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VESF Summer School 2012:
Squeezing and QND Techniques

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LIGO-G1200700

What do I want you to take with you from this lecture?

- How can we actually trick Heisenberg?
- Get an intuitive understanding of Quantum noise and how to reduce it in our instruments.
- An easy tool ‘graphical quadrature picture’.

- *What I want to avoid is filling the blackboard only with complicated Quantum mechanics.*

References / Literature

- References for individual topics, specialised articles, as well as and pictures I have 'stolen', are given on the slides.
- If you are looking for a more completeor more detailed material have a look at the following:
  - V. Braginsky and F. Khalili: "Quantum Measurement", Cambridge University Press, 1992.
  - S. Danilishin and F. Khalili, “Quantum Measurement Theory in Gravitational-Wave Detectors”, Living Rev. Relativity, 15, (2012), 5. http://www.livingreviews.org/lrr-2012-5
  - H. Müller-Ebhard et al: “Review of quantum non-demolition schemes for the Einstein Telescope” 2009, ET-note, available at https://tds.ego-gw.it/itf/tds/file.php?callFile=ET-010-09.pdf
  - H. Miao et al: "Comparison of Quantum Noise in 3G Interferometer Configurations" https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=78229

Two Basic principles of GW detector

- 1. You need to make your test masses very quiet (quieter than the signal you want detect).
- 2. You need to read out the test mass positions with very high accuracy, without introducing ‘too much’ noise due to measurement itself.

- Photon shot noise is a sensing noise, photon radiation pressure noise is a back-action-noise.

The Standard quantum limit

- Standard Quantum Limit of a free mass is equivalent to Heisenberg uncertainty.
- Arises when one tries to detect gravitational wave by continuously measuring free-mass displacement, since

$$[\hat{x}(t), \hat{x}(t')] \neq 0$$

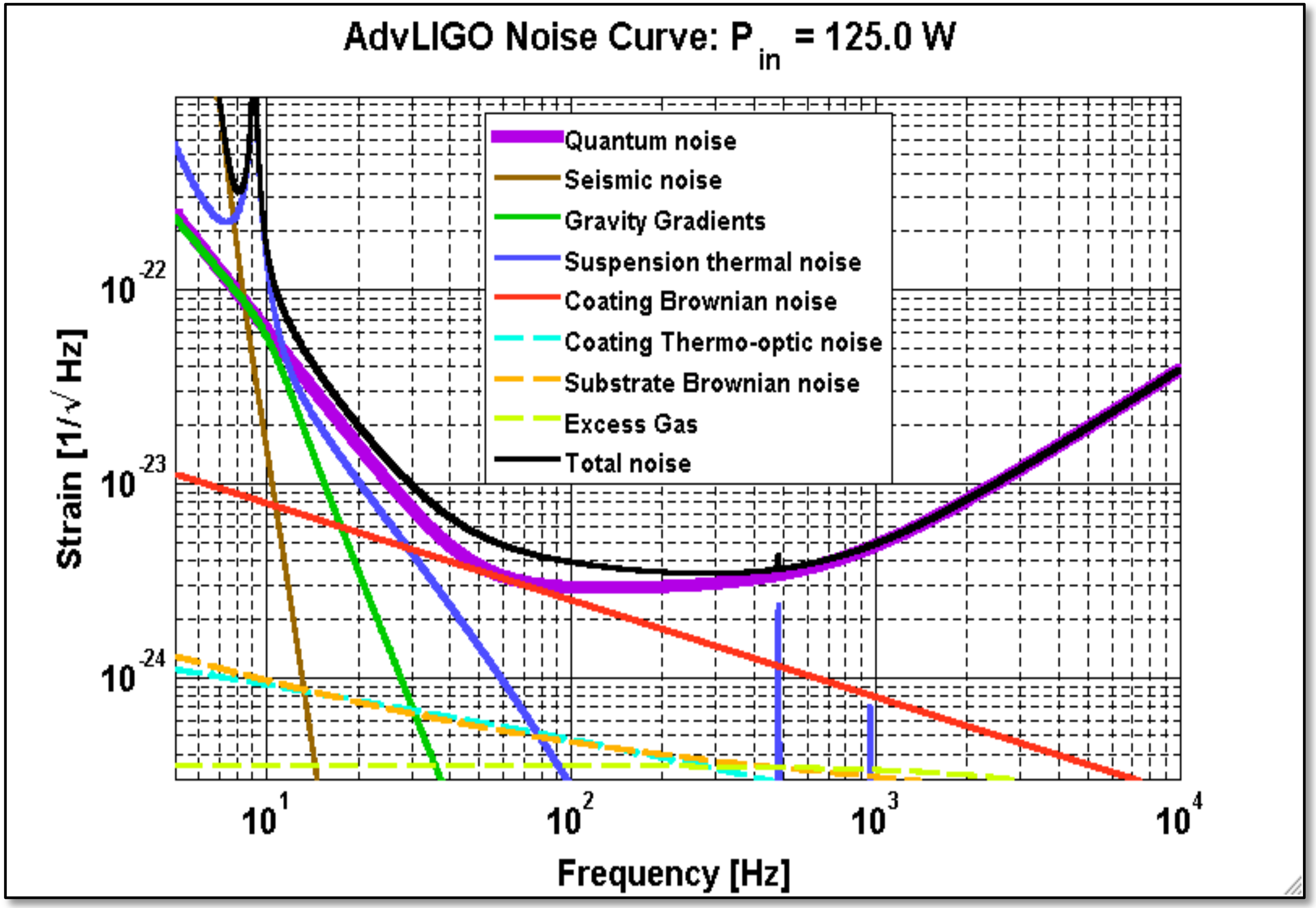
  - Precise measurement on x has to perturb p. Perturbation in p converts to future error in x.
- In our case: Light fields enforce Heisenberg uncertainty through complementarity between Shot noise and Radiation Pressure noise.

Overview

- Introduction: Quantum noise, Standard Quantum Limit, Vacuum fluctuations, 'Ball-on-a-stig', Quadrature Picture.
- The easiest way to beat the SQL: Varaiational Readout
  - Example: AEI-10m interferometer
- The best quantum noise reduction technique for second generation GW detectors: Squeezed light injection
  - How to make squeezed light?
  - Example: GEO-HF and Einstein telescope
- Optical rigidity
  - What is an optical spring?
  - Optical Bar and Optical Lever schemes
  - Local Readout scheme for Advanced LIGO
- Speed meter configurations

Why is Quantum Noise Reduction *'soooo'* important?

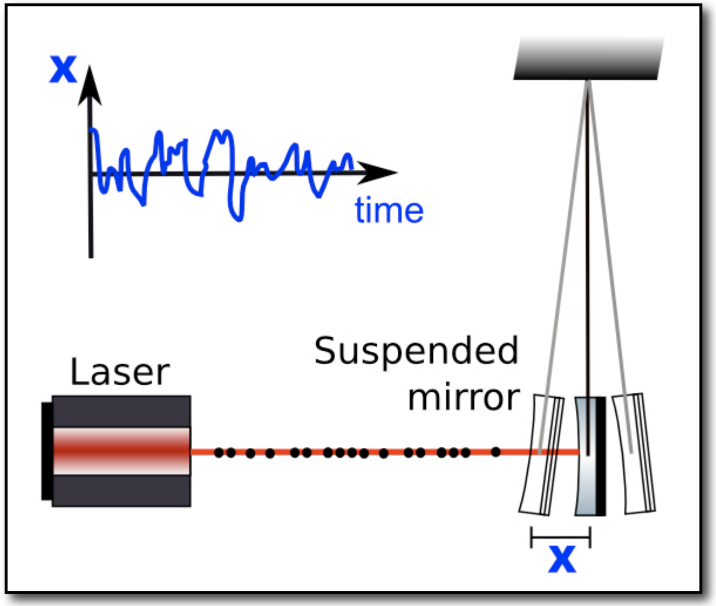
- For future interferometers Quantum noise is the ***main*** limiting noise source.
- Example: Broadband (tuned Signal Recycling) configuration of Advanced LIGO
- For all frequencies above 12Hz Quantum noise is the limiting noise.



Scenario 1b from LIGO-T070247-01-I

What is quantum noise?

- Quantum noise is comprised of **photon shot noise** at high frequencies and **photon radiation pressure noise** at low frequencies.
- The photons in a laser beam are not equally distributed, but follow a Poisson statistic.



wavelength

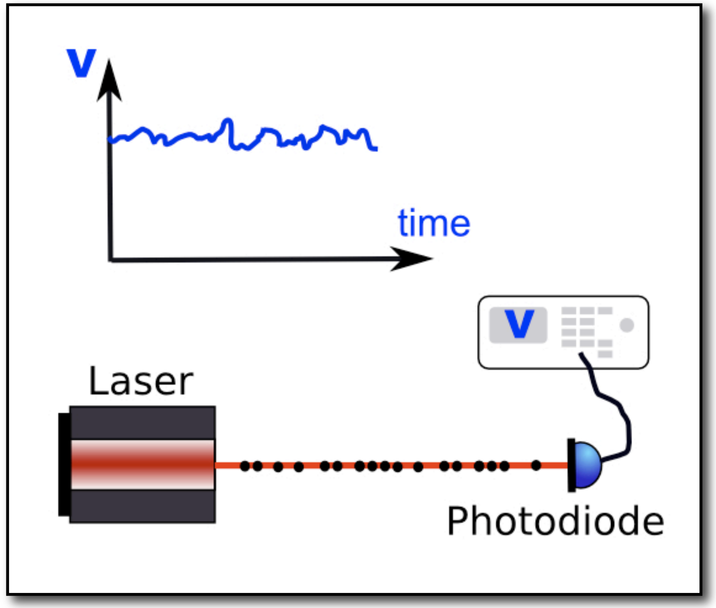
$$h_{\mathrm{sn}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

optical power

Mirror mass

Arm length

$$h_{\mathrm{rp}}(f) = \frac{1}{m f^2 L}\sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$



photon radiation pressure noise

photon shot noise

The Standard Quantum Limit (SQL)

- While shot noise contribution decreases with optical power, radiation pressure level increases:

wavelength

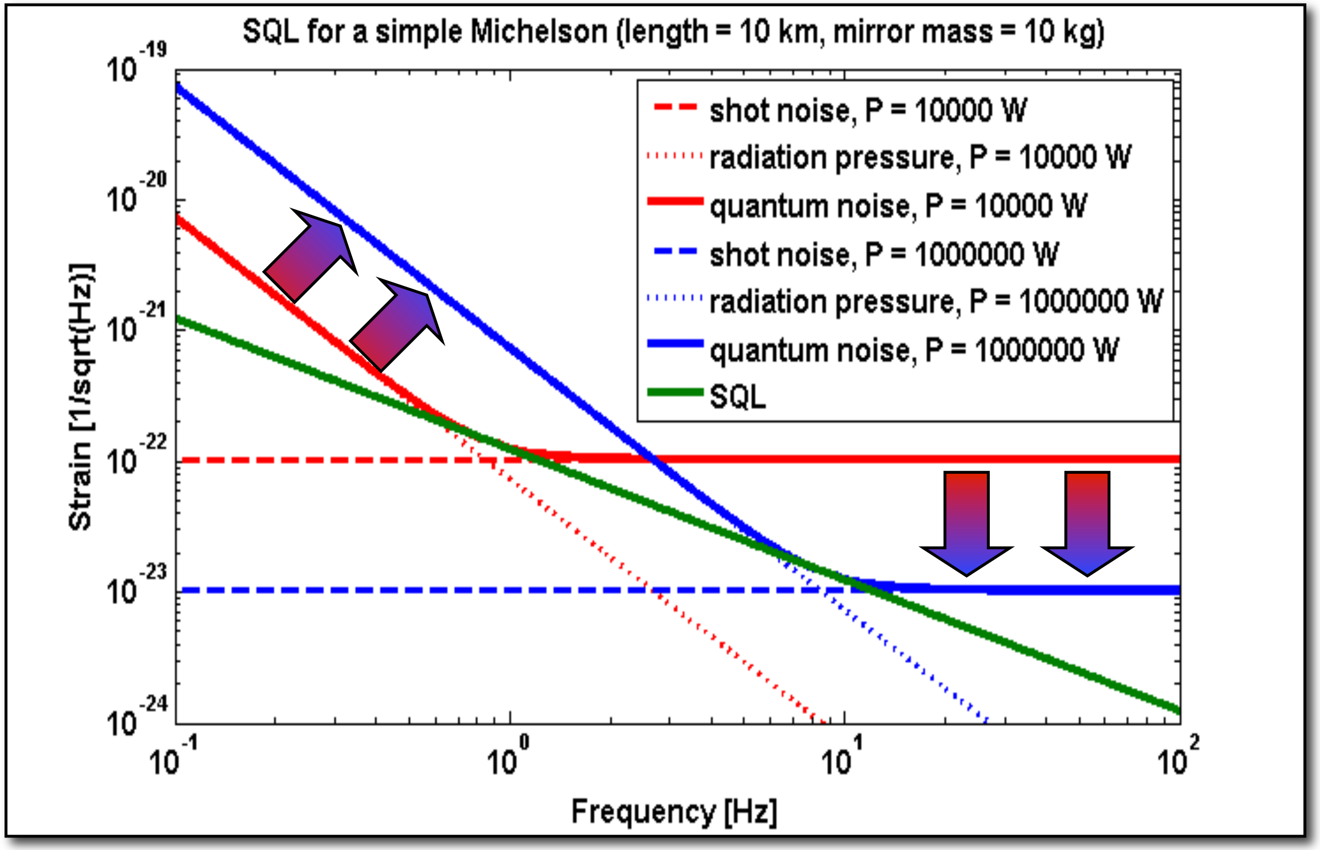
$$h_{\mathrm{sn}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

optical power

Arm length

Mirror mass

$$h_{\mathrm{rp}}(f) = \frac{1}{m f^2 L}\sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$



- The SQL is the minimal sum of shot noise and radiation pressure noise.
- Using a classical quantum measurement the SQL represents the lowest achievable noise.

V.B. Braginsky and F.Y. Khalili: Rev. Mod. Phys. 68 (1996)

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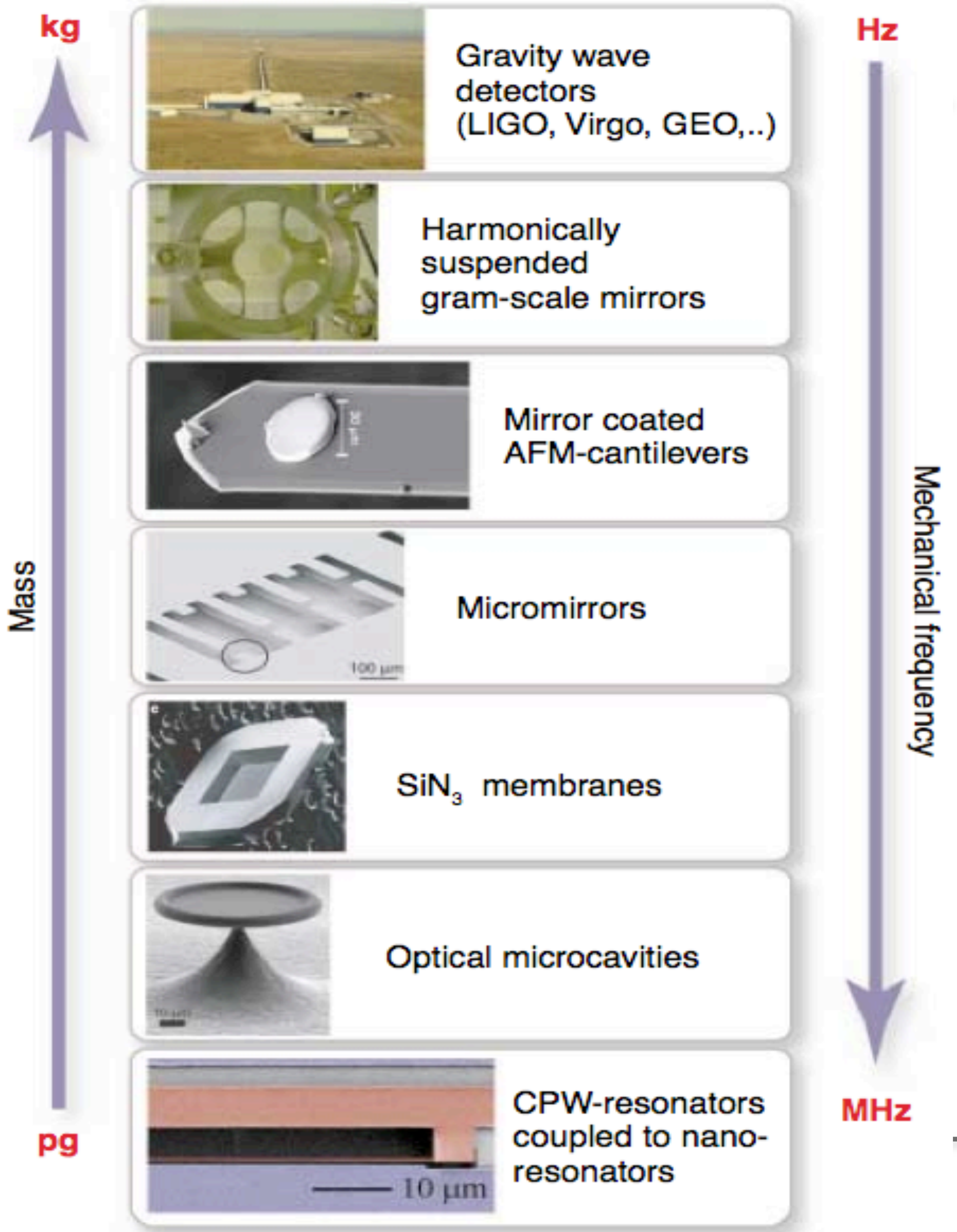
Scales of SQL Experiments

Fig. 3. Experimental cavity optomechanical systems. (**Top** to **Bottom**) Gravitational wave detectors [photo credit LIGO Laboratory], harmonically suspended gram-scale mirrors (*28*), coated atomic force microscopy cantilevers (*29*), coated micromirrors (*14*, *15*), SiN_3 membranes dispersively coupled to an optical cavity (*31*), optical microcavities (*13*, *16*), and superconducting microwave resonators coupled to a nanomechanical beam (*33*). The masses range from kilograms to picograms, whereas frequencies range from tens of megahertz down to the hertz level. CPW, coplanar waveguide.

T.J. Kippenberg and K. J. Vahala, Science 321 (2008) 1172



The Mathematical approach

- Quantitative description of an electrical field ***E*** at the position ***r*** and time *t*:

$$\boxed{\boldsymbol{E}(\boldsymbol{r},t) = E_0\left[a(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}\omega t} - a(\boldsymbol{r})^*\mathrm{e}^{+\mathrm{i}\omega t}\right]\boldsymbol{p}(\boldsymbol{r})}$$

$$\boxed{a(\boldsymbol{r}) = a_0(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r})}}$$

complex amplitude

phase

angular frequency

polarisation

- We can now introduce two new 'properties':

$$X_1(\boldsymbol{r}) = a^*(\boldsymbol{r}) + a(\boldsymbol{r})$$

$$X_2(\boldsymbol{r}) = \mathrm{i}\left[a^*(\boldsymbol{r}) - a(\boldsymbol{r})\right]$$

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The Mathematical approach (2)

$$X_1(\boldsymbol{r}) = a^*(\boldsymbol{r}) + a(\boldsymbol{r})$$ amplitude quadrature

$$X_2(\boldsymbol{r}) = \mathrm{i}\,[a^*(\boldsymbol{r}) - a(\boldsymbol{r})]$$ phase quadrature

- Using X_1 and X_2 we can rewrite the electrical field:

$$\boxed{\boldsymbol{E}(\boldsymbol{r},t) = E_0\,[X_1 \cos(\omega t) - X_2 \sin(\omega t)]\,\boldsymbol{p}(\boldsymbol{r},\mathrm{t})}$$

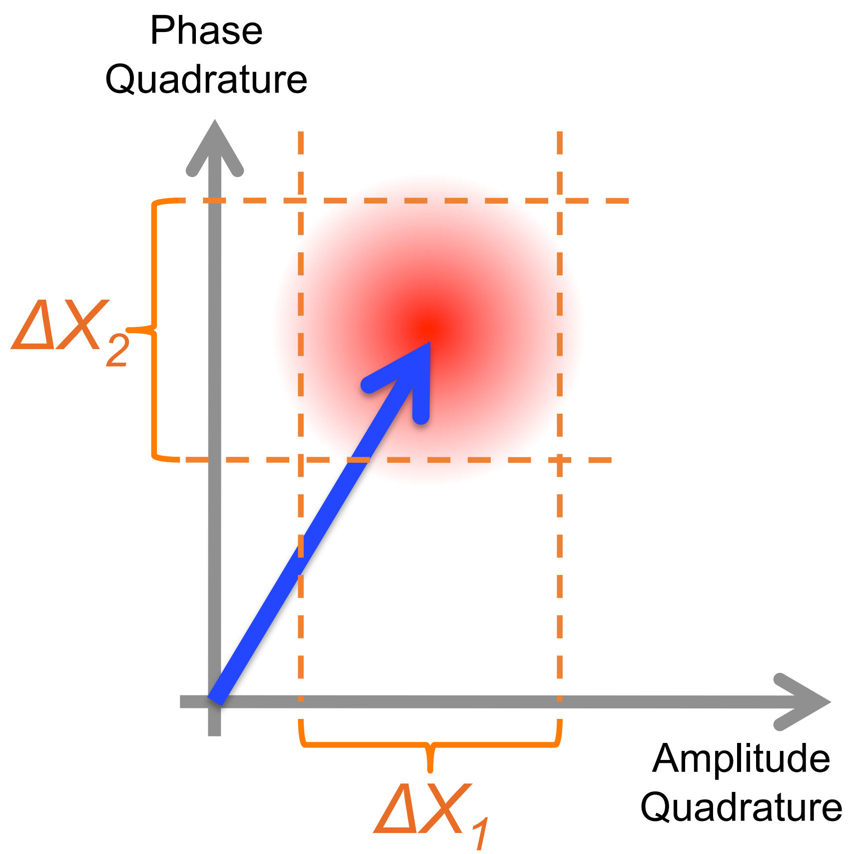
- Finally, we have to introduce a quantisation of the electrical field:

$$\hat{X}_1 = \hat{a}^\dagger + \hat{a}$$ amplitude quadrature operator

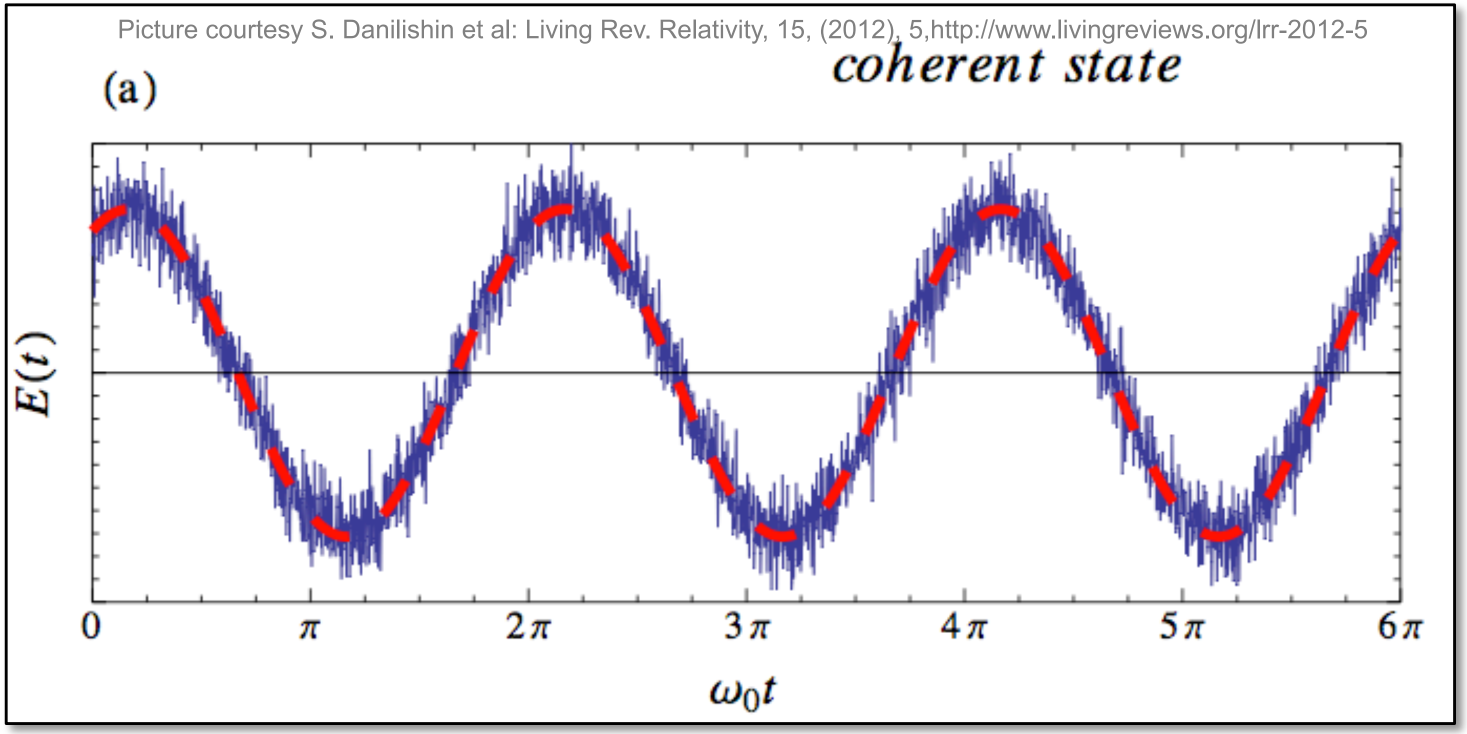
$$\hat{X}_2 = \mathrm{i}\left(\hat{a}^\dagger - \hat{a}\right)$$ phase quadrature operator

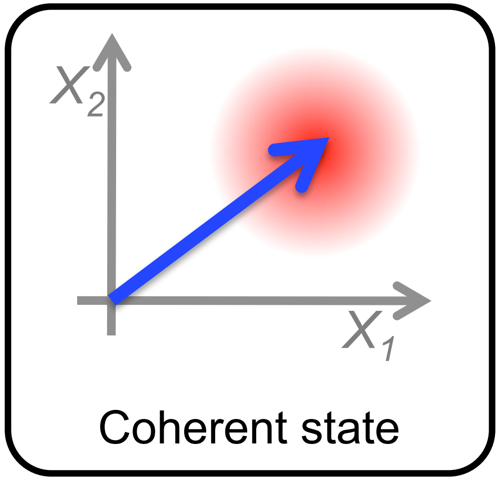
Introducing the Ball-on-a-stick concept

- Let's try to convert the quadrature idea into a picture:
- A laser sends out light with an average frequency and amplitude. => This is the blue arrow.
- However, individual photons have a uncertainty, i.e. may have slightly different frequency or amplitude. => This is indicated by the red ball.



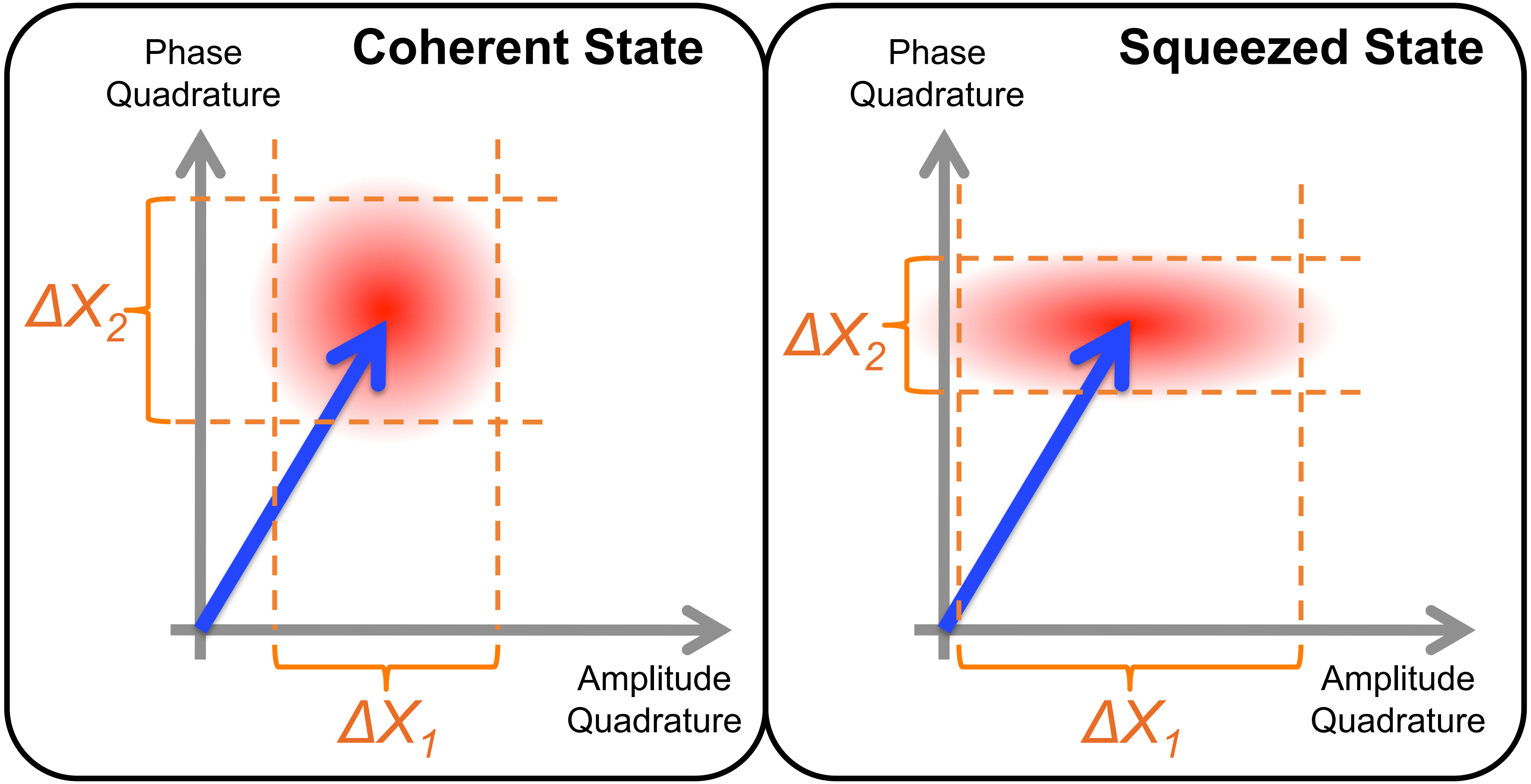
Ball-on-a-Stick vs Timeseries





- The red dashed line in left hand picture corresponds to the blue arrow on the right hand plot.
- The fluctuations of the blue trace around the red-dashed curve in the left hand plot correspond to the red area in the right plot.

Ball on a stick and Heisenberg



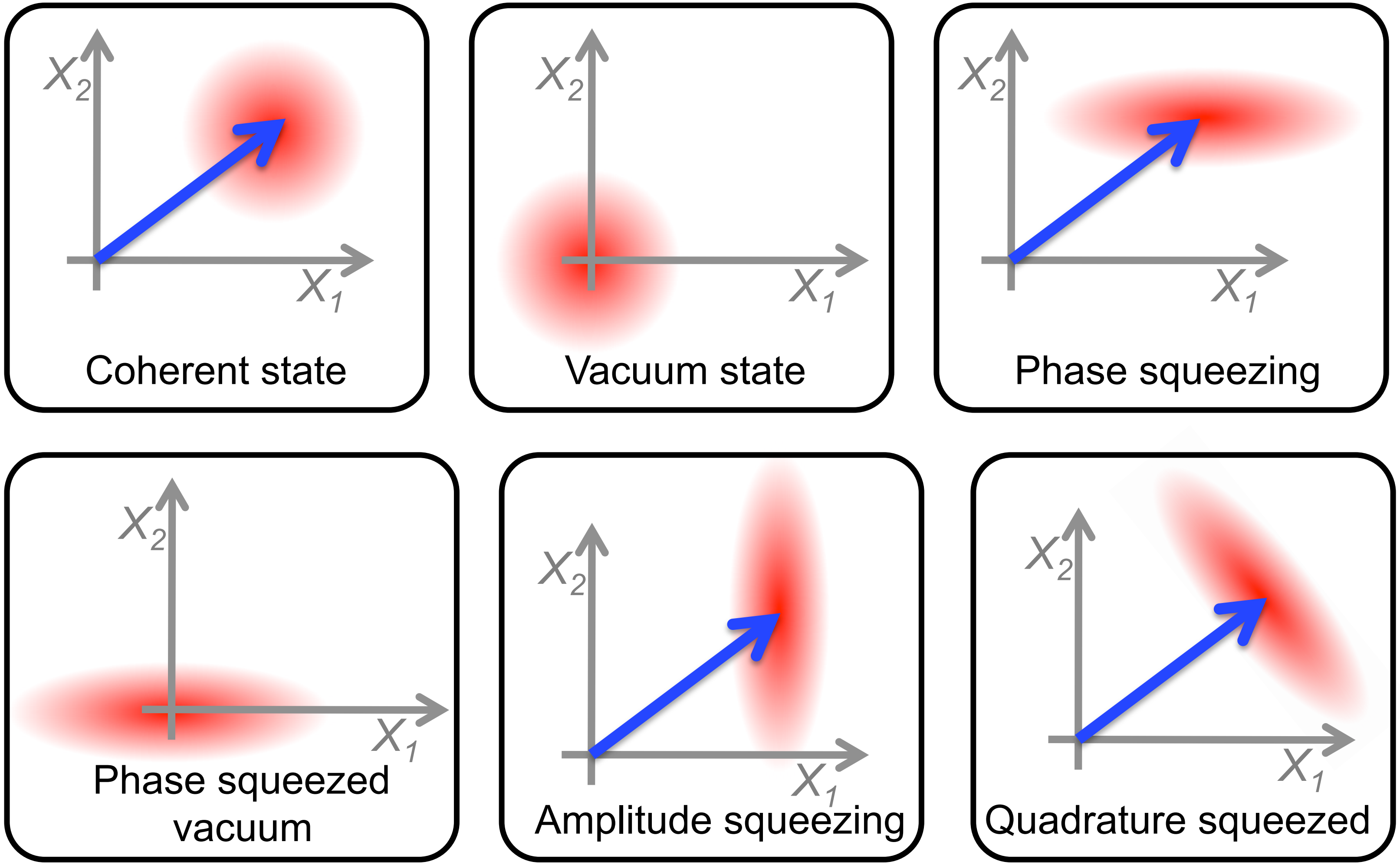
Heisenberg uncertainty principle: there is a minimal area of ball

$$\Delta\hat{X}_1 \Delta\hat{X}_2 \geq 1$$

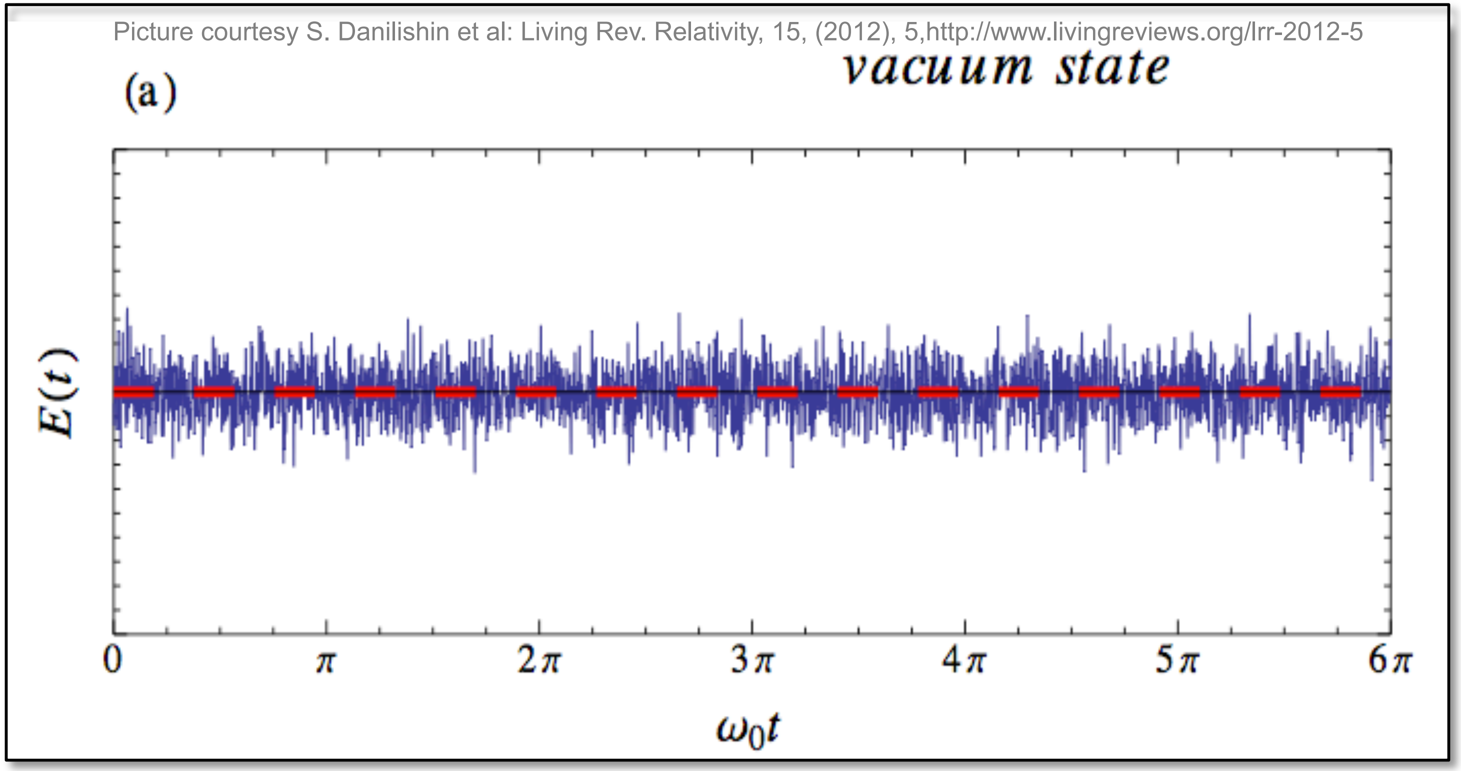
Time series of phase and amplitude squeezing

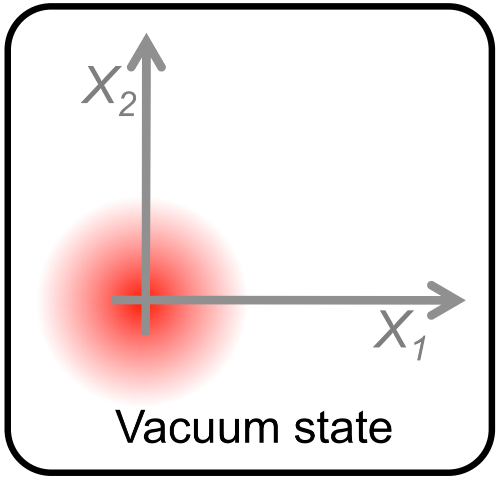
Task for you:
Sketch a time series of a) a phase squeezed state and b) of an amplitude squeezed state.

Examples of ball on the stick



Vacuum fluctuations

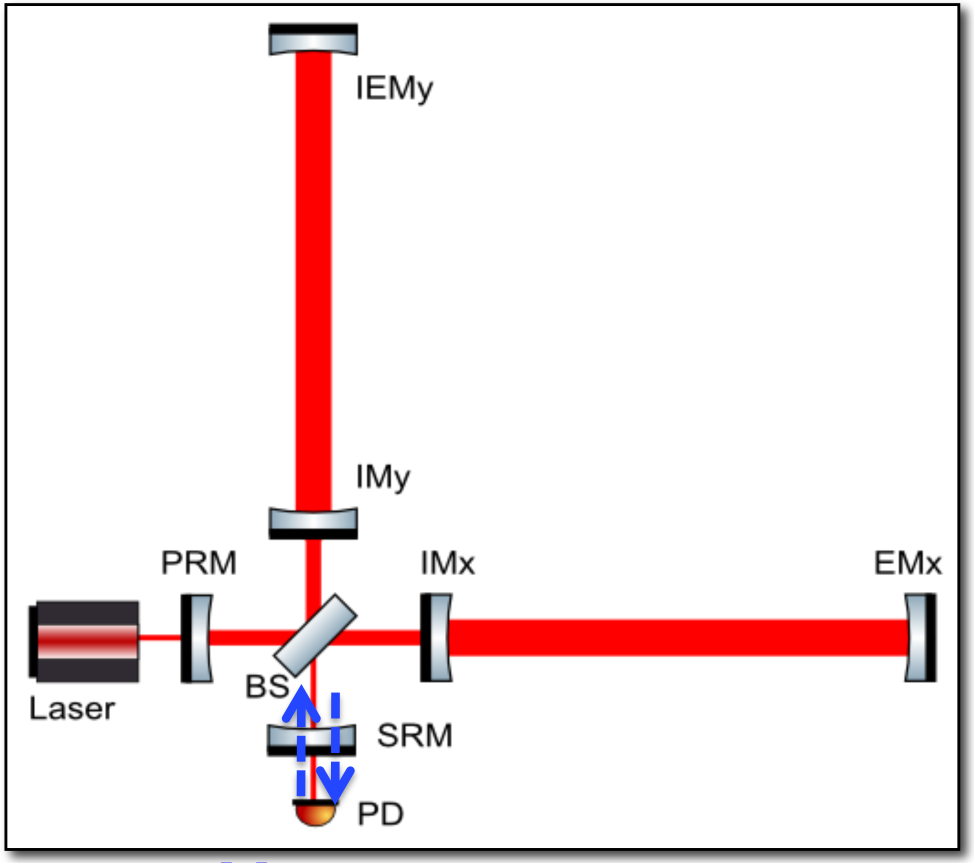




- Even if there is 'no' light, vacuum fluctuations are always present.
- Vacuum fluctations enter our systems via any 'open port'.

Vacuum Fluctuations

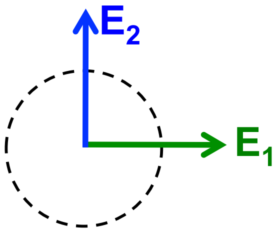
- How does 'Ball-on-the-stick' fits into the interferometer picture?
- An intuitive picture is to consider:
  - A vacuum state is entering the interferometer from the photodiode.
  - It is then 'reflected from the interferometer' and is detected together with the GW signal on the main photodiode.



Vacuum fluctuations

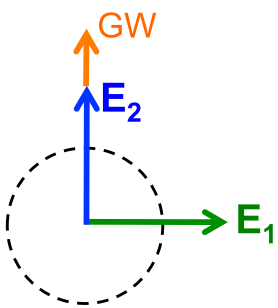
GW signal in Quadrature picture

Shot noise shows up in both quadratures

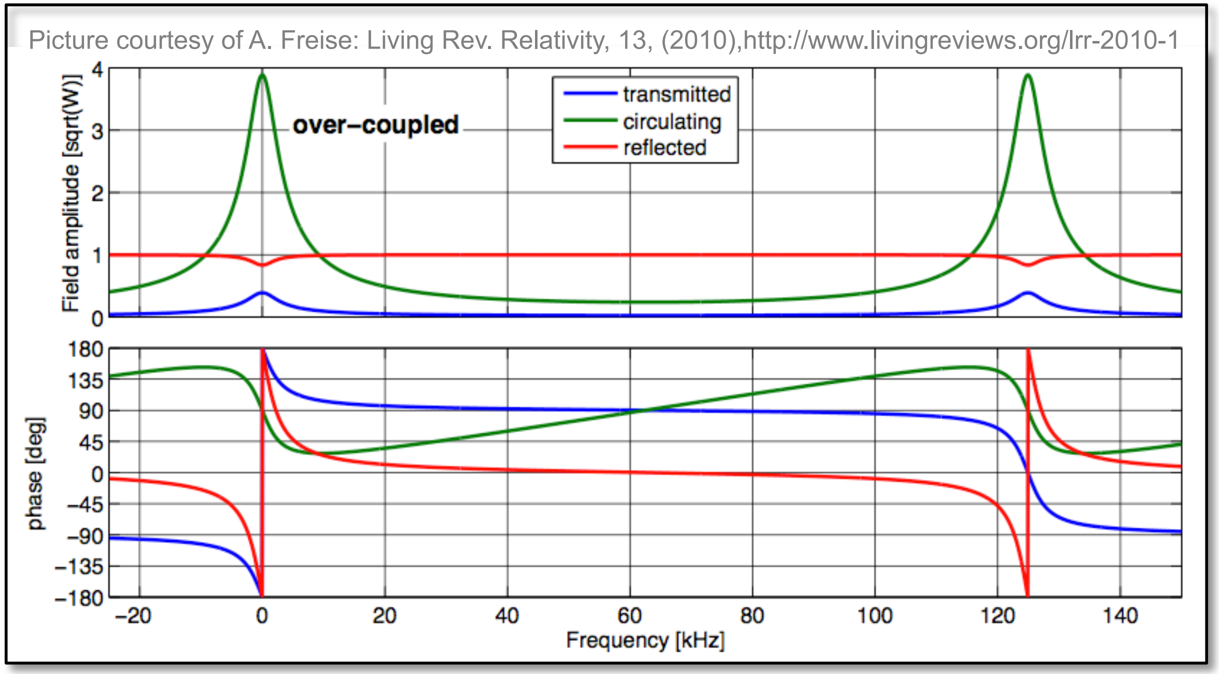


No correlations

GW signal shows up in phase quadrature



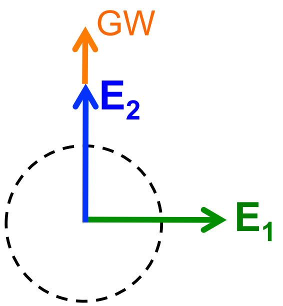
No correlations



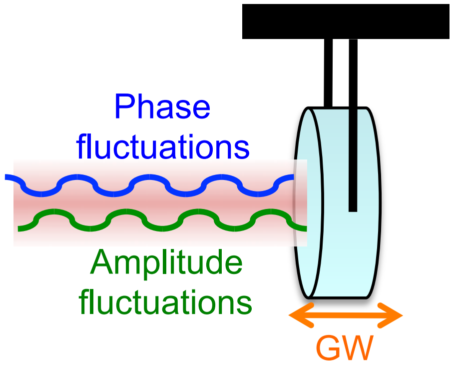
- If you change the length of a cavity around its resonance, the slope of the amplitude is zero, while the slope of the phase is maximal. Therefore, GW signal adds to phase quadrature.

Where does Radiation pressure noise come from?

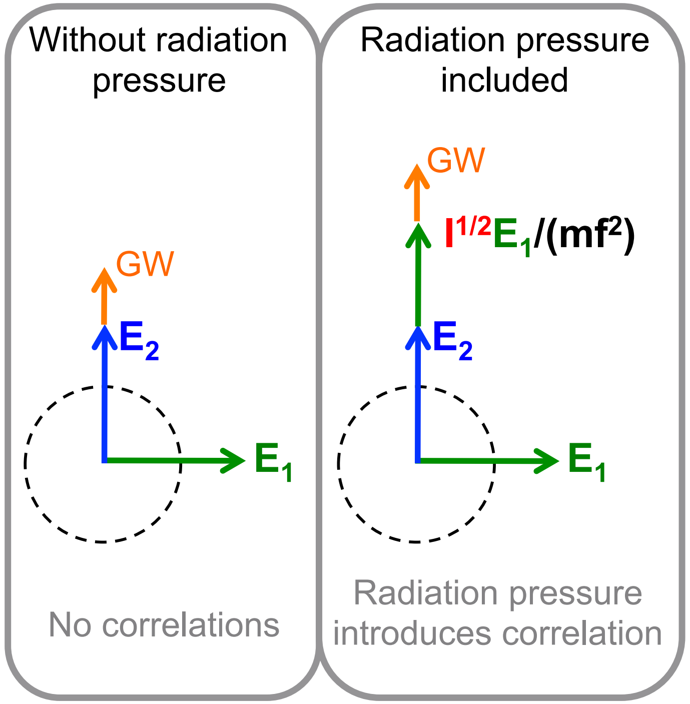
Without radiation pressure

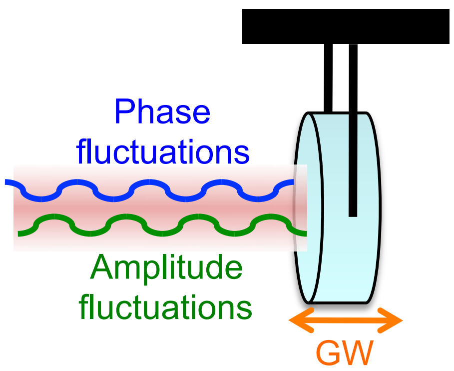


No correlations



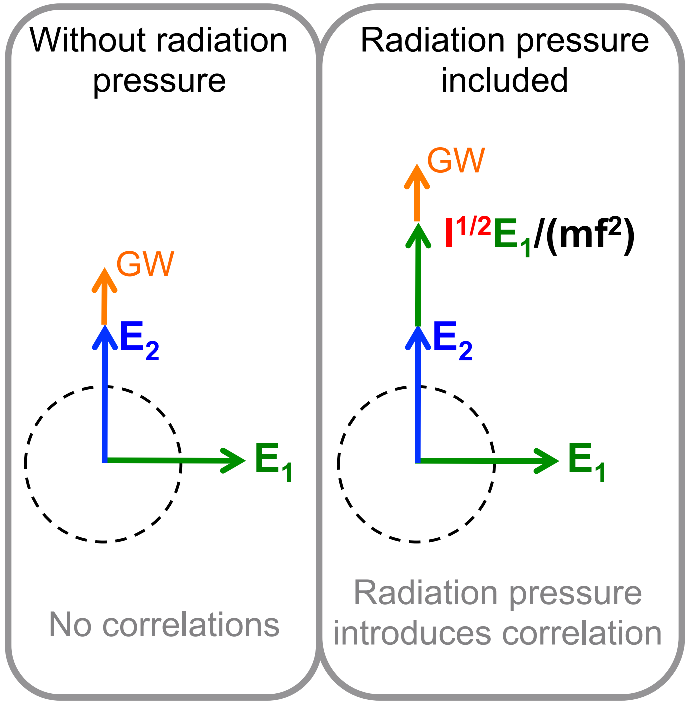
Where does Radiation pressure noise come from?

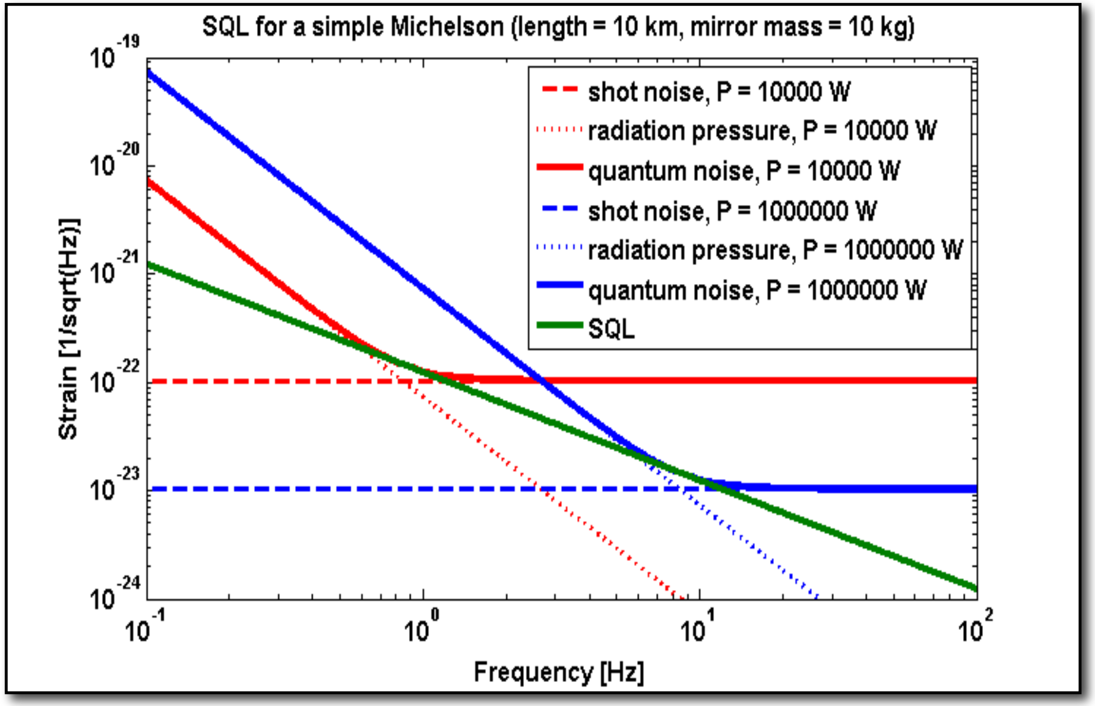




- Amplitude fluctuations act onto suspended mirror.
- Mirror is moved and gives contribution in the phase quadrature.
- This new contribution:
  - is correlated to E_1
  - depends on the mirror mass
  - Its magnitude goes with $1/f^2$

Where does Radiation pressure noise come from?





- At high frequencies radiation pressure is negilicable (due to $1/f^2$).
- At low frequencies radiation pressure is dominant

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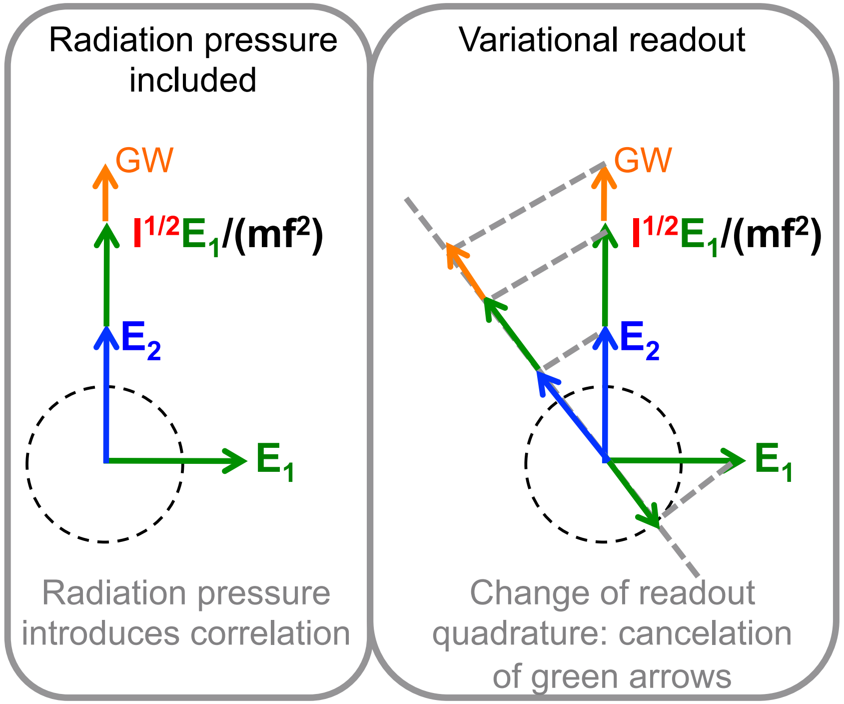
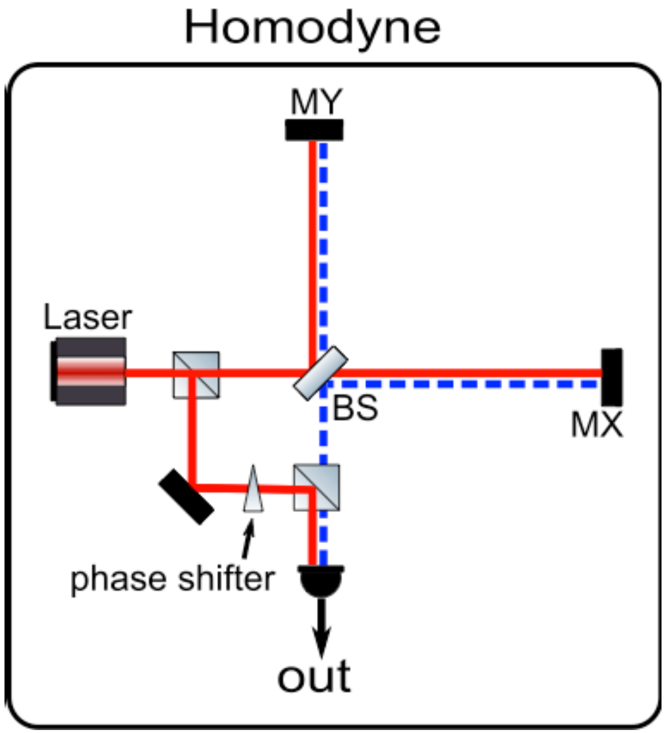
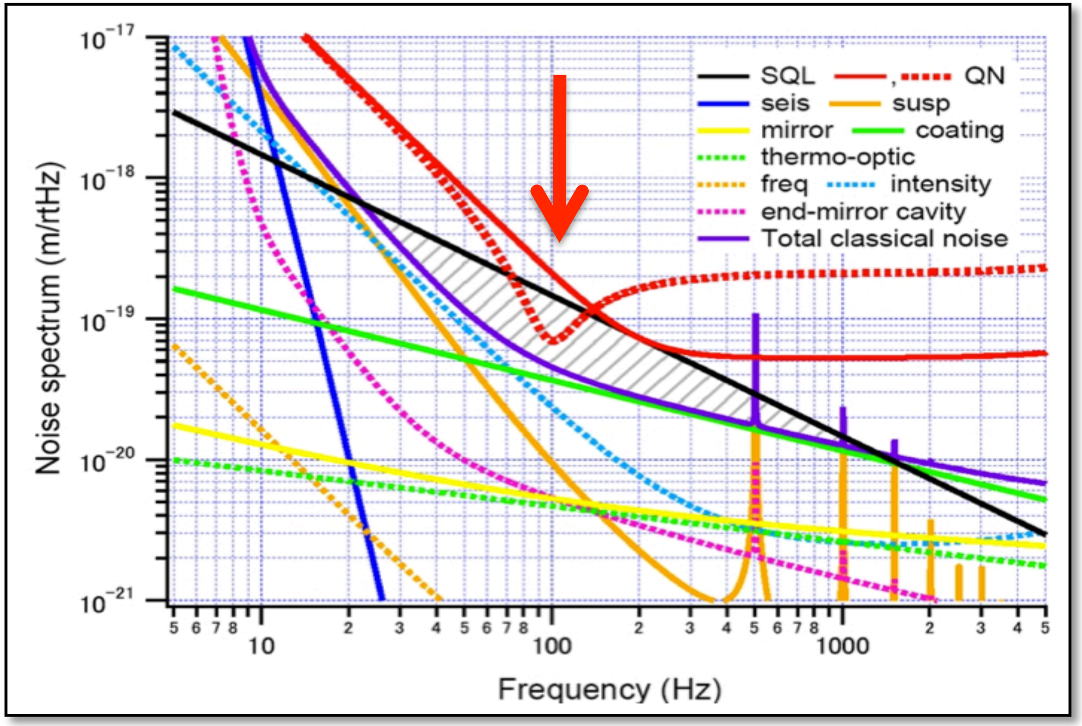
Surpassing the SQL at AEI-10m: Variational Readout

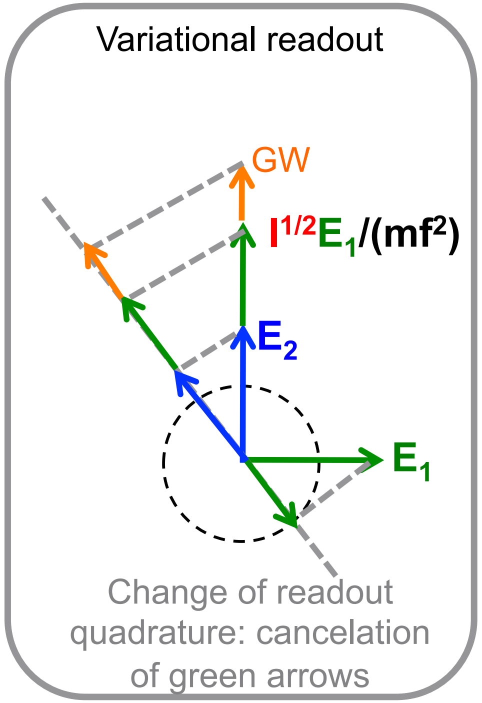




- Variational Readout enables us to partly cancel radiation pressure noise by selecting the appropriate readout quadrature.
- Readout quadarture (angle) can be chosen by phase shifter

Surpassing the SQL at AEI-10m-Interferometer





- Using Variational readout, we will be able to completely cancel the radiation pressure noise at **ONE** frequency (100Hz) and surpass the SQL a factor 2 to 3.
- To cancel the radiation pressure noise a all frequencies, we would need a frequency dependent angle of the readout quadrature

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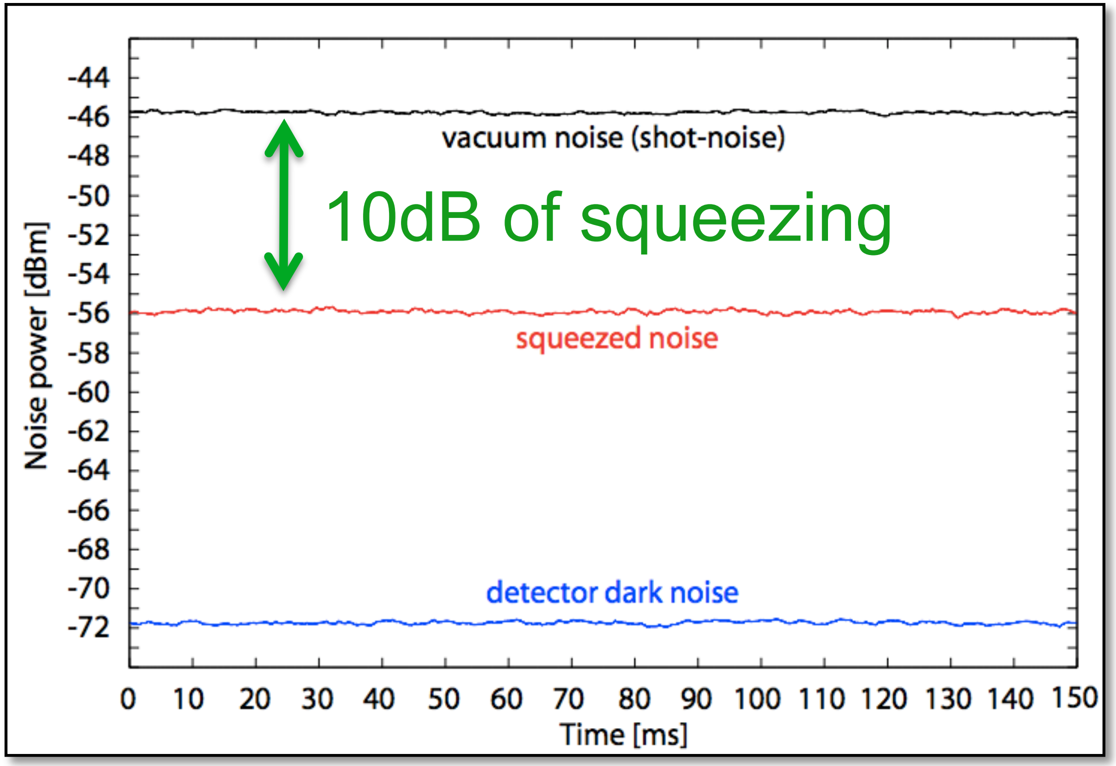
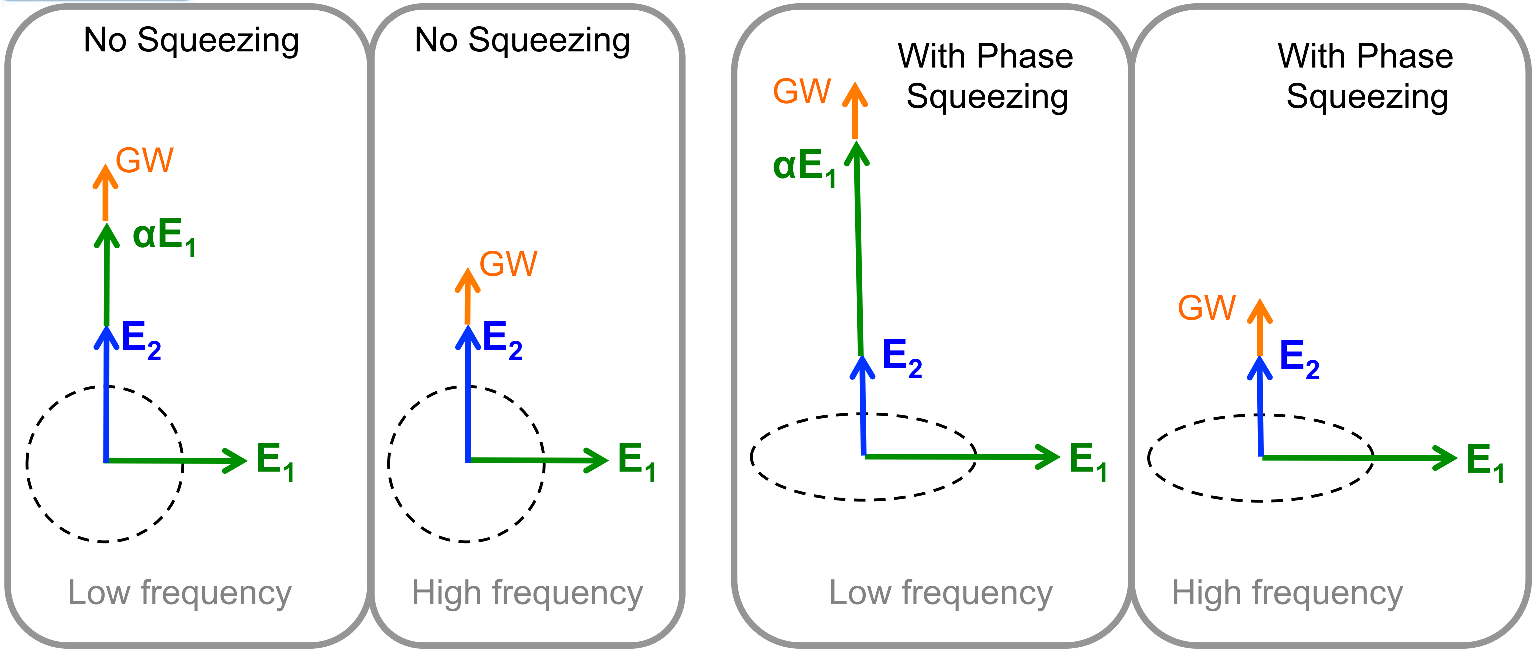
What squeezed light is available ?

- Over the past decade, squeezing made incredible progress.
- We have now all we need:
  - Squeezing at all frequencies of interest (as low as 1 Hz)
  - Squeezing factors > 10dB, improves the quantum noise by a factor 3 (or is equivalent to a power increase of 10)

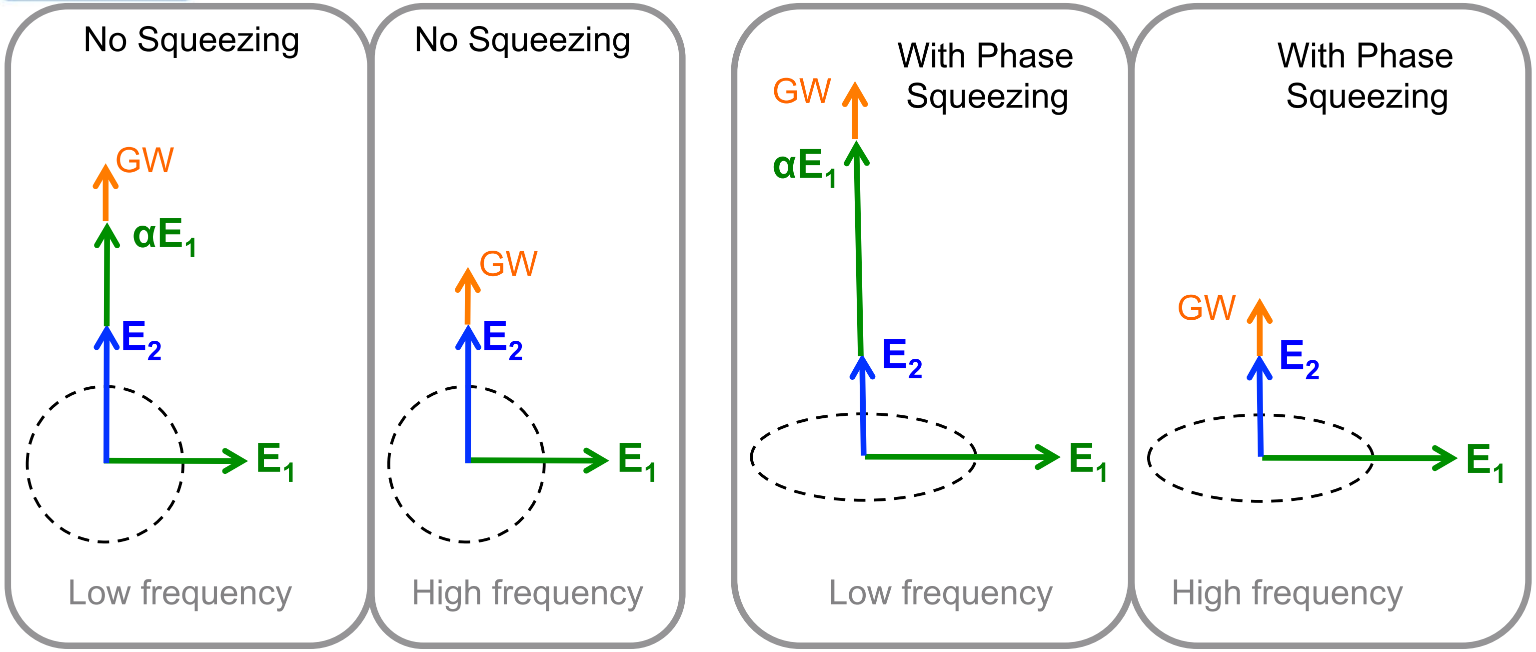


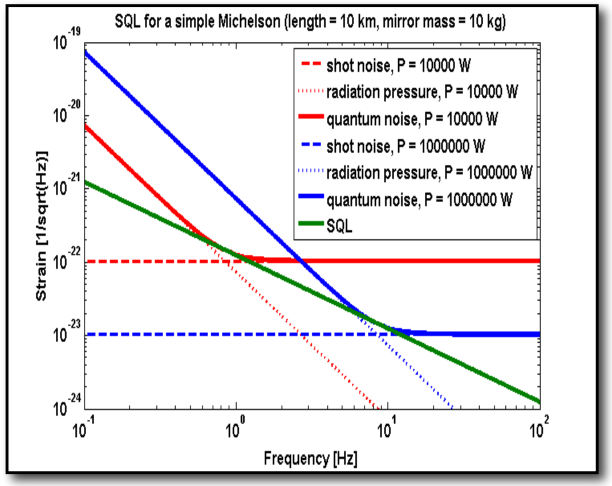
Vahlbruch *et al.*, PRL. 100, 033602

Injecting squeezed light into an interferometer



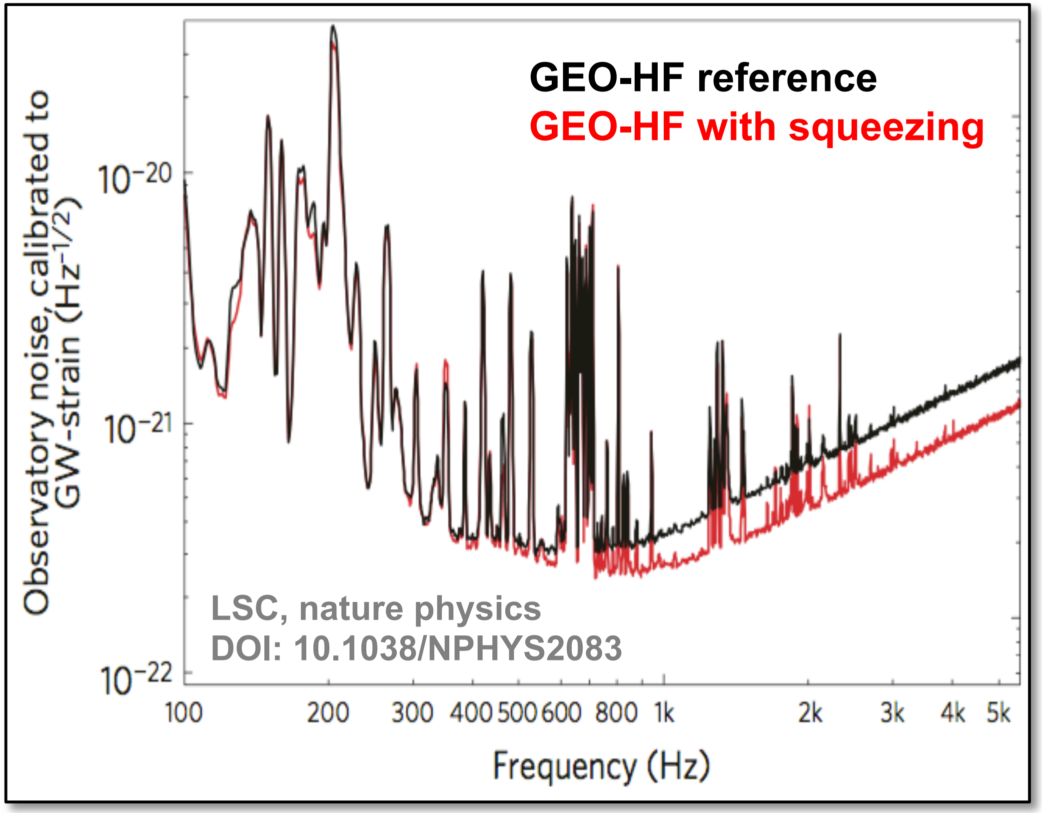
Injecting squeezed light into an interferometer

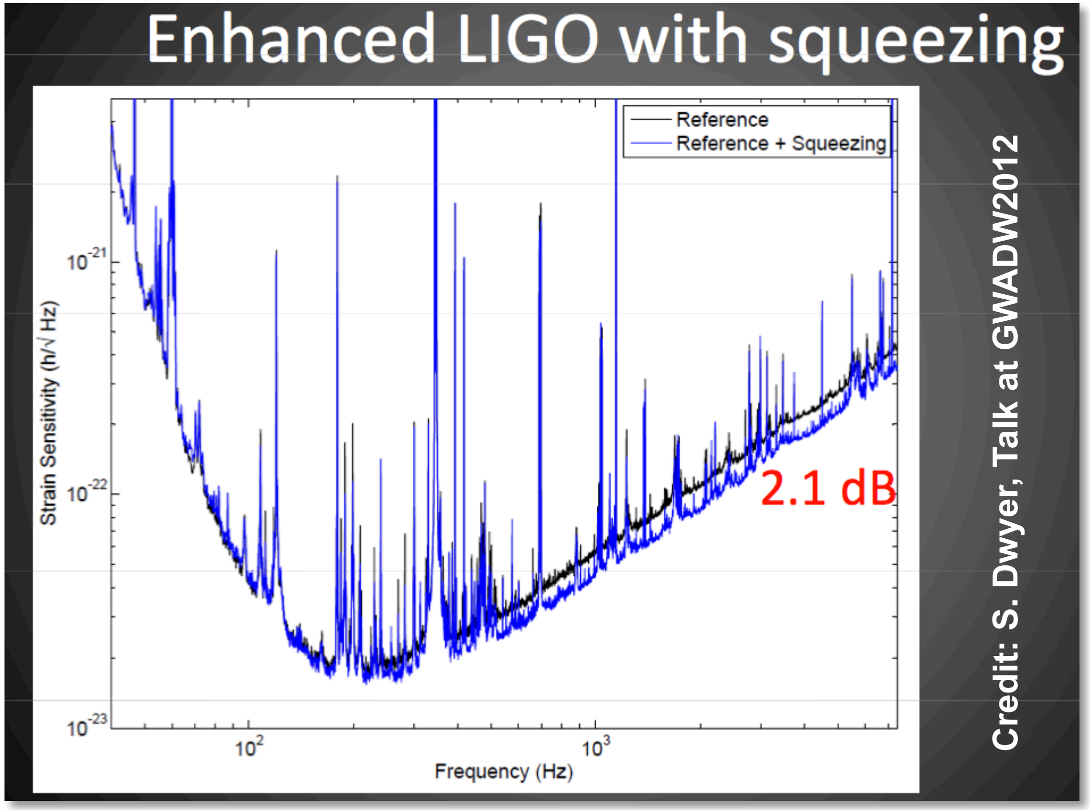




- Injecting phase squeezing into detector output:
  - High frequency sensitivity improved ☺
  - Low frequency sensitivity decreased ☹
- Phase squeezing gives in principle the same as a power increase.
- **With pure phase squeezing you cannot beat SQL!**

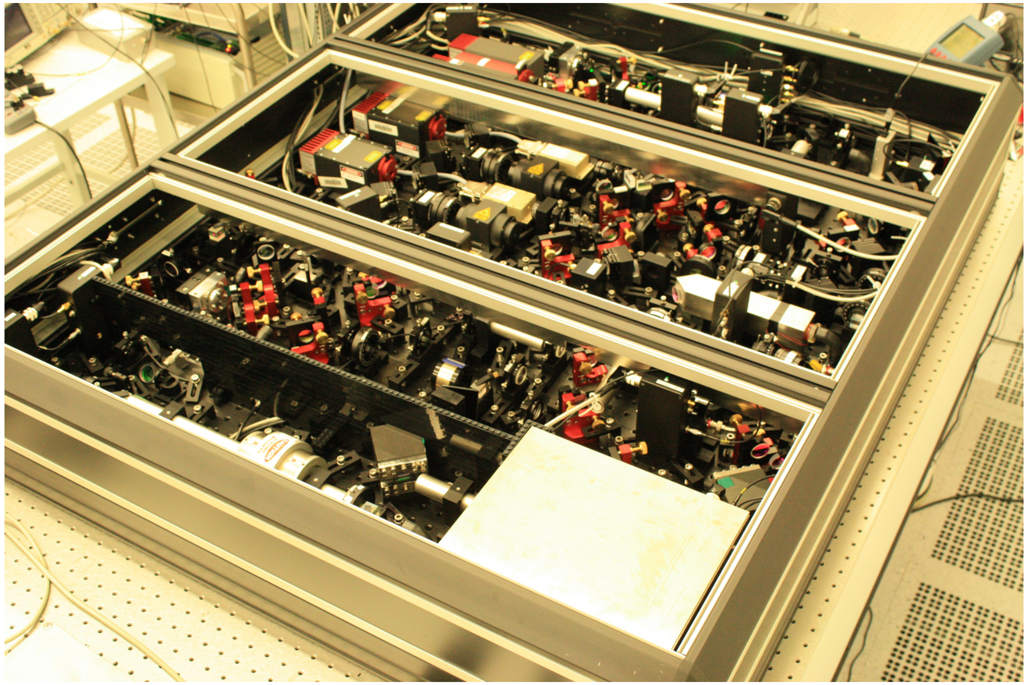
Squeezing is no Scifi. IT WORKS !!



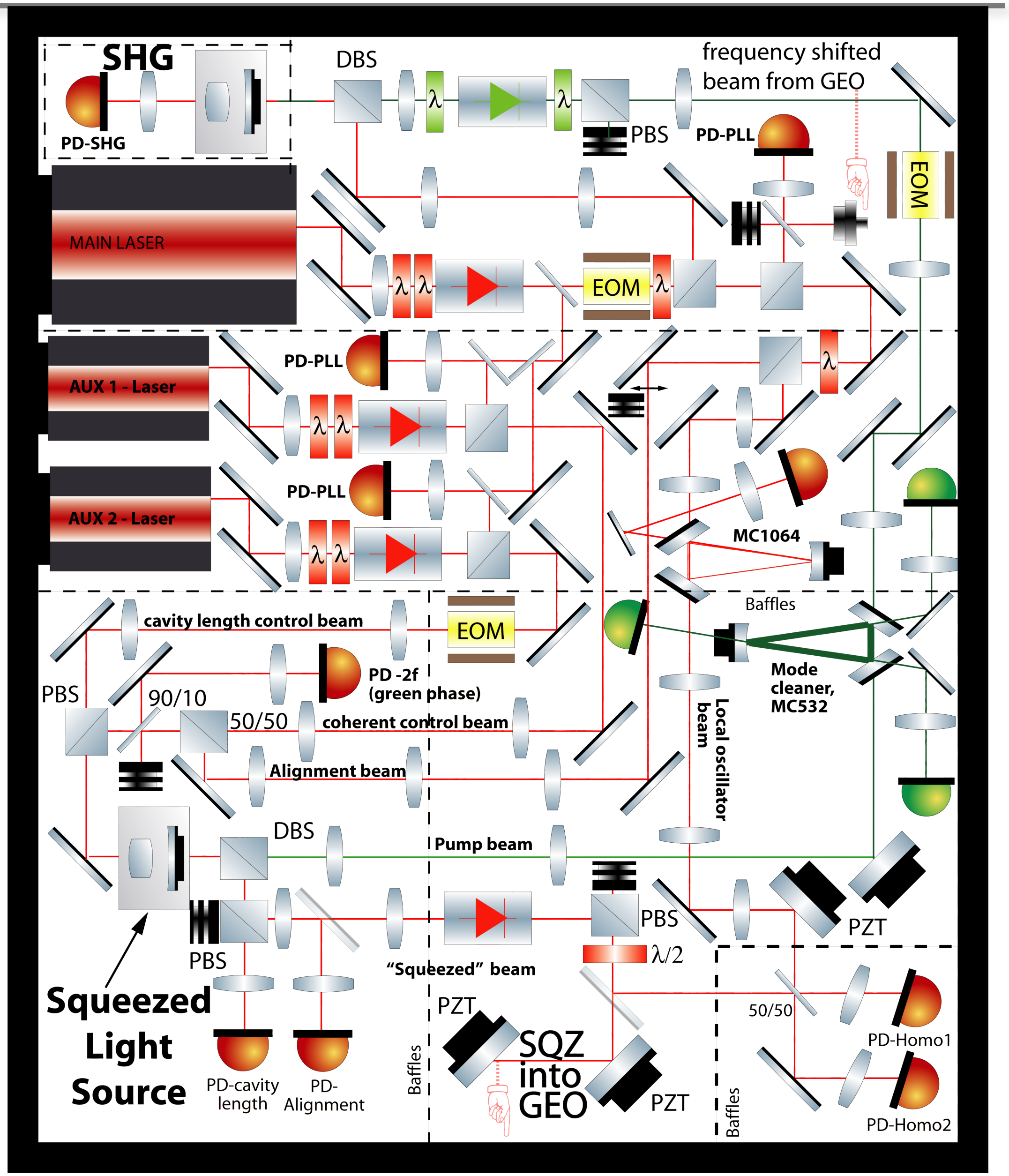


- Recently squeezing of up to 3.5dB and at frequencies as low as a few 100Hz have been demonstarted in GEO600 and Enhanced LIGO.

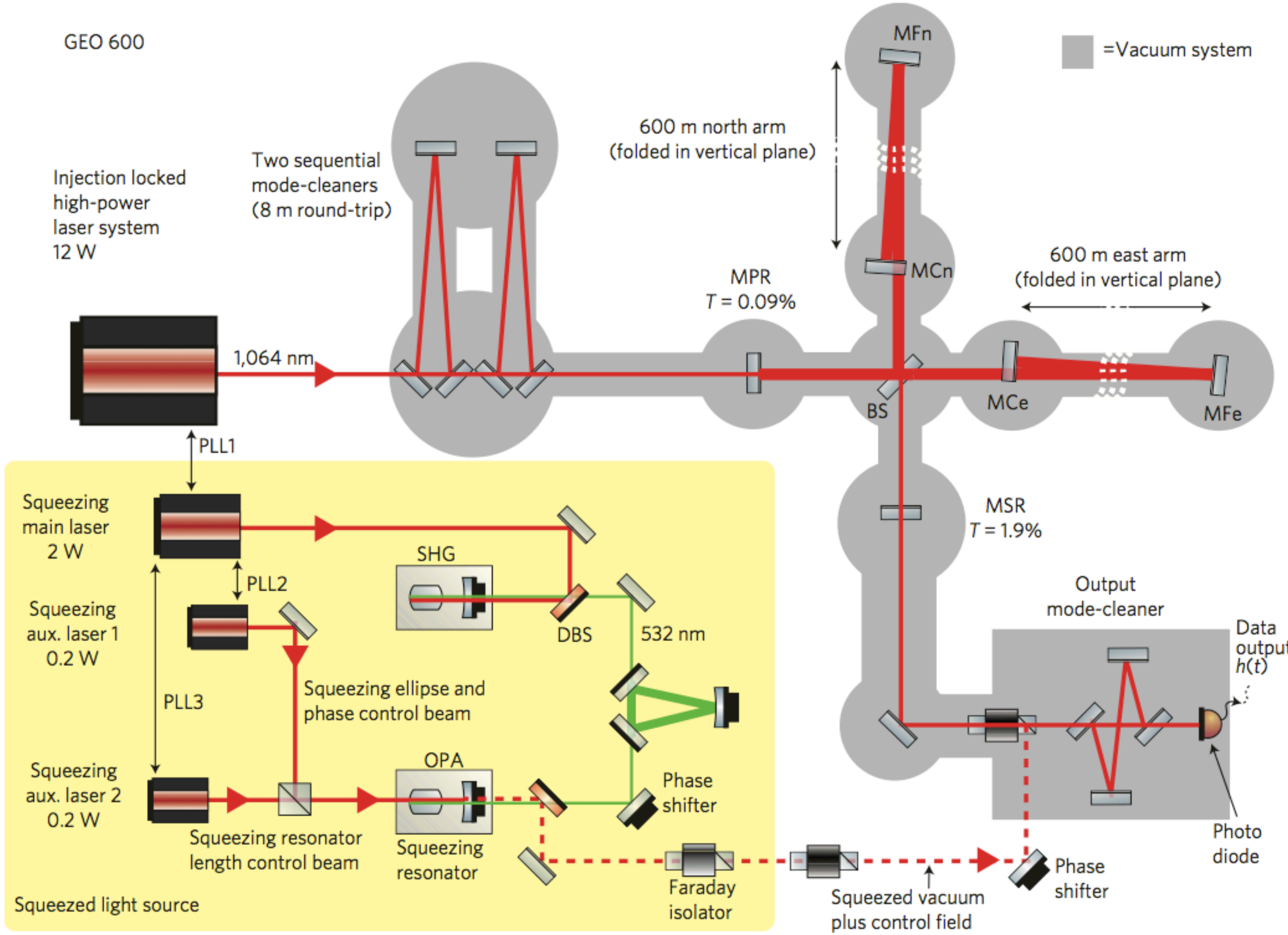
The GEO600 squeezer (schematic)



Images courtesy to GEO600 squeezing Group



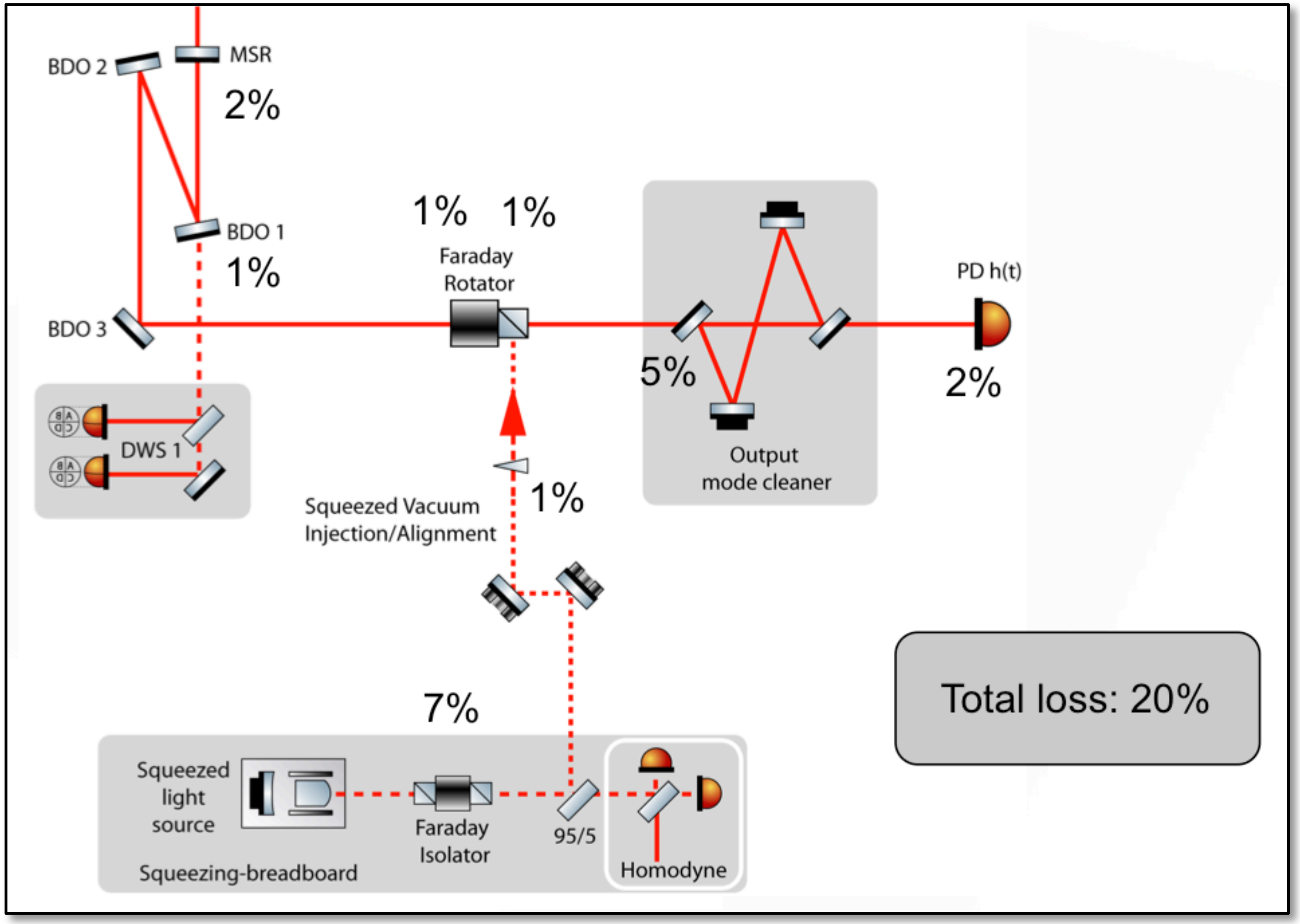
Squeezing injection in GEO



Picture taken from:
LSC, nature physics
DOI: 10.1038/NPHYS2083

How to inject squeezed light into GEO??

- Squeezed light is injected via a Faraday rotator into the back of the interferometer.
- It is then reflected from the signal recycling mirror (MSR) and detected at the main photodiode (PD).
- Squeezing requires low losses.



A. Khalaidovski: http://gw.icrr.u-tokyo.ac.jp/gwadw2010/program/2010_GWADW_Khalaidovski.ppt

Squeezing and losses

- If squeezed light is lost it is replaced again by vacuum fluctuations.
- For GEO-HF we start with 10dB. => 20% loss => effective quantum noise reduction of 6dB

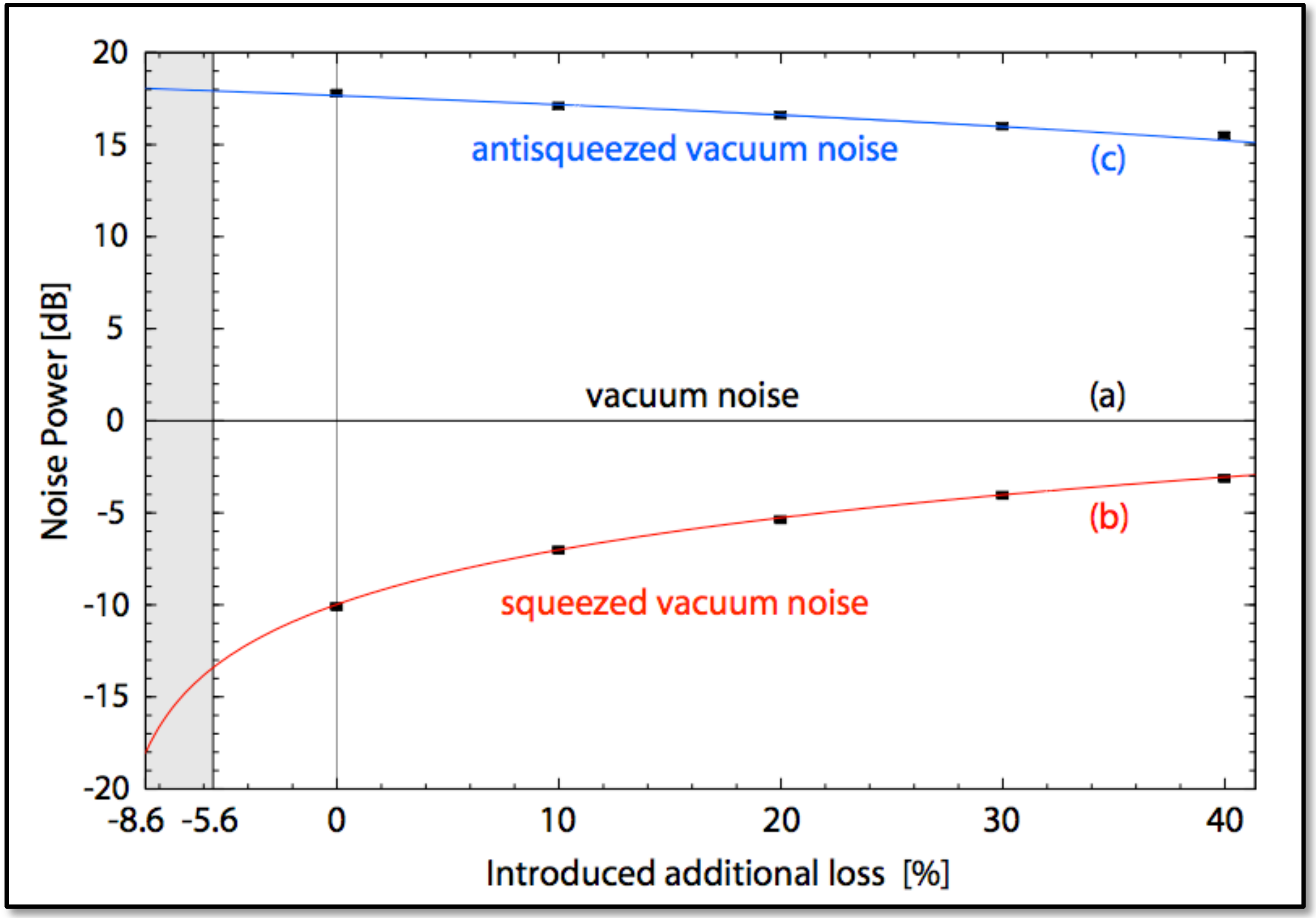
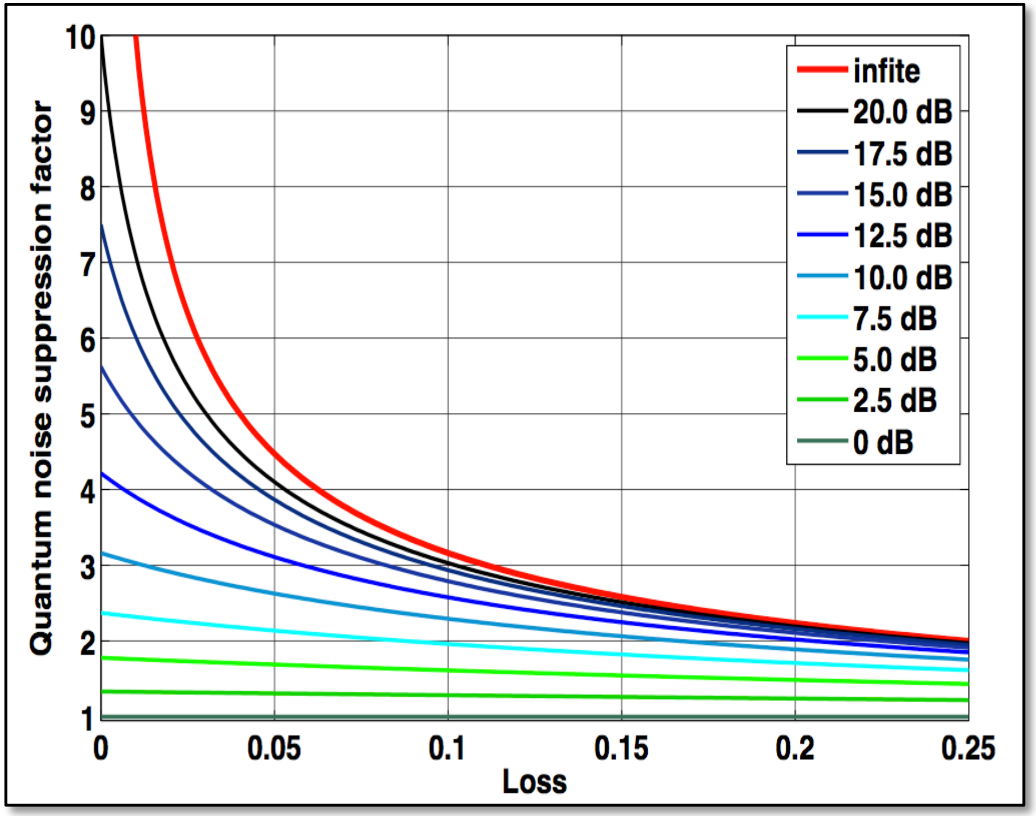


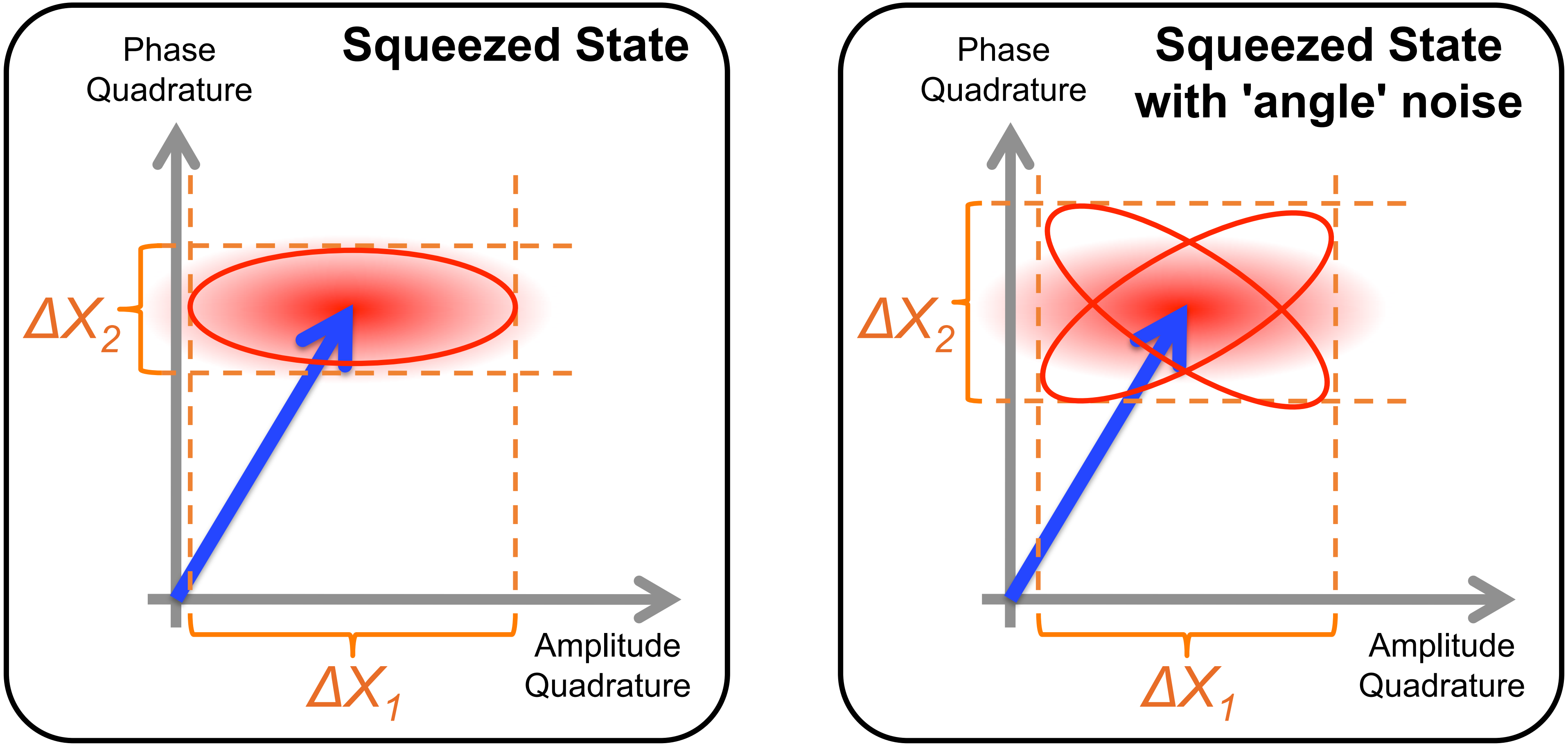
Image: H. Vahlbruch, PhD thesis.

What is the maximum Squeezing ?

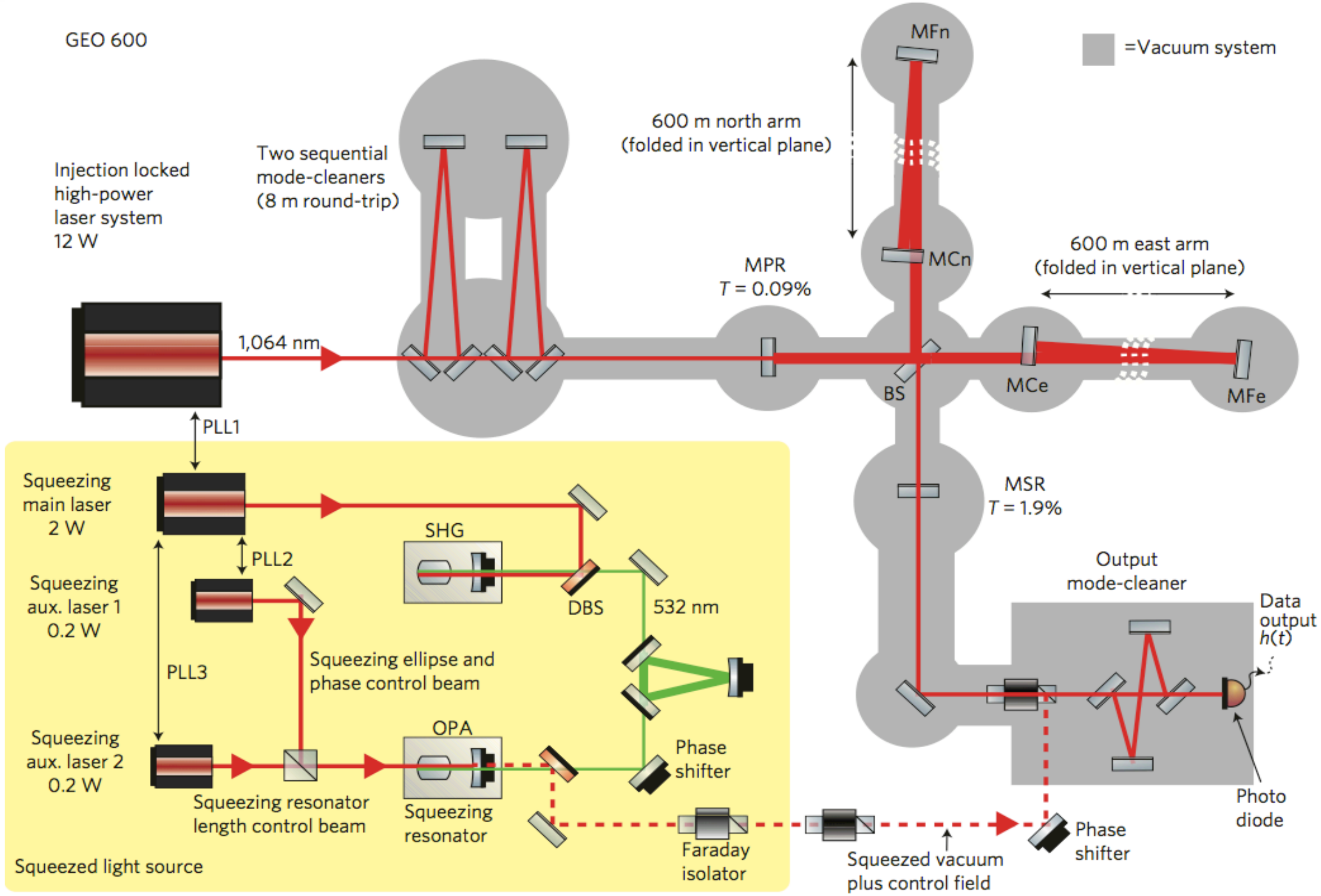
- What is the maximal noise reduction we will ever get from squeezing?
- With the current losses >0.2 we are not limited by the achievable squeezing of the sources.
- Even with an infinite squeezing level it seems hardly possible to get an improvement better than a factor 3.



Wobbeling of the squeezing angle



Squeezing injection in GEO



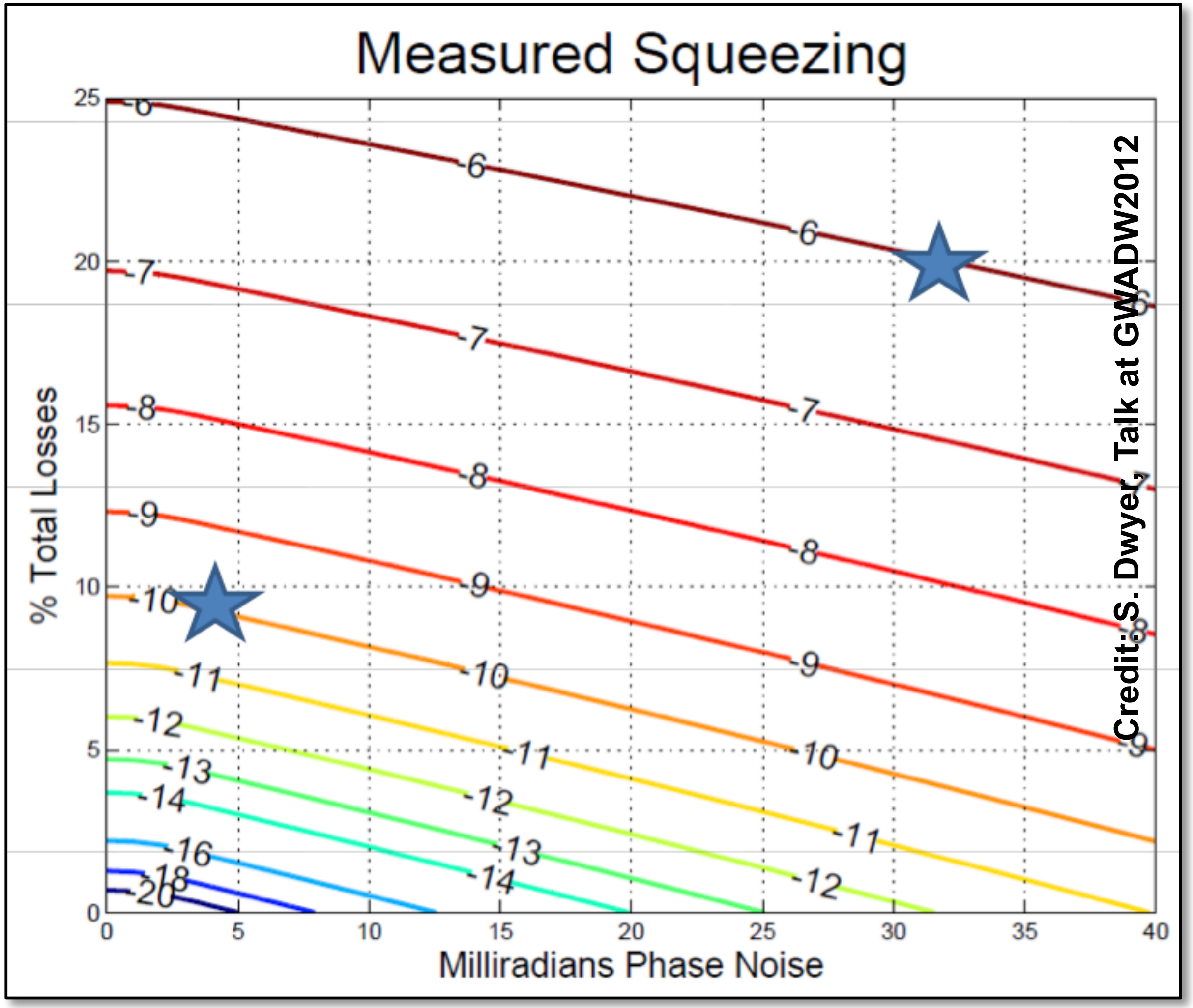
Picture taken from: LSC, nature physics DOI: 10.1038/NPHYS2083

Tasks for you:

1. Where in the drawing above do you think the phase noise is introduced?
2. Where do you think you can find the best error signal to correct it?

Phase noise on squeezed light

- Need to stabilise the squeezing angle very accurately.
- PLEASE NOTE: In this case it is not the audioband frequencies that are important, but the rms!
- Key-problem will be to find good error signals for this.



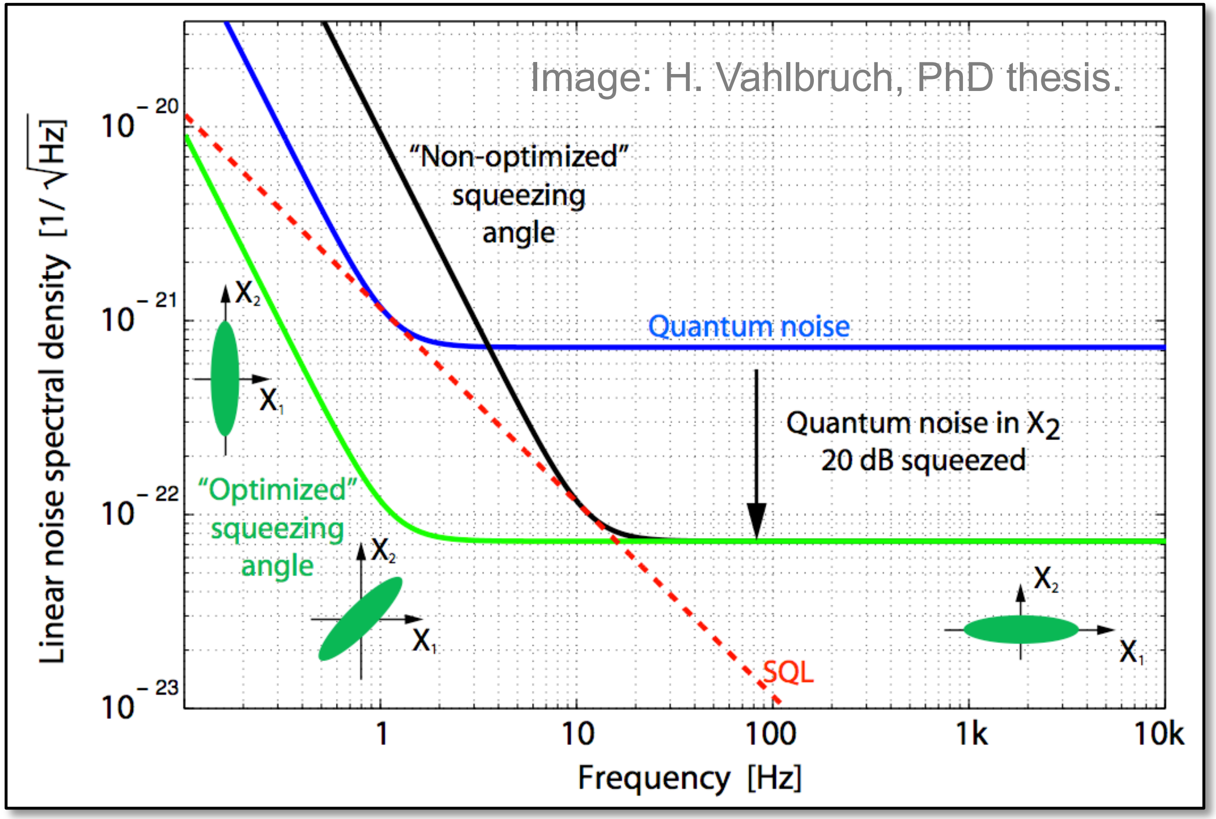
Can squeezing be used to surpass the SQL?

- So far the squeezing we have considered only helped for shot noise, but increased the radition pressure noise. ☹
- Such squeezing provides the same sensitivity improvement as an increase in optical power would do.
- So is there any chance to improve the sensitivity at alll frequencies by injecting squeezed states? That would allow us to achieve what we are 'really' after, i.e. surpass the SQL and do QND measurements

Frequency dependent Squeezing

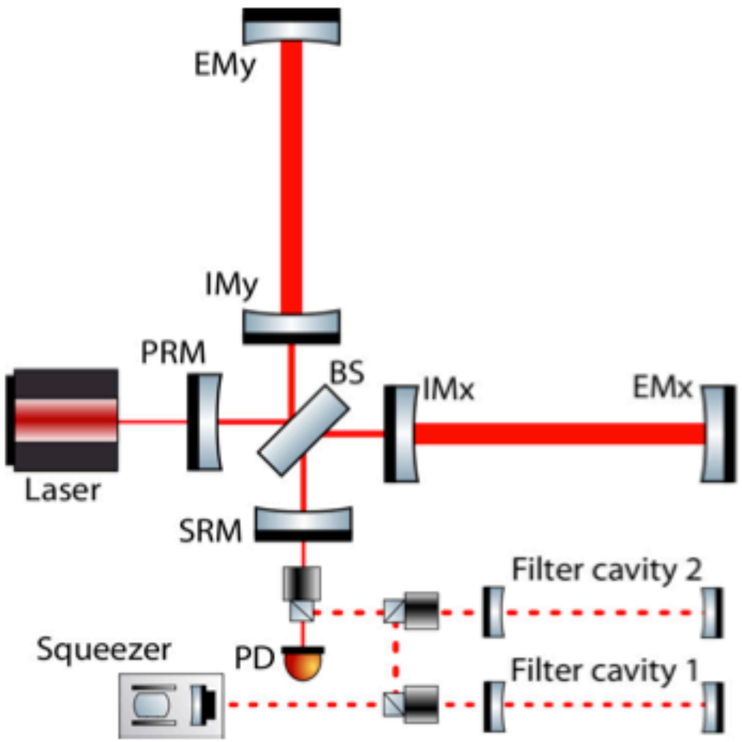
If squeezed light should reduce shot noise and radiation pressure noise, you need a frequency dependent squeezing angle:

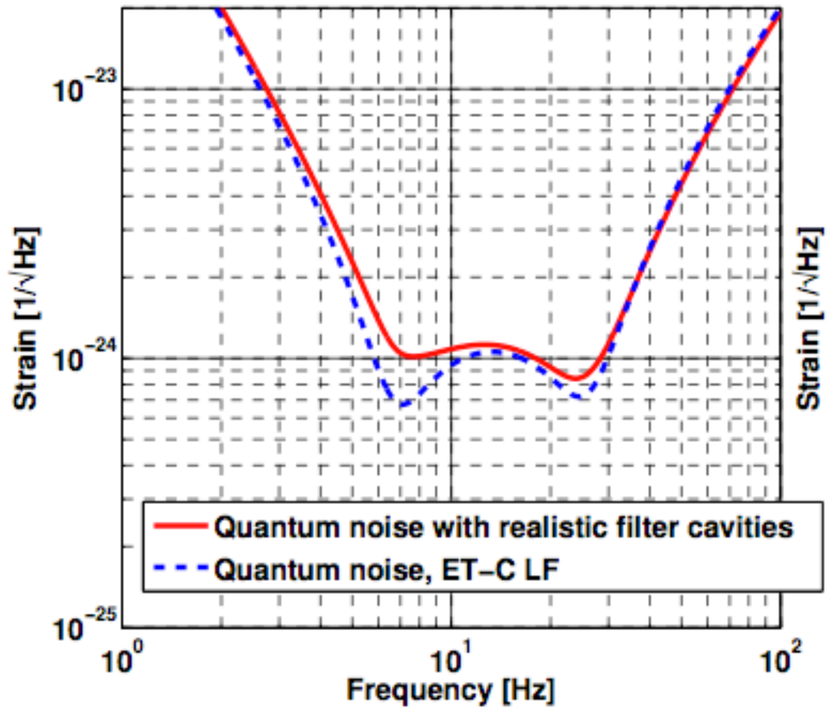
- Phase squeezing at high frequencies.
- Amplitude squeezing at low frequencies



With frequency dependent squeezing it is possible to surpass the SQL.

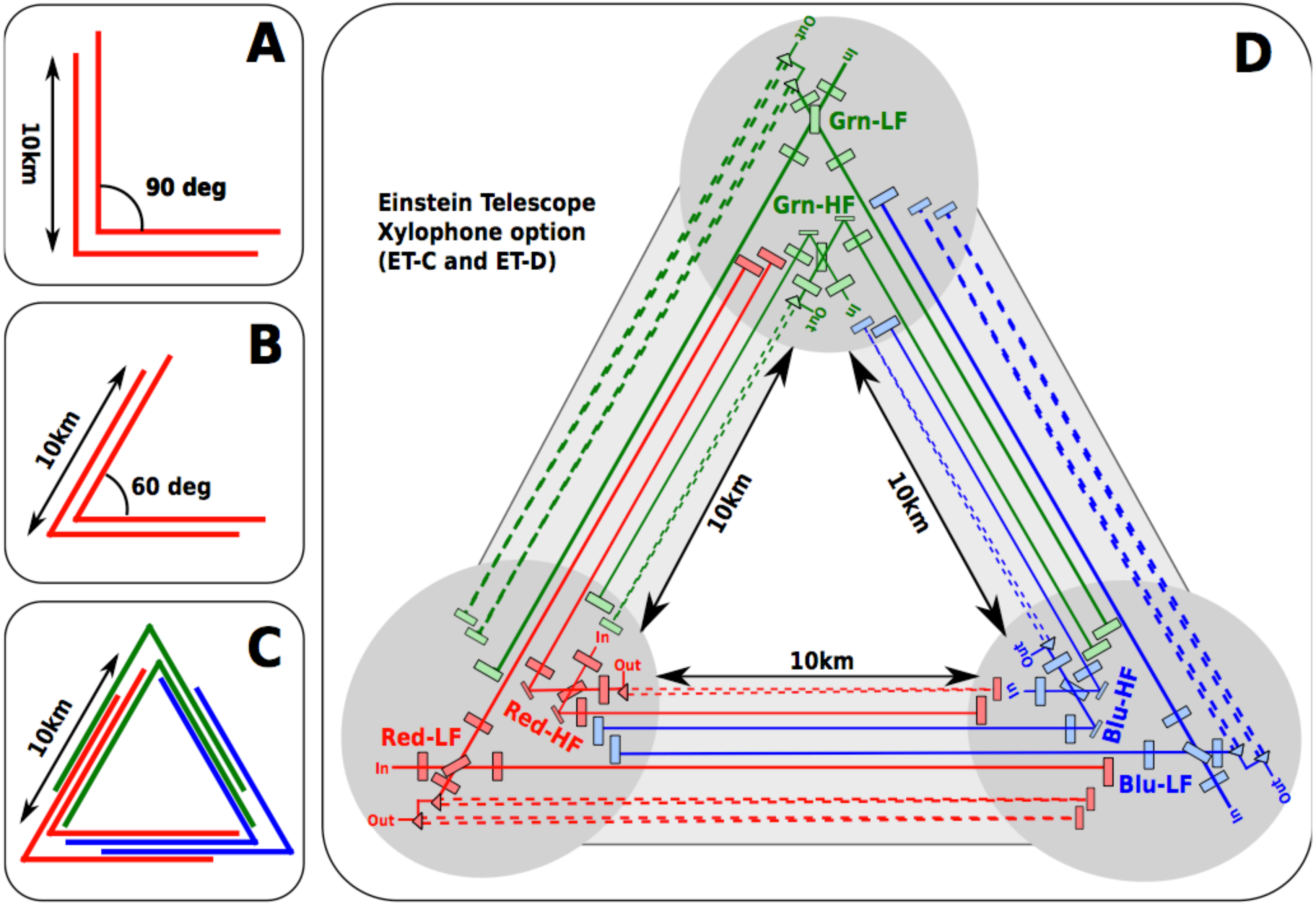
Creating frequency dependent Squeezing with filter cavities





- We can realise frequency dependent squeezing by reflecting it on a cavity, i.e. making use of the cavity's dispersion.

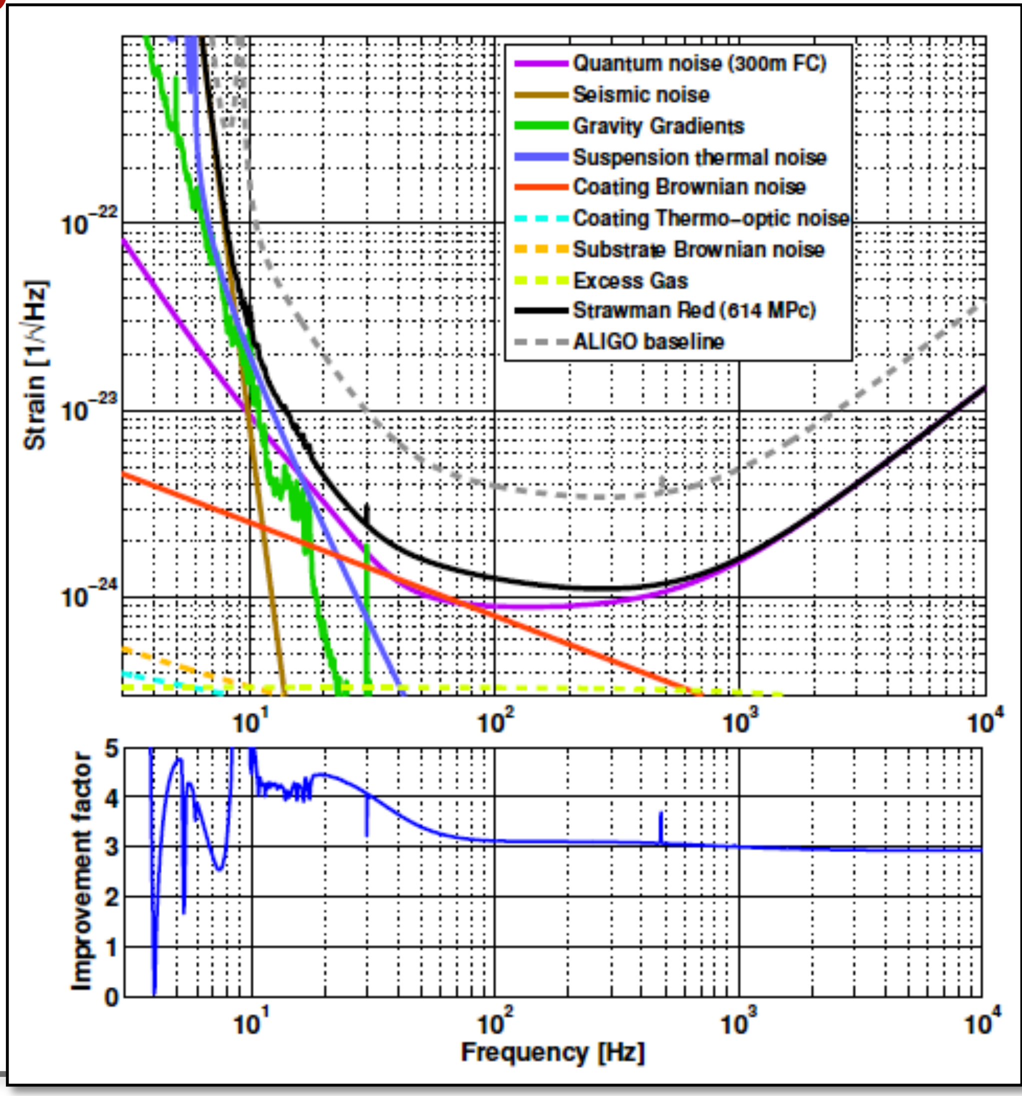
ET layout



A
10km
90 deg
B
10km
60 deg
C
10km
D
Einstein Telescope
Xylophone option
(ET-C and ET-D)
Grn-LF
Grn-HF
Red-LF
Red-HF
Blu-HF
Blu-LF
10km
10km
10km
In
Out

Example of frequency dependent Squeezing: LIGO3 Red

- During the last year people have started to investigate potential upgrades to the Advanced LIGO detectors.
- For details please see:
  - LIGO-T1200005-v2
  - LIGO-T1200031–v3
- Here we just want to take it as an example to look at the design of the frequency dependent squeezing



LIGO3-Red Quantum noise

- We kept the interferometer configuration and the mirror reflectivities the same as in aLIGO baseline.
- **Introduced frequency dependent input squeezing.**
- Key aspects: achievable squeezing level & required length of filter cavity

Laser and Optical Parameters		
Laser Wavelength	1064 nm	1064 nm
Optical Power at Test Masses	730 kW	730 kW
Arm Cavity Finesse	450	450
Signal Recycling	$T = 20\,\%$, tuned	$T = 20\,\%$, tuned
Squeezing Factor	n.a.	20 dB
Filtercavity (FC) length	n.a.	300 m
FC Detuning	n.a.	-16.8 Hz
FC Input Mirror Transmittance	n.a.	425 ppm
Squeezing Losses	n.a.	$9\,\% + 30$ ppm roundtrip in FC

Example: Squeez losses LIGO3 Red

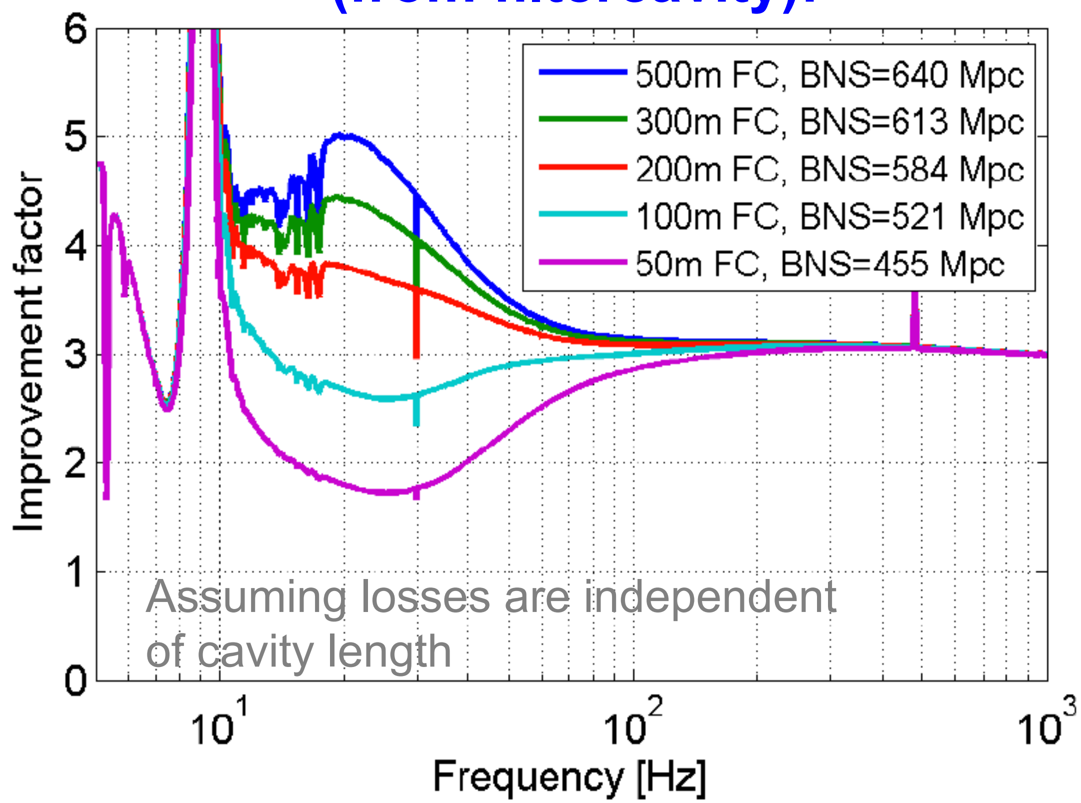
Frequency independent losses:

- Generation of squeezing: 3 %
- Optical isolation: 3 x 0.8 %
- Mode matching to IFO and to OMC: 2 x 1 %
- OMC loss and QE of PD: 2 x 0.5 %
- Mode matching to filter cavity: 1 %

= 9% in total

+

Frequency dependent loss (from filtercavity):



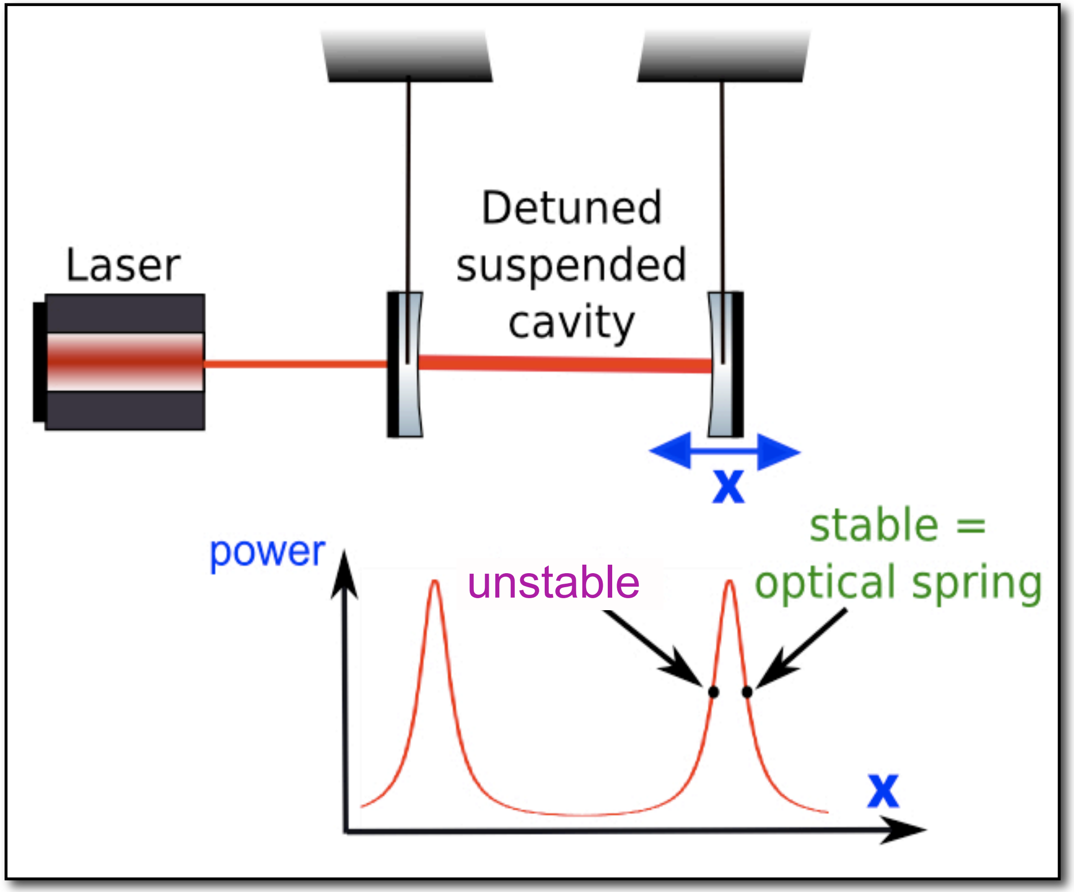
Starting from 20dB squeezing inside the squeezing crystal the losses reduce the observed squeezing to about 9-10dB

Overview

- Introduction: Quantum noise, Standard Quantum Limit, Vacuum fluctuations, 'Ball-on-a-stig', Quadrature Picture.
- The easiest way to beat the SQL: Varaiational Readout
  - Example: AEI-10m interferometer
- The best quantum noise reduction technique for second generation GW detectors: Squeezed light injection
  - How to make squeezed light?
  - Example: GEO-HF and Einstein telescope
- Optical rigidity
  - What is an optical spring?
  - Optical Bar and Optical Lever schemes
  - Local Readout scheme for Advanced LIGO
- Speed meter configurations

What is an Optical Spring?

- Detuned cavities can be used to create optical springs.
- Position change of the mirror => power changes => radiation pressure force changes.
- Optical springs couple the mirrors of a detuned cavity with a spring constant that can be as stiff as diamond.
- Can be used as low-noise transducer for GW signals to mirror movement in the local frame.



Beating the SQL with advanced detectors?

- Detuned Signal Recycling also creates a optical spring resonance => quantum noise shows two 'bumps', the optical spring (at low frequencies) and the pure optical resonance (at high frequencies).
- Actually advanced LIGO and advanced VIRGO could beat the SQL, if the quantum noise at low frequencies would not be coverd by other noise sources.

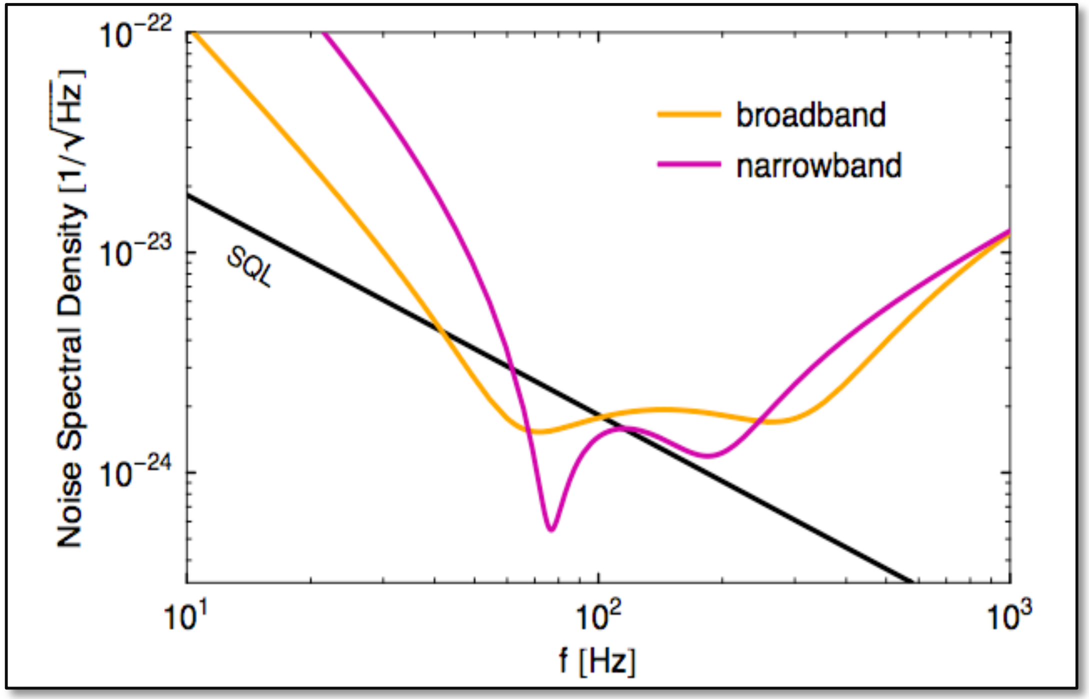
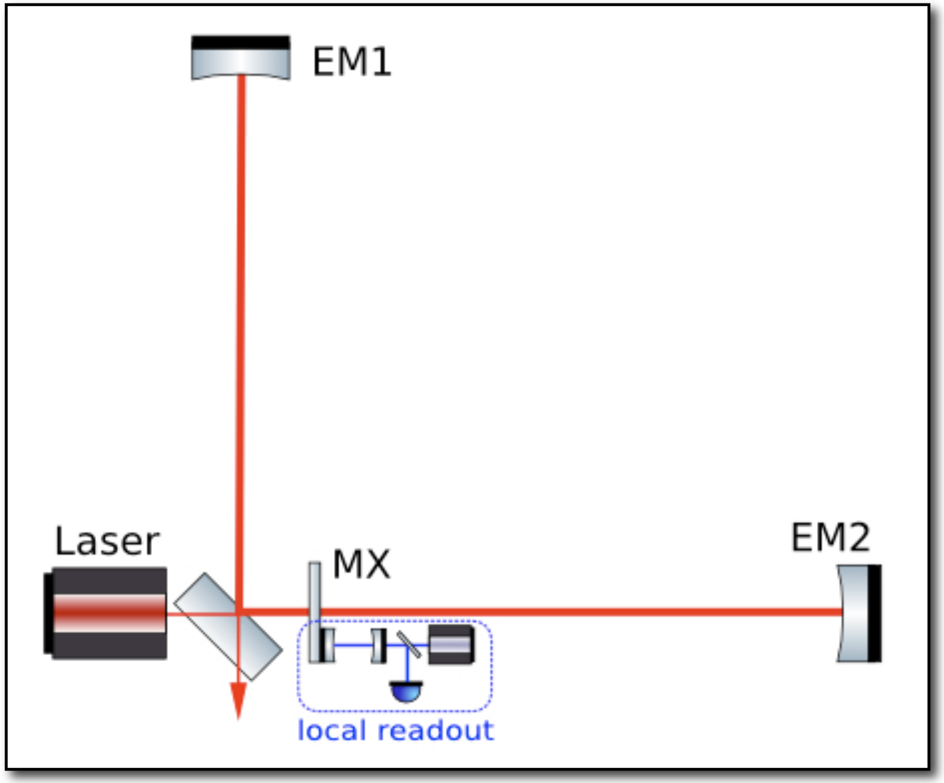


Image H. Mueller-Ebhardt, PhD thesis

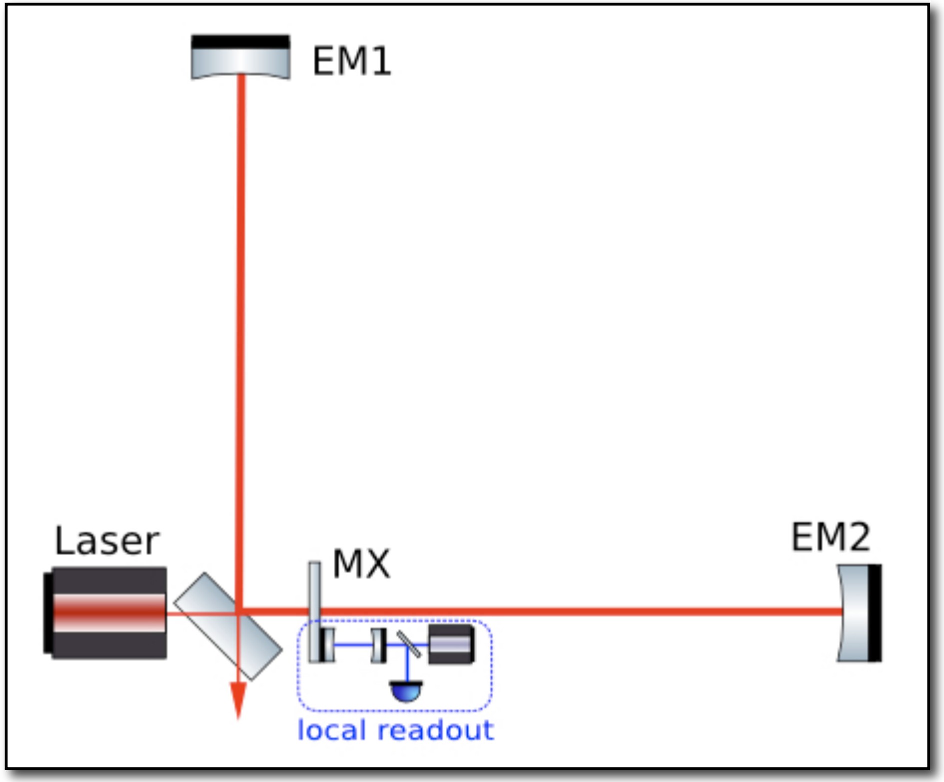
Optical Bar configurations

- Very light mirror (MX) is coupled to the movement of EM1 and EM2 via optical springs.
- MX can then locally read out by a small local meter without disturbing the quantum states in the main instrument (QND measurement).
- Split between GW transducer and readout allows separate optimisation of these two systems.

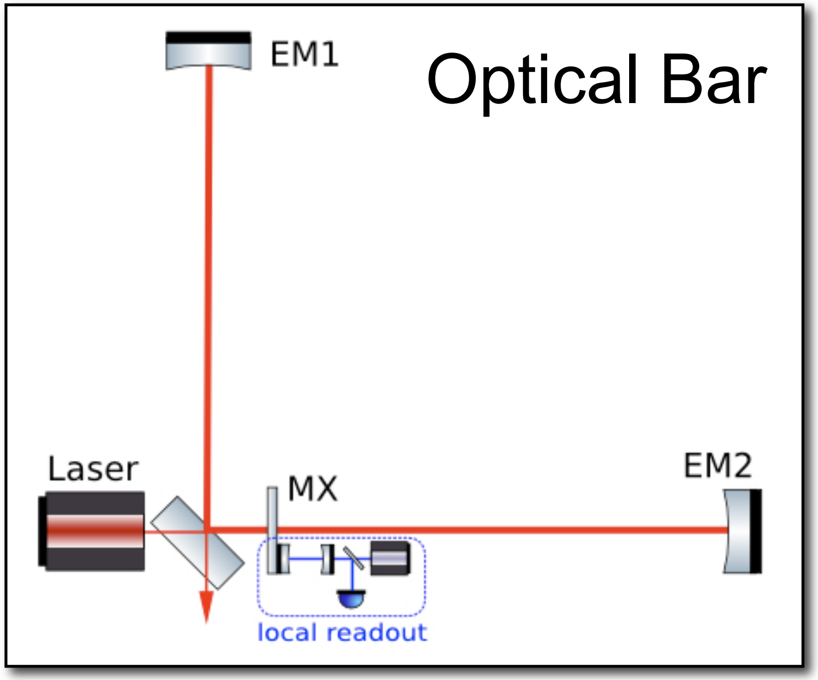


Optical Bars and 'moving mirrors'

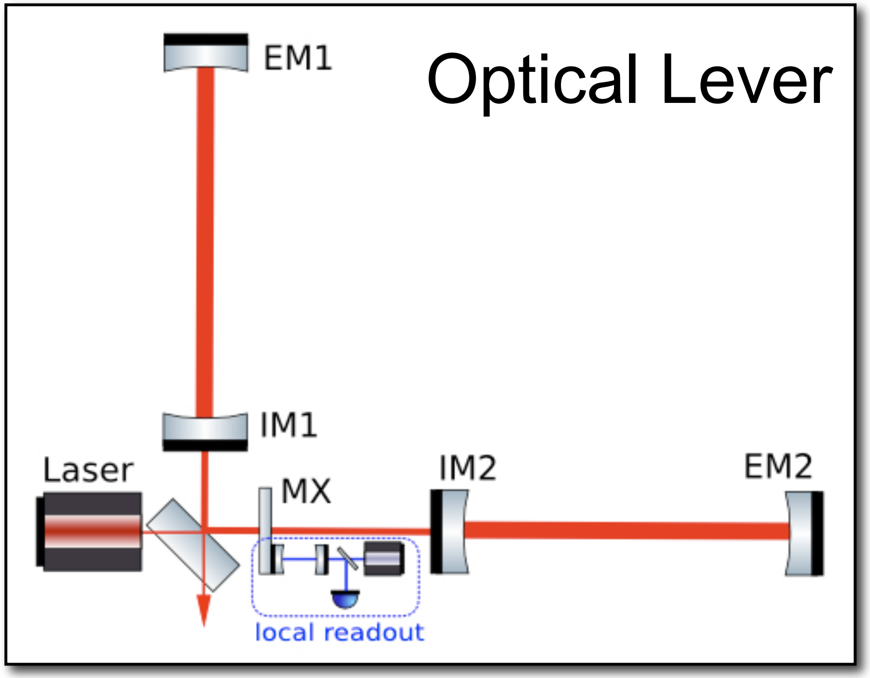
Task for you:
Image there are 3 observers standing each next to EM1, EM2 and MX. A gravitational wave comes by. Which observer can 'see' which mirrors moving?



Optical Bars and Optical Levers



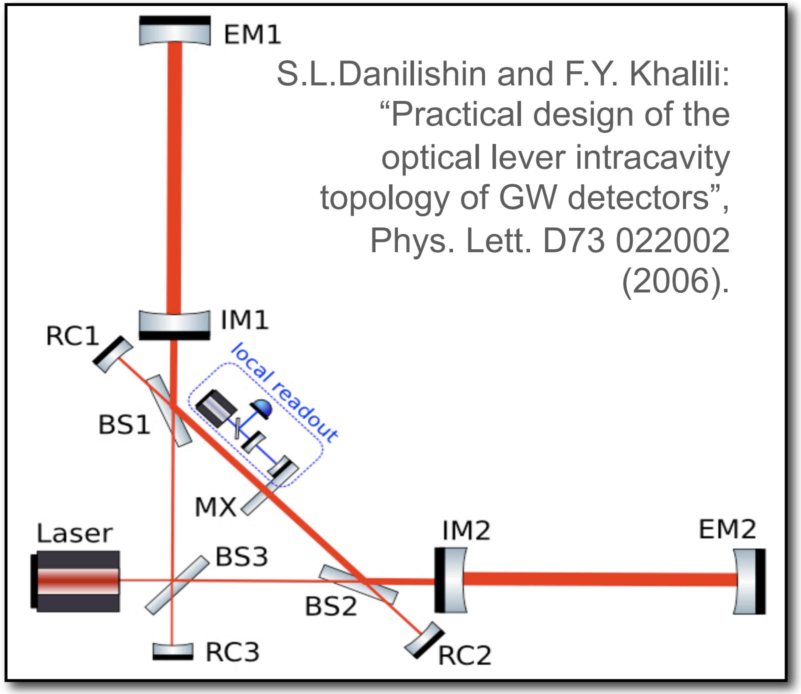
V.B. Braginsky and F.Y. Khalili: "Nonlinear meter for the gravitational wave antenna",Phys. Lett. A 218 (1996).

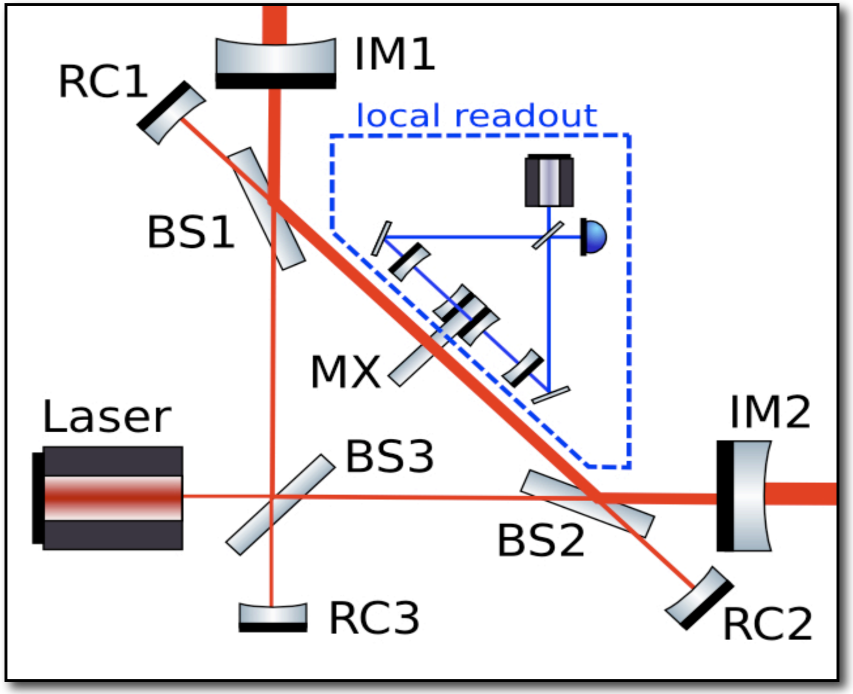


F.Y. Khalili: "The 'ptical lever' intracavity readout scheme for gravitational-wave antennae",Phys. Lett. A 298 (2002).

- Optical lever: introducing arm cavities increases the movement of MX by the Finesse of the arm cavity.

More realistic Designs for an Optical Lever

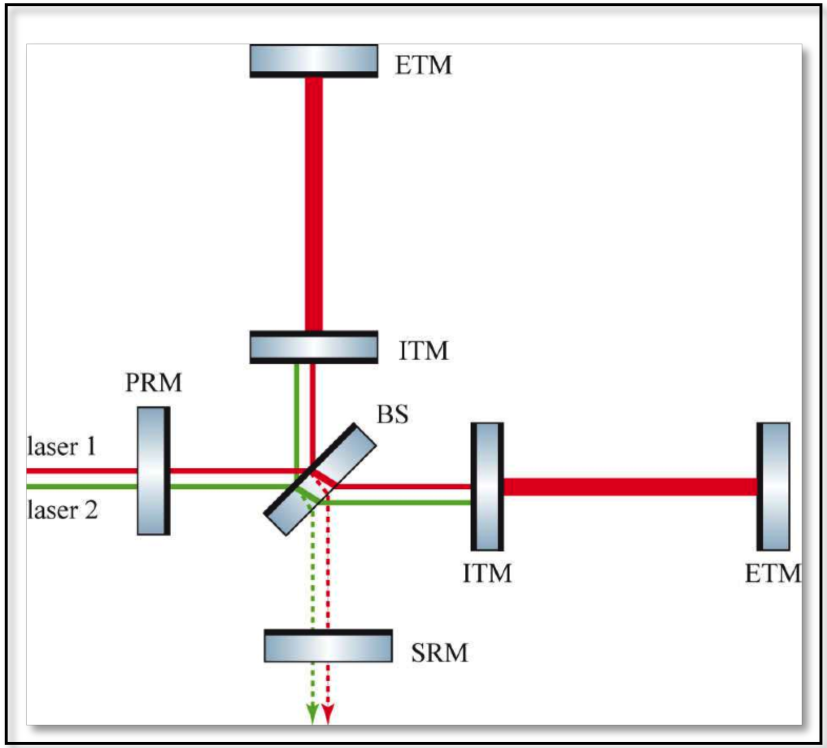




- Build symmetric optical levers => make use of common mode noise rejection.
- Use recycling techniques, such as power recycling.
- Increased number of components => more complex couplings => harder to control.
- KEYPOINT: Get the local readout to the required sensitivity.

Local Readout for Advanced LIGO

- While optical Bars and levers require a complete redesign of the interferometers, so-called 'local readout' is compatible within advanced LIGO infrastructure
- At low frequencies ITM and ETM are rigidly connected.
- At low frequencies GW signal is not in differential arm length, but in ITM movement (local frame).
- Use a separate laser system to read out the position of the ITM.

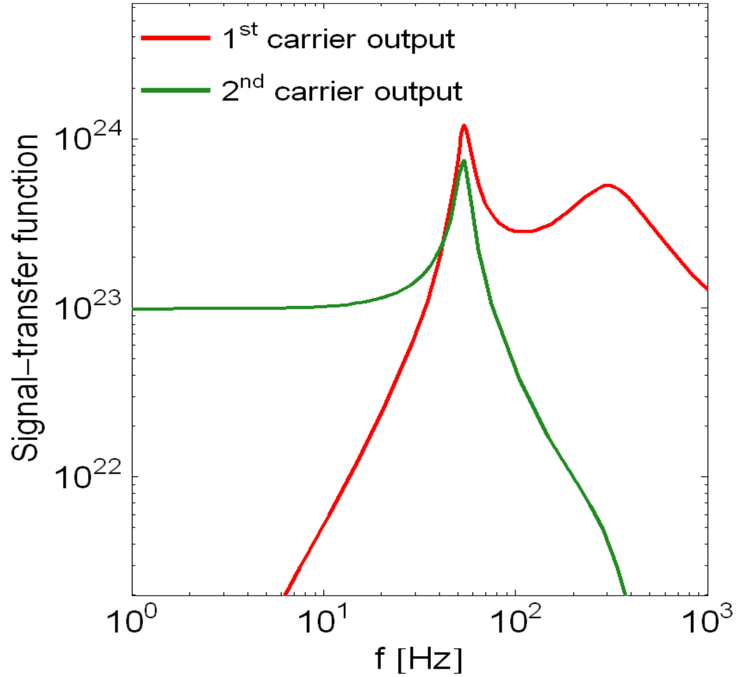


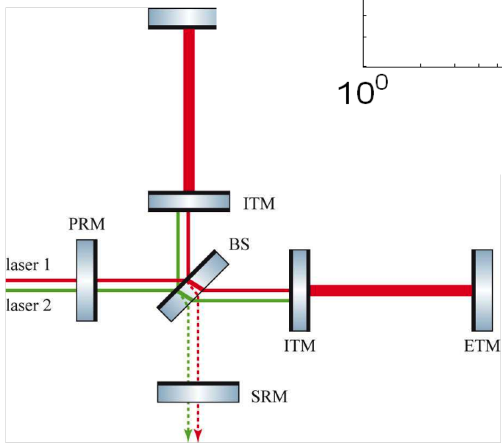
*Rehbein et al: PRD **76, 062002 (2007)***

Local Readout for Advanced LIGO (2)

How does local readout for Advanced LIGO work?

- **At low frequencies:** the arm cavity mirrors are 'rigidly' connected by optical springs => GW does not change the distance between ITM and ETM. However, GW signal is imprinted on ITM movement (in respect to BS), which and can be read out by additional green laser.
- **At high frequencies:** no optical spring present => ITM and ETM can move independently.

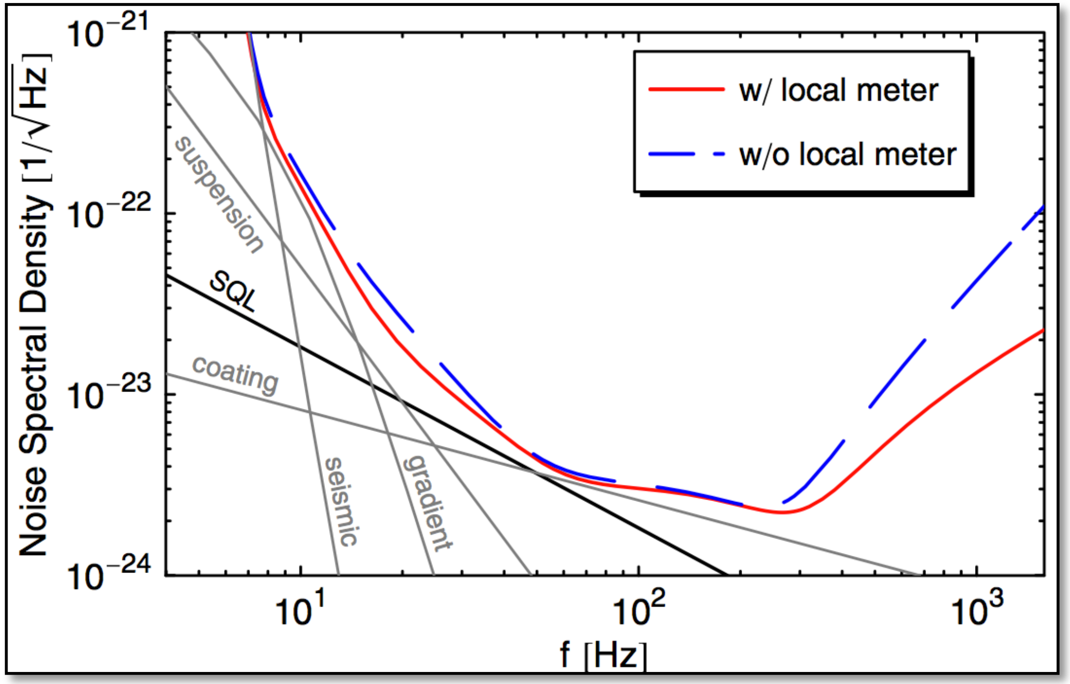




*Rehbein et al: PRD **76, 062002 (2007)***

Local Readout for Advanced LIGO (3)

- Technique allows to increase low frequency sensitivity.
- In a second step the Signal-Recycling can then be re-tuned to slightly higher frequency.
- Win at low and high frequencies. =>



Rehbein et al: PRD **76, 062002 (2007)**

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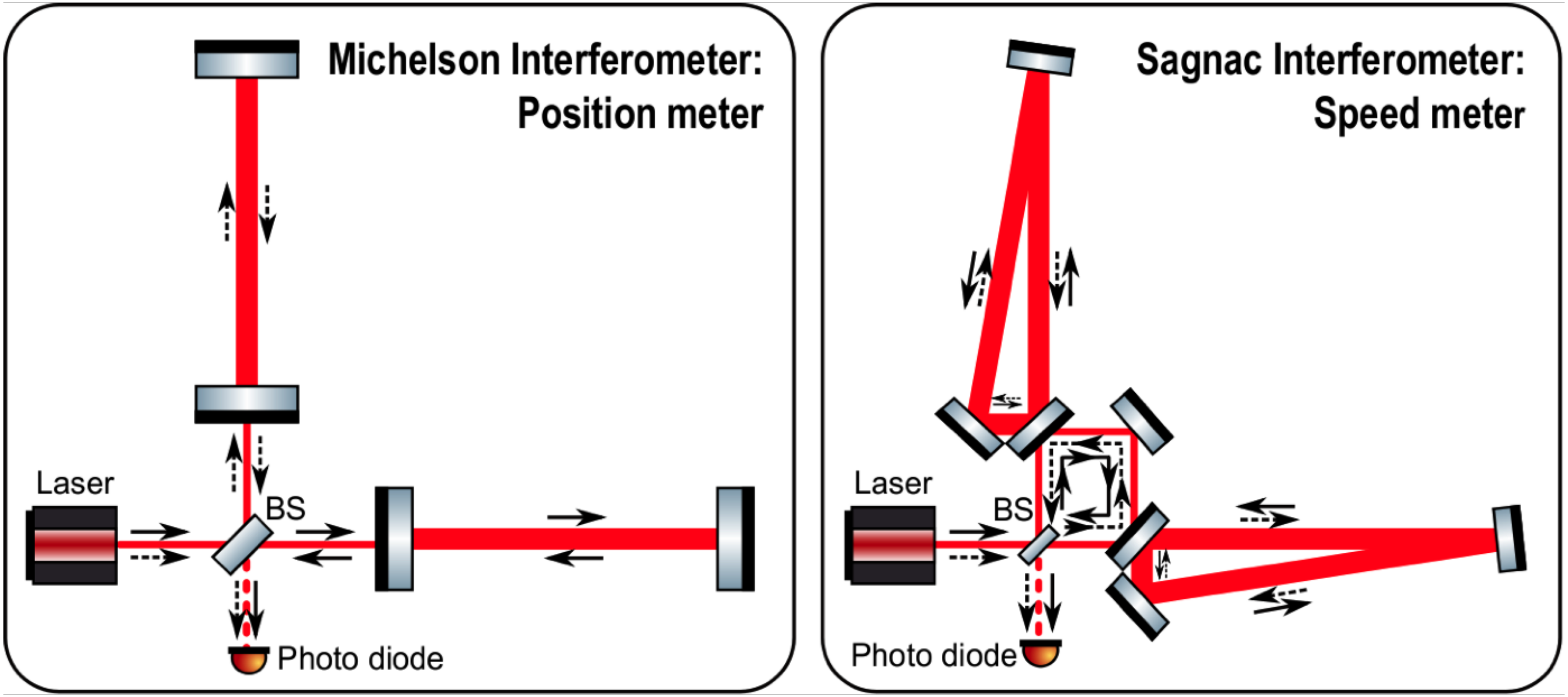
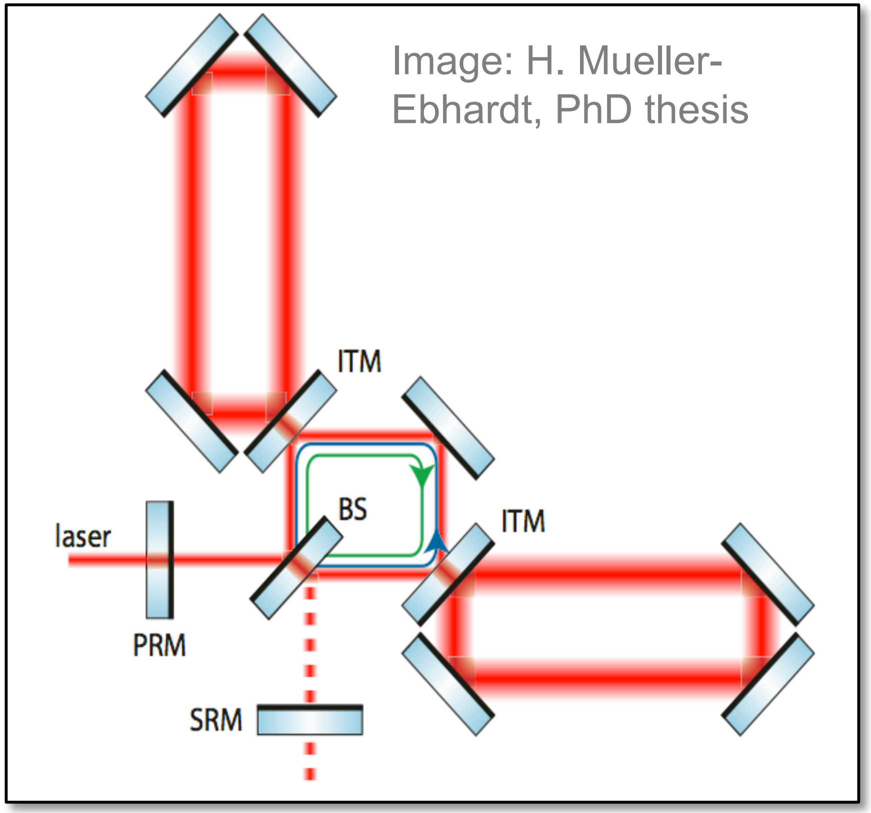
How can a Speedmeter help?

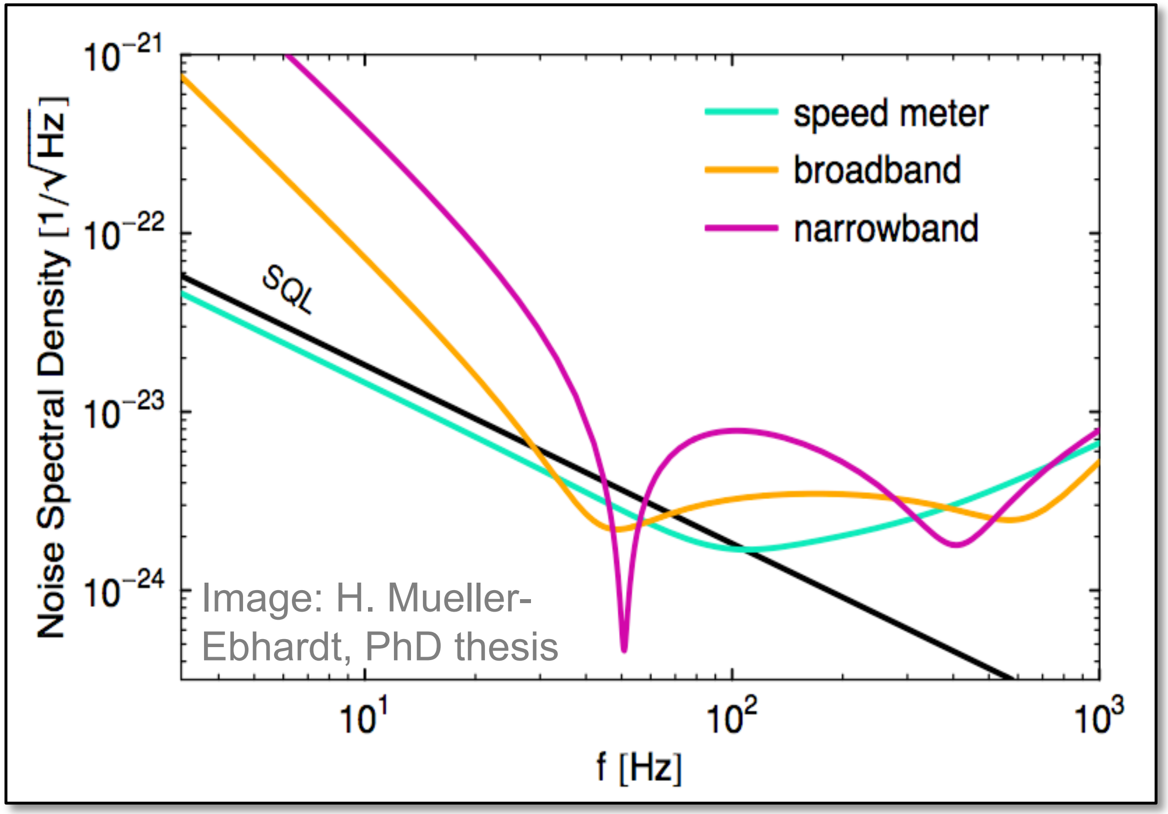
- So far we used Michelson interferometers to derive strain, by continuously measuring the displacement of the mirrors.
- However, quantum mechanics limits the accuracy of the measurement: $[\hat{x}(t), \hat{x}(t')] \neq 0$
- **The Standard Quantum Limit is the equivalent of the Heisenberg Uncertainty Principle.**
- However, already in 1930s John von Neumann told us that there are observables which can measured continuously without encountering the Heisenberg uncertainty. For example the momentum or speed of a testmass in our case.
- **A Sagnac interferometer measures only (!) the speed of the mirrors, from which we can obtain the strain h(t). => No quantum limit.**

Speedmeter vs Positionmeter



Speed meter 2

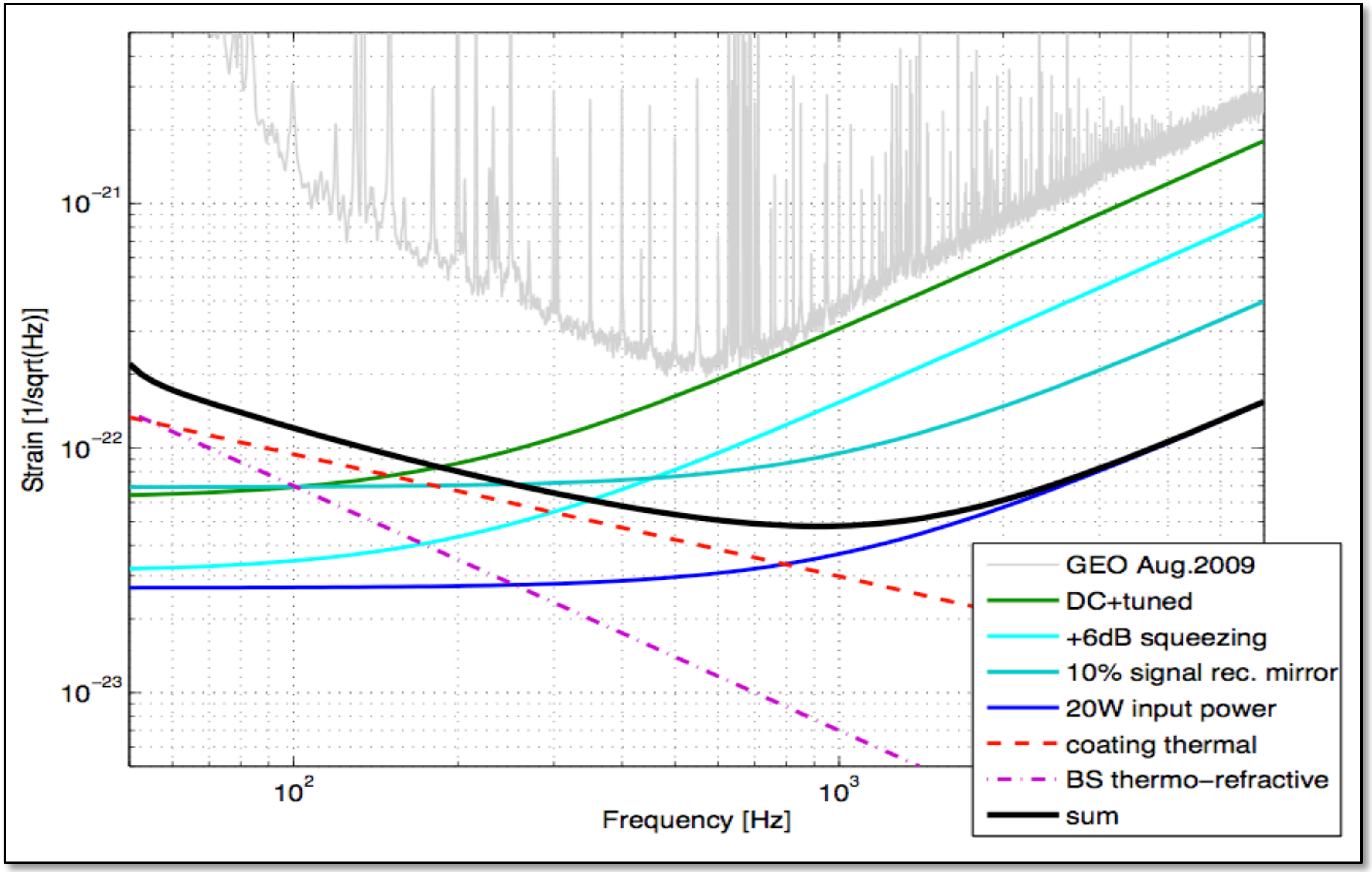




- Speedmeter can in principles cancel radiation pressure noise to a large extend and surpass the SQL over a wide frequency range

EXTRA SLIDES

GEO-HF sensitivity



Readout Methods