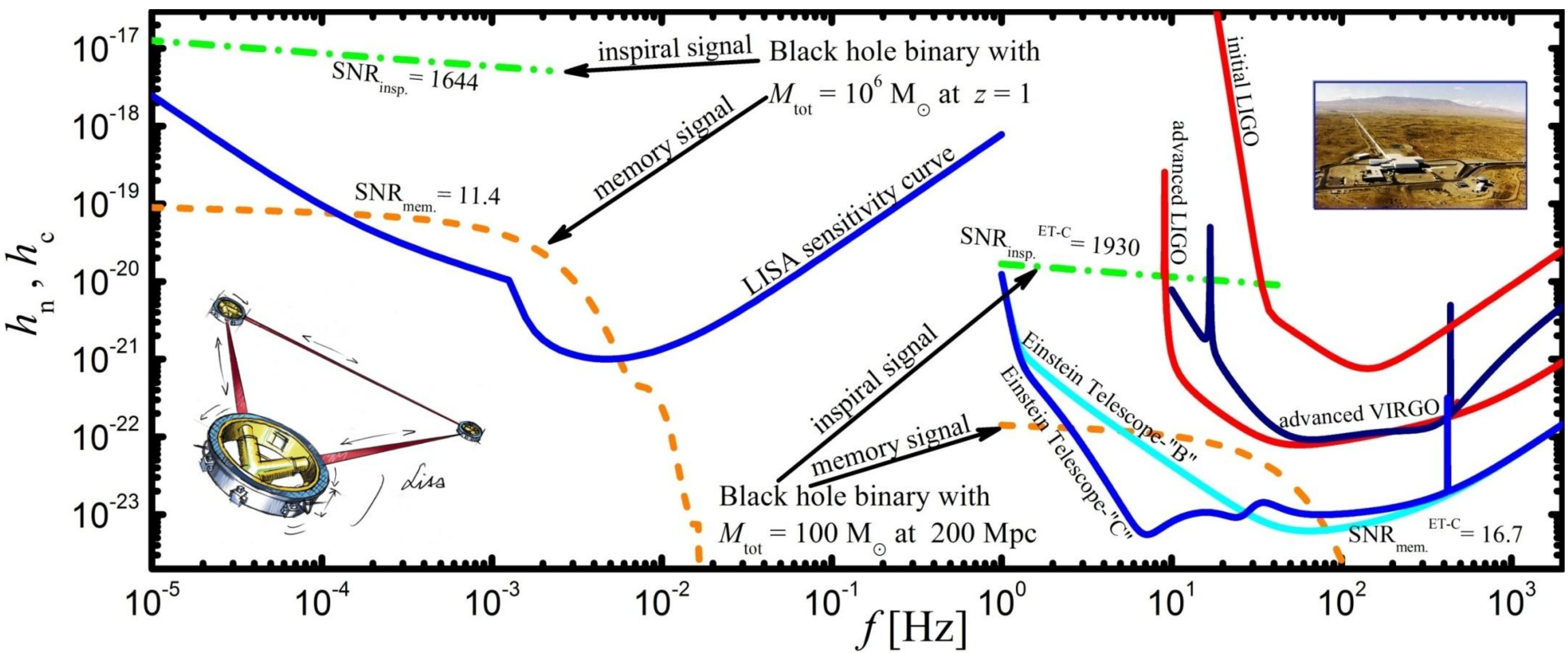


Memory sources: binary BHs

Nonlinear memory from binary BHs computed analytically in the inspiral [Wiseman & Will ’91, MF ’09a, MF ‘11, Guo & MF ‘12], and numerically during the merger/ringdown [MF ‘09, Pollney & Reisswig ‘11]



- Detectable to z~2 with LISA for a wide range of SMBH masses
- Out to ~ 20Mpc w/ aLIGO, ~ 1Gpc w/ ET



Memory sources: GRBs

GRBs are known to accelerate matter to high Lorentz factor (≥100), resulting in a GW w/ memory. [Segalis & Ori '01; Piran '01; Sago et al '04]

For a single jet [Sago et al '09]:

$$\Delta h^{(\mathrm{mem})} \sim 1.6 \times 10^{-22} \left(\frac{2\gamma m}{3 \times 10^{51}\,\mathrm{erg}} \right) \left(\frac{1\,\mathrm{Mpc}}{R} \right)$$

$$\text{with } f_c \sim \frac{1}{10^{-3}\,\mathrm{sec}}$$

Unfortunately, GRBs are usually at R>>100 Mpc.

Concluding remarks:

- Memory arises from sources with unbound matter or energy (including GWs).

- Observing the memory would allow us to probe particular aspects of a source:
  - Two-body scattering: asymptotic velocity of the stars
  - Supernovae: information about the ejected matter and neutrinos
  - GRBs: nature of the jet
  - Binary BHs: probe a particular nonlinear aspect of GR (the GW stress-energy tensor)

- In the low-frequency limit, the memory signal is particularly simple (depending only on an amplitude and cutoff frequency) and might be easy to search for.

- Memory signal also has a different angular dependence than other parts of the GW (or EM) signal.

- Prospects for detecting the memory with the upcoming generation of detectors is poor, but not substantially worse than other classes of sources that we routinely try to detect. Prospects are better for future ground and spaced-based detectors.