VESF Summer School 2010:

Quantum Noise and QND

Stefan Hild

LIGO-G1000723

What do I want you to take with you from this lecture?

- How can we actually trick Heisenberg?
- Get an intuitive understanding of Quantum noise and how to reduce it in our instruments.
- An easy tool 'graphical quadrature picture'.

- *What I want to avoid: 1 hour of filling the blackboard with complicated Quantum mechanics.*

Two Basic principles of GW detector

- 1. You need to make your test masses very quiet (quieter than the signal you want detect).
- 2. You need to read out the test mass positions with very high accuracy, without introducing ‘too much’ noise due to measurement itself.

- Photon shot noise is a sensing noise, photon radiation pressure noise is a back-action-noise.

The Standard quantum limit

- Standard Quantum Limit of a free mass is equivalent to Heisenberg uncertainty.
- Arises when one tries to detect gravitational wave by continuously measuring free-mass displacement, since

$$[\hat{x}(t), \hat{x}(t')] \neq 0$$

  - Precise measurement on x has to perturb p. Perturbation in p converts to future error in x.
- In our case: Light fields enforce Heisenberg uncertainty through complementarity between Shot noise and Radiation Pressure noise.

Overview

- Introduction: Quantum noise, Standard Quantum Limit, Vacuum fluctuations, 'Ball-on-a-stig', Quadrature Picture.
- The easiest way to beat the SQL: Varaiational Readout
  - Example: AEI-10m interferometer
- The best quantum noise reduction technique for second generation GW detectors: Squeezed light injection
  - How to make squeezed light?
  - Example: GEO-HF and Einstein telescope
- Optical rigidity
  - What is an optical spring?
  - Optical Bar and Optical Lever schemes
  - Local Readout scheme for Advanced LIGO
- Speed meter configurations

Some remarks ahead …

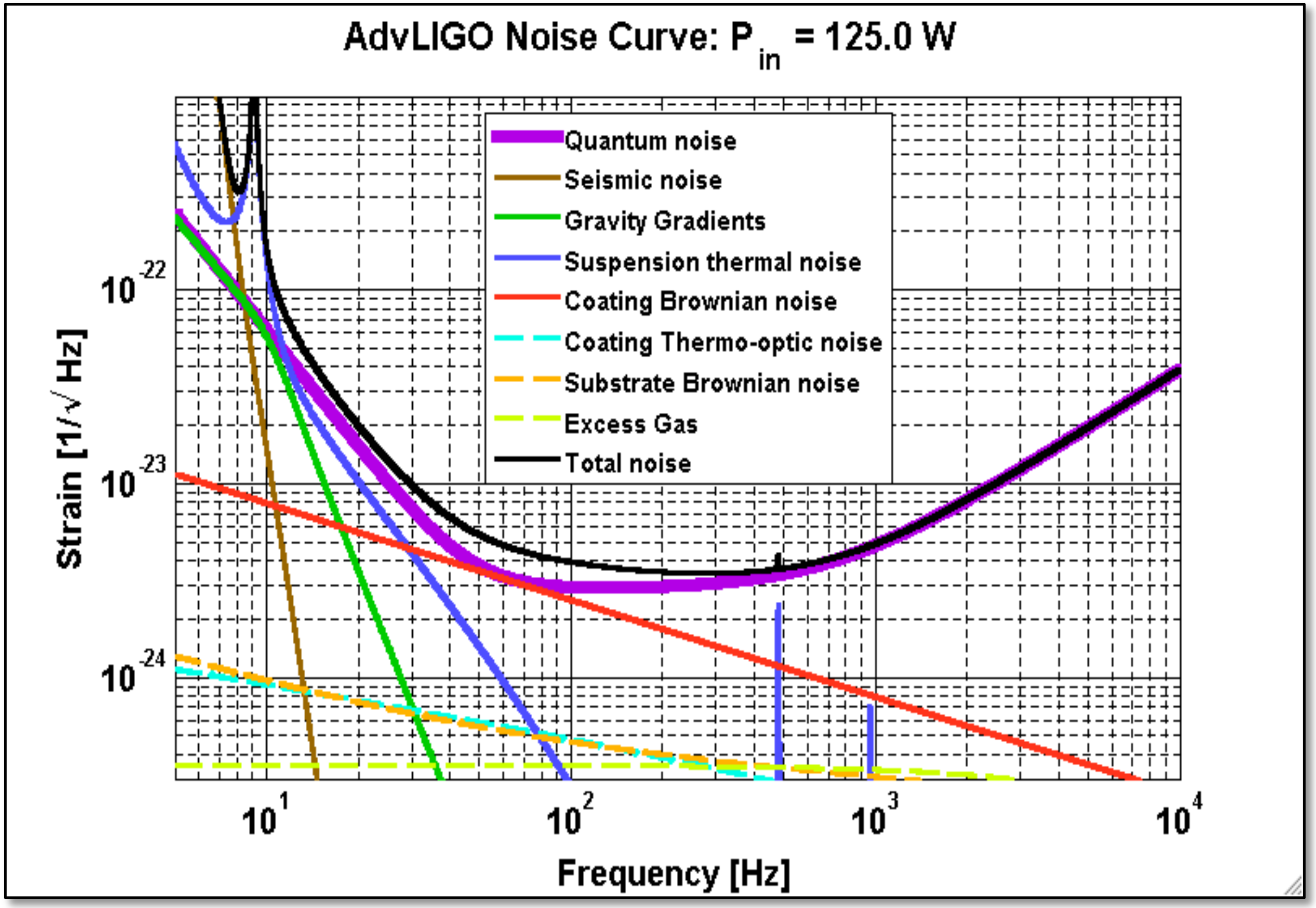
- ‘Quantum-non-demolition’ measurements are not a new topic (Braginski, Khalili and others theoretically developed QND-techniques already decades ago), but it becomes more and more a hot topic, as the experimental realisation starts to come true. ☺

- During the next hour I would like to give you an overview of the QND techniques that I believe are relevant for GW observatories. (of course this reflects my personal view and other people might set the focus on a different subset of configurations …)

- Please note: not all of the concepts I will talk about are considered to be real ‘QND’ experiments. However, all of them reduce Quantum noise and are therefore also called ‘Quantum noise reduction’ (QNR) techniques.

Why is Quantum Noise Reduction ‘*soooo*’ important?

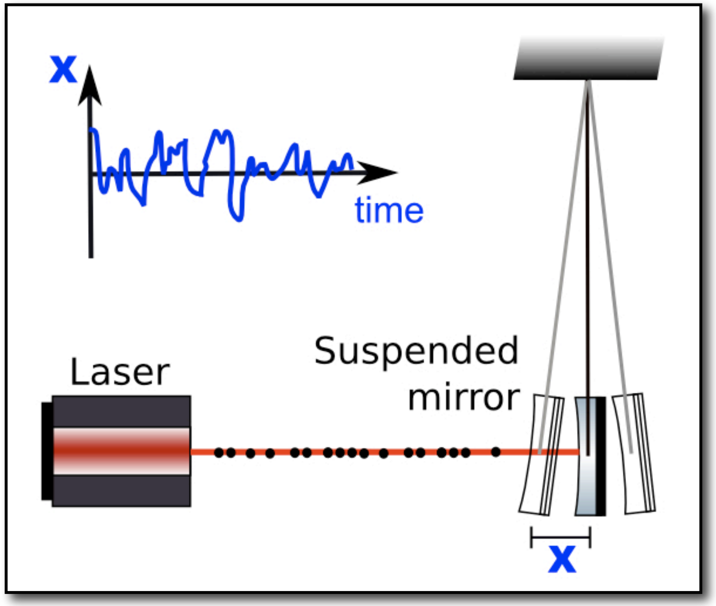
- For future interferometers Quantum noise is the ***main*** limiting noise source.
- Example: Broadband (tuned Signal Recycling) configuration of Advanced LIGO
- For all frequencies above 12Hz Quantum noise is the limiting noise.



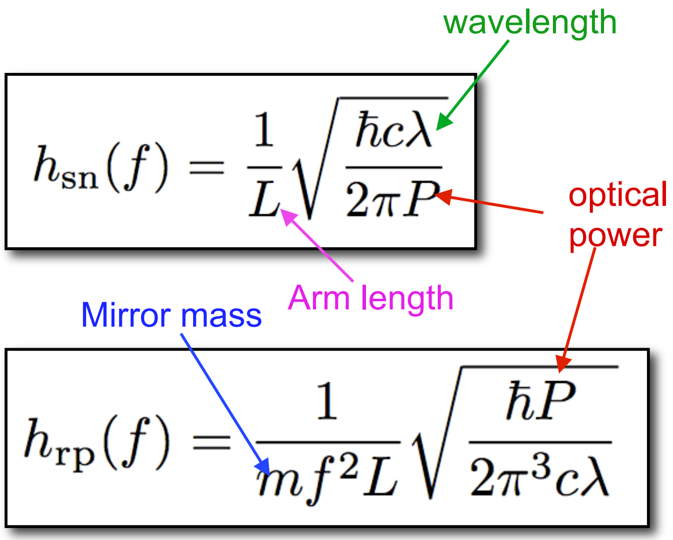
Scenario 1b from LIGO-T070247-01-I

What is quantum noise?

- Quantum noise is comprised of **photon shot noise** at high frequencies and **photon radiation pressure noise** at low frequencies.
- The photons in a laser beam are not equally distributed, but follow a Poisson statistic.

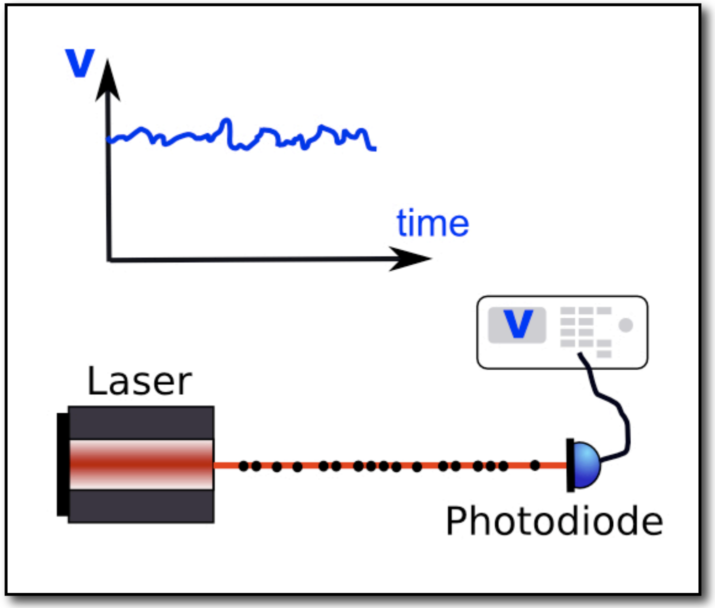


photon radiation pressure noise



$$h_{\mathrm{sn}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

$$h_{\mathrm{rp}}(f) = \frac{1}{m f^2 L}\sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$



photon shot noise

The Standard Quantum Limit (SQL)

- While shot noise contribution decreases with optical power, radiation pressure level increases:

wavelength

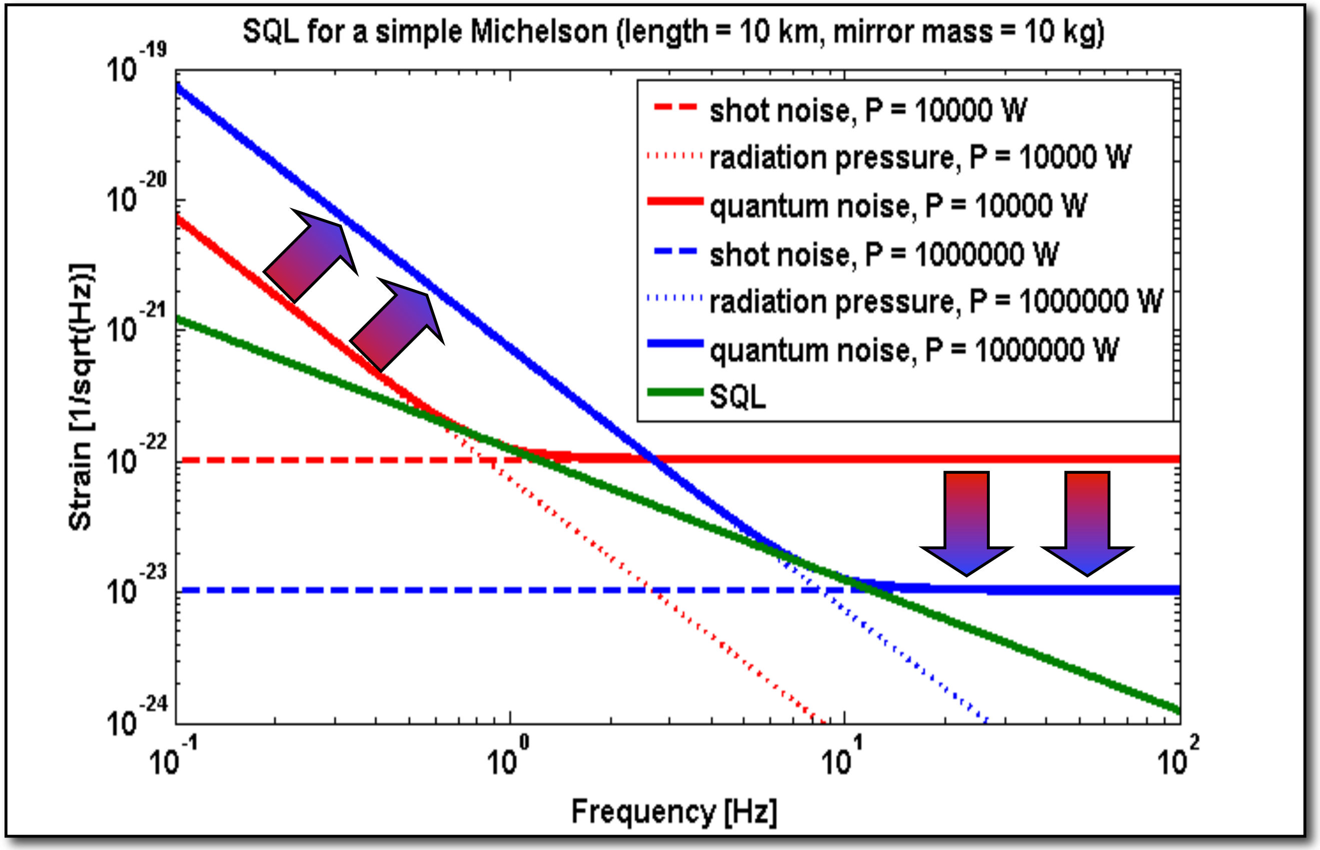
$$h_{\mathrm{sn}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

optical power

Arm length

Mirror mass

$$h_{\mathrm{rp}}(f) = \frac{1}{m f^2 L}\sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$

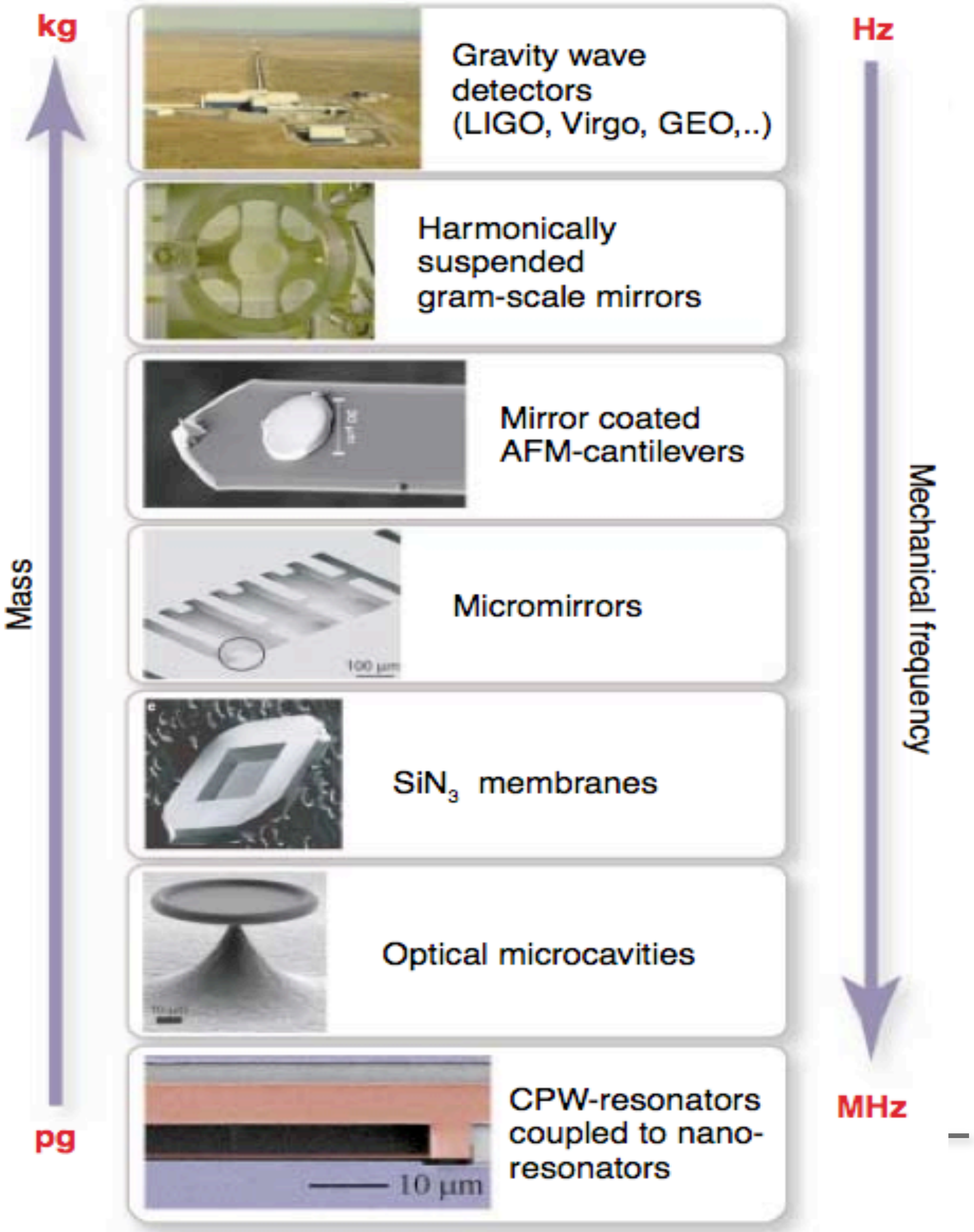

SQL for a simple Michelson (length = 10 km, mirror mass = 10 kg)

- The SQL is the minimal sum of shot noise and radiation pressure noise.
- Using a classical quantum measurement the SQL represents the lowest achievable noise.

V.B. Braginsky and F.Y. Khalili: Rev. Mod. Phys. 68 (1996)

Scales of SQL Experiments

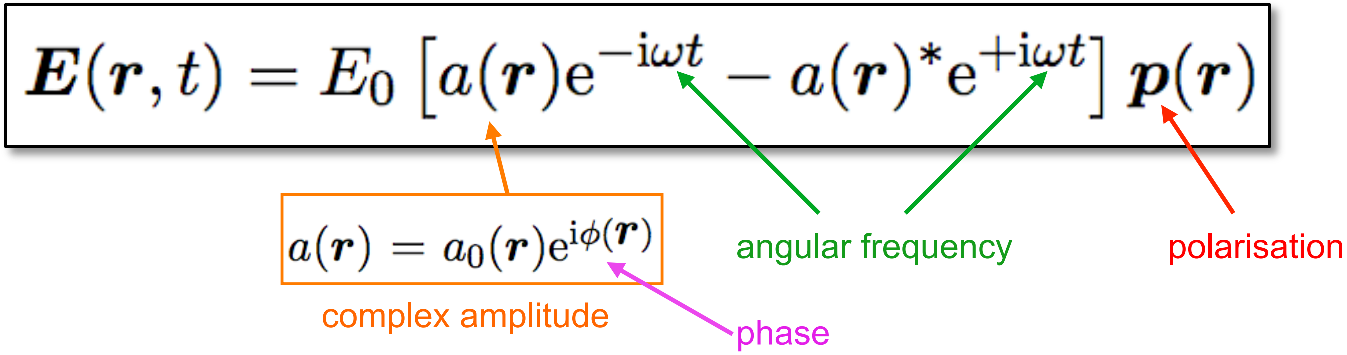
Fig. 3. Experimental cavity optomechanical systems. (**Top** to **Bottom**) Gravitational wave detectors [photo credit LIGO Laboratory], harmonically suspended gram-scale mirrors (*28*), coated atomic force microscopy cantilevers (*29*), coated micromirrors (*14*, *15*), SiN$_3$ membranes dispersively coupled to an optical cavity (*31*), optical microcavities (*13*, *16*), and superconducting microwave resonators coupled to a nanomechanical beam (*33*). The masses range from kilograms to picograms, whereas frequencies range from tens of megahertz down to the hertz level. CPW, coplanar waveguide.



T.J. Kippenberg and K. J. Vahala, Science 321 (2008) 1172

The Mathematical approach

- Quantitative description of an electrical field ***E*** at the position ***r*** and time *t*:



$$\boldsymbol{E}(\boldsymbol{r},t) = E_0\left[a(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}\omega t} - a(\boldsymbol{r})^*\mathrm{e}^{+\mathrm{i}\omega t}\right]\boldsymbol{p}(\boldsymbol{r})$$

$$a(\boldsymbol{r}) = a_0(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r})}$$

complex amplitude

phase

angular frequency

polarisation

- We can now introduce two new 'properties':

$$X_1(\boldsymbol{r}) = a^*(\boldsymbol{r}) + a(\boldsymbol{r})$$

$$X_2(\boldsymbol{r}) = \mathrm{i}\left[a^*(\boldsymbol{r}) - a(\boldsymbol{r})\right]$$

The Mathematical approach (2)

$$X_1(\boldsymbol{r}) = a^*(\boldsymbol{r}) + a(\boldsymbol{r})$$ amplitude quadrature

$$X_2(\boldsymbol{r}) = \mathrm{i}\,[a^*(\boldsymbol{r}) - a(\boldsymbol{r})]$$ phase quadrature

- Using X_1 and X_2 we can rewrite the electrical field:

$$\boxed{\boldsymbol{E}(\boldsymbol{r},t) = E_0\,[X_1 \cos(\omega t) - X_2 \sin(\omega t)]\,\boldsymbol{p}(\boldsymbol{r},\mathrm{t})}$$

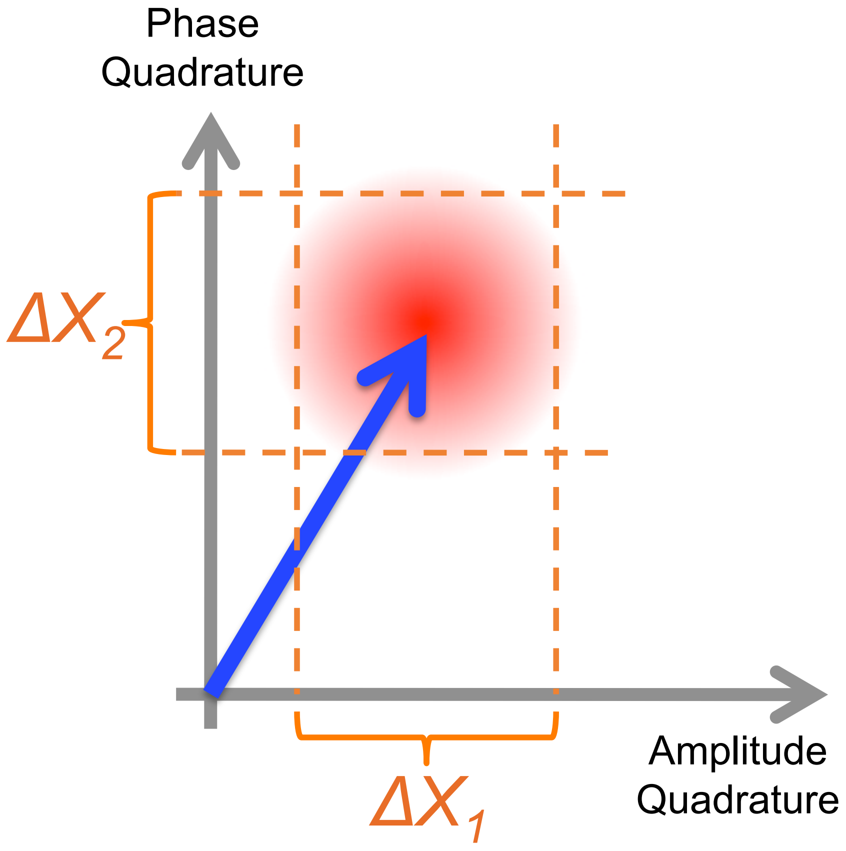
- Finally, we have to introduce a quantisation of the electrical field:

$$\hat{X}_1 = \hat{a}^\dagger + \hat{a}$$ amplitude quadrature operator

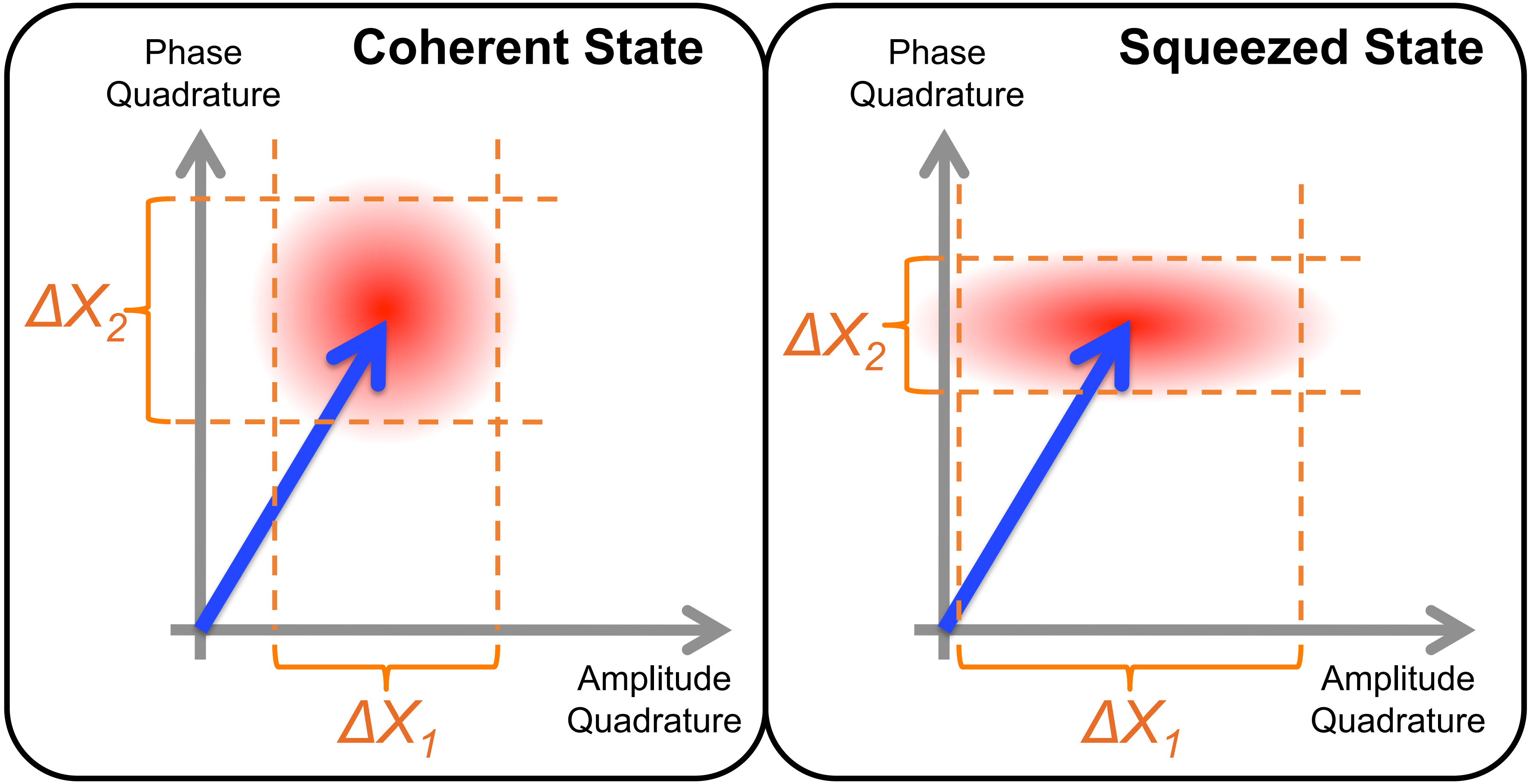
$$\hat{X}_2 = \mathrm{i}\left(\hat{a}^\dagger - \hat{a}\right)$$ phase quadrature operator

Introducing the Ball-on-a-stick concept

- Let’s try to convert the quadrature idea into a picture:
- A laser sends out light with an average frequency and amplitude. => This is the blue arrow.
- However, individual photons have a uncertainty, i.e. may have slightly different frequency or amplitude. => This is indicated by the red ball.



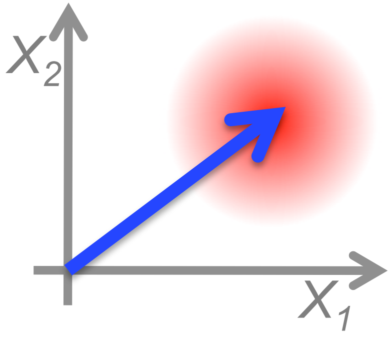
Ball on a stick and Heisenberg



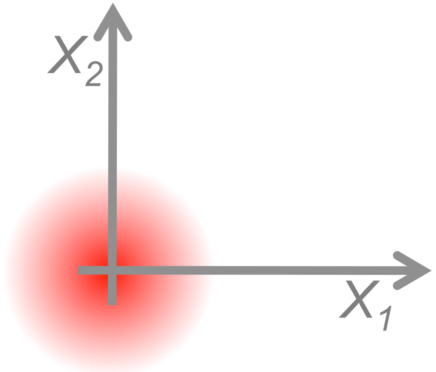
Heisenberg uncertainty principle: there is a minimal area of ball

$$\Delta\hat{X}_1 \Delta\hat{X}_2 \geq 1$$

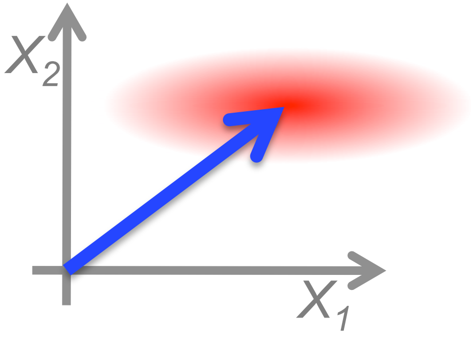
Examples of ball on the stick



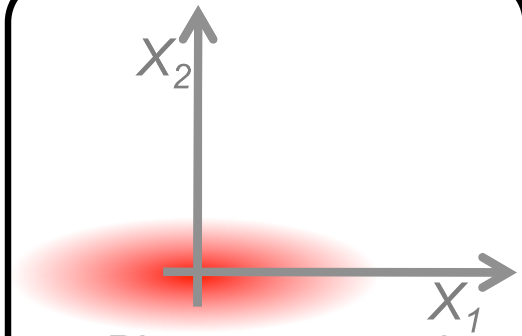
Coherent state



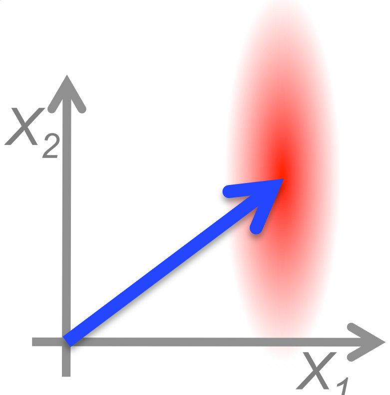
Vacuum state



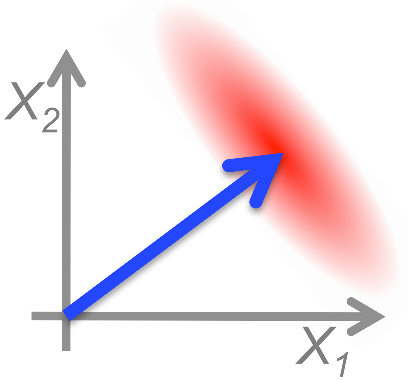
Phase squeezing



Phase squeezed vacuum



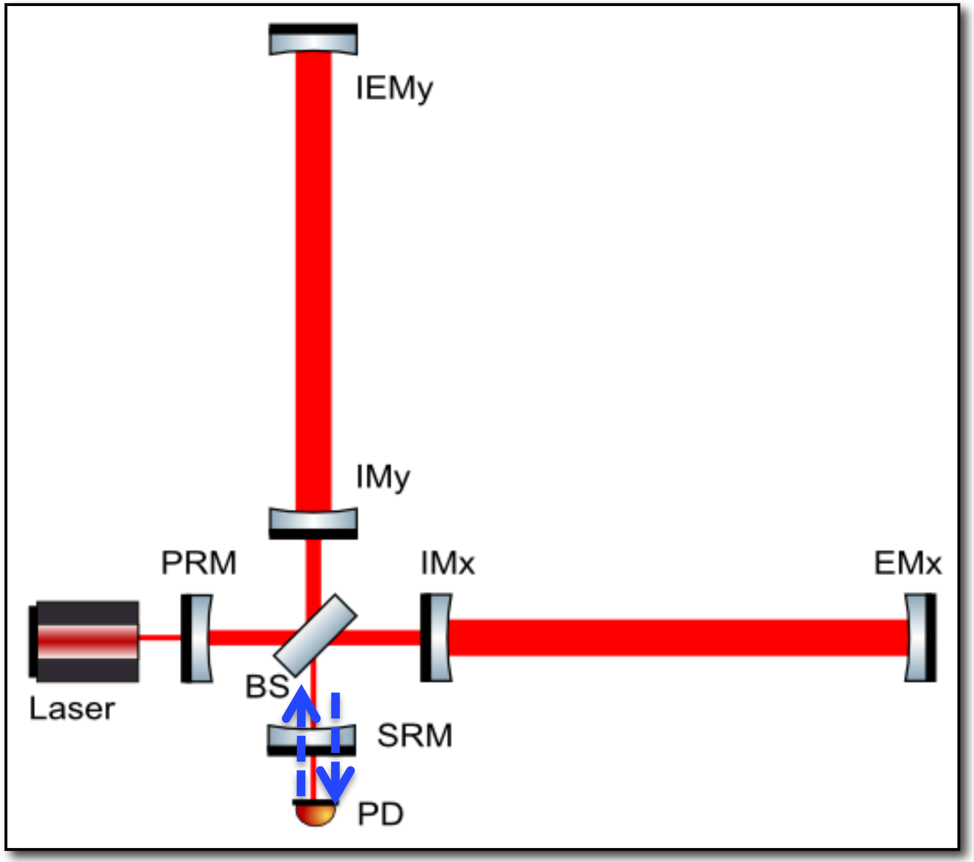
Amplitude squeezing



Quadrature squeezed

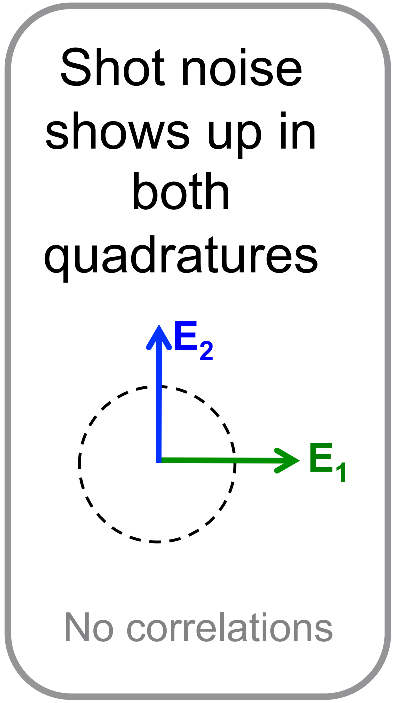
Vacuum Fluctuations

- How does ‘Ball-on-the-stick’ fits into the interferometer picture?
- An intuitive picture is to consider:
  - A vacuum state is entering the interferometer from the photodiode.
  - It is then ‘reflected from the interferometer’ and is detected together with the GW signal on the main photodiode.



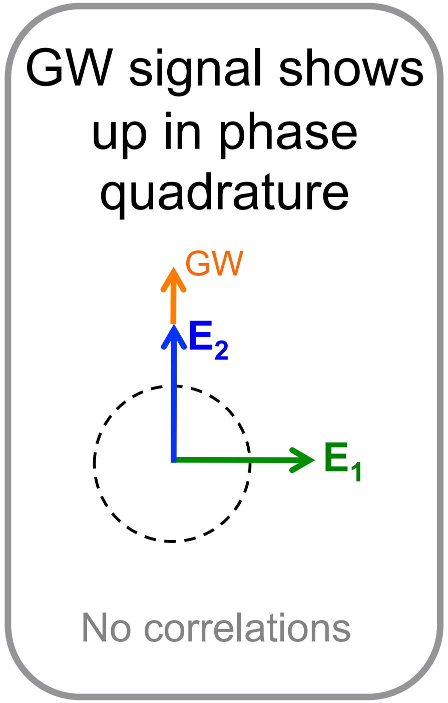
Vacuum fluctuations

GW signal in Quadrature picture



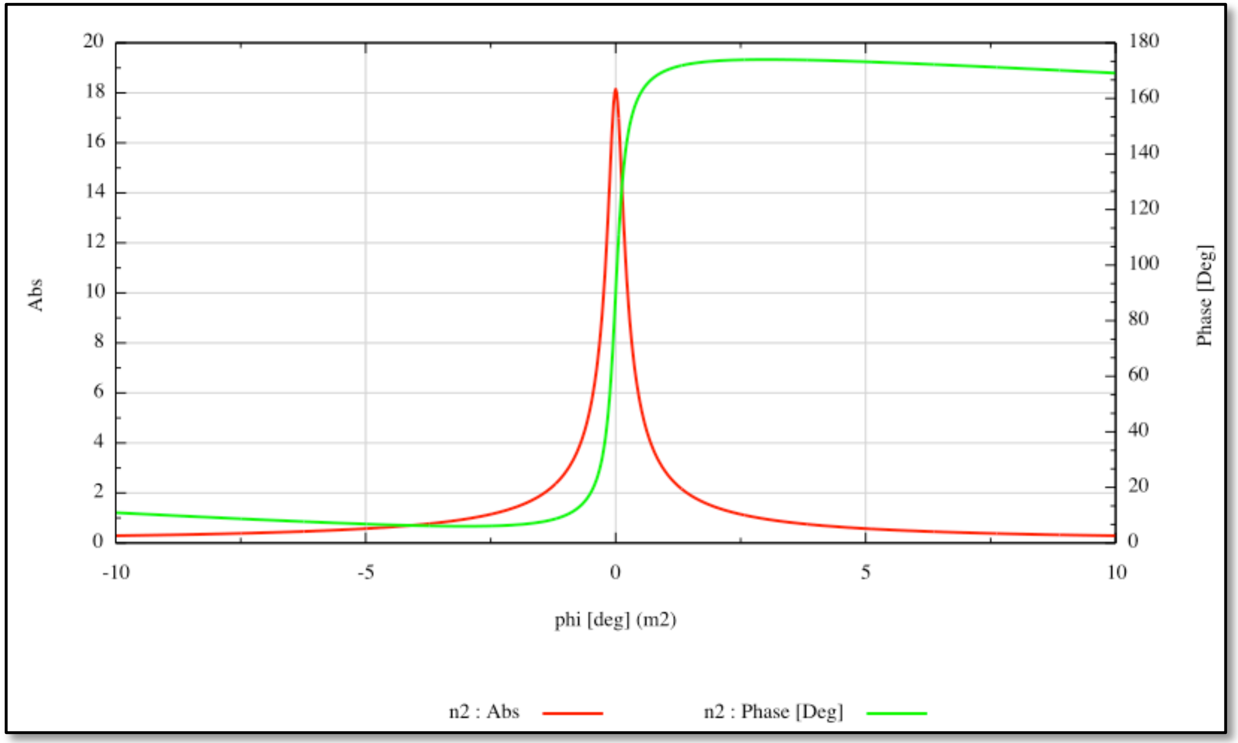
Shot noise shows up in both quadratures

No correlations



GW signal shows up in phase quadrature

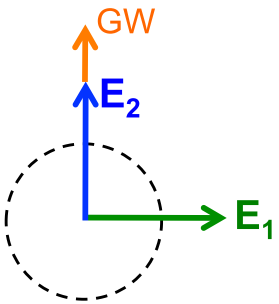
No correlations



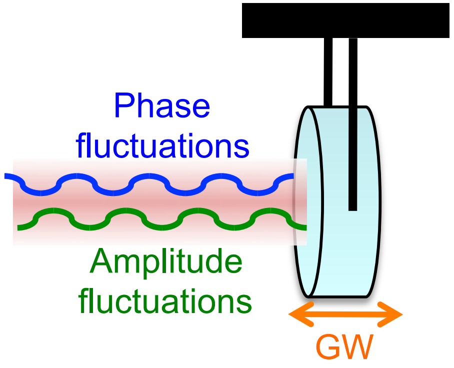
- If you change the length of a cavity around its resonance, the slope of the amplitude is zero, while the slope of the phase is maximal. Therefore, GW signal adds to phase quadrature.

Where does Radiation pressure noise come from?

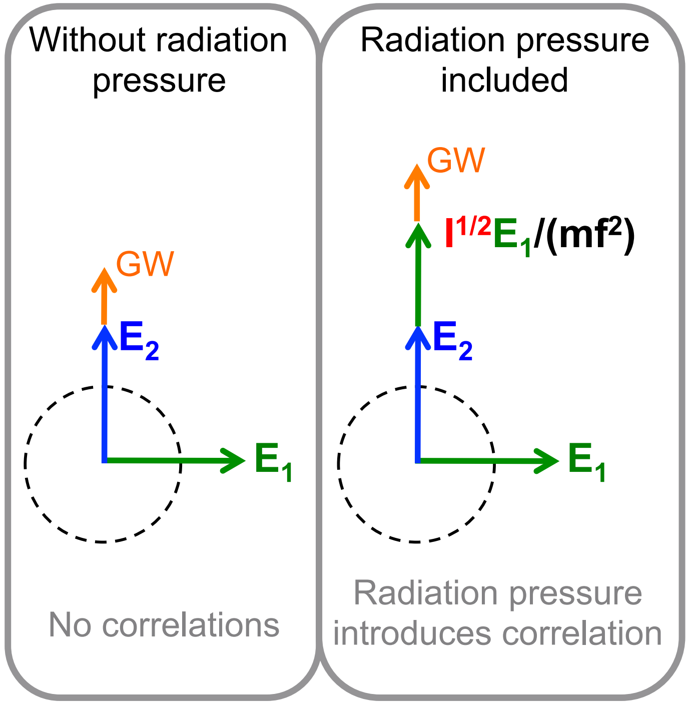
Without radiation pressure



No correlations



Where does Radiation pressure noise come from?

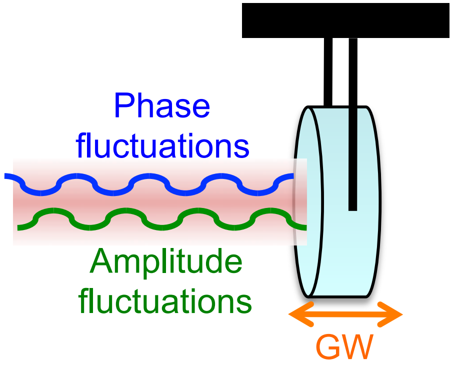


Without radiation pressure

No correlations

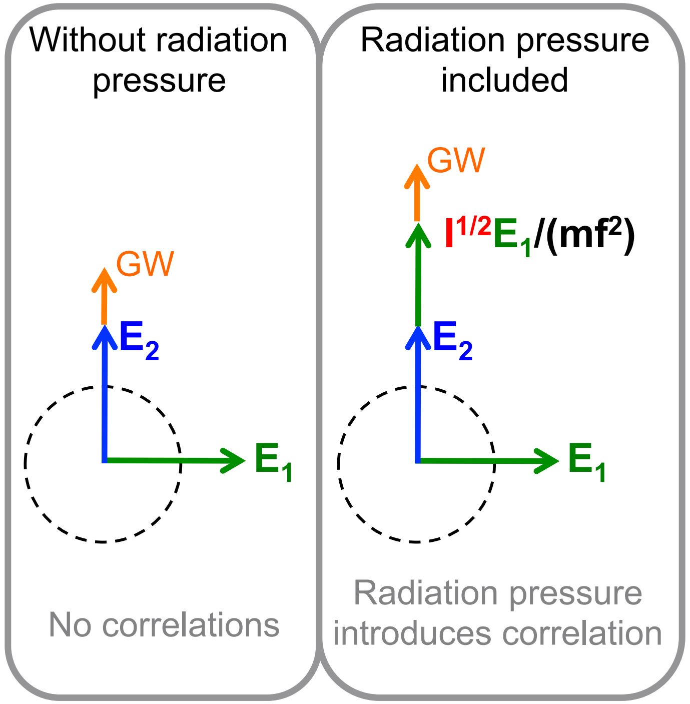
Radiation pressure included

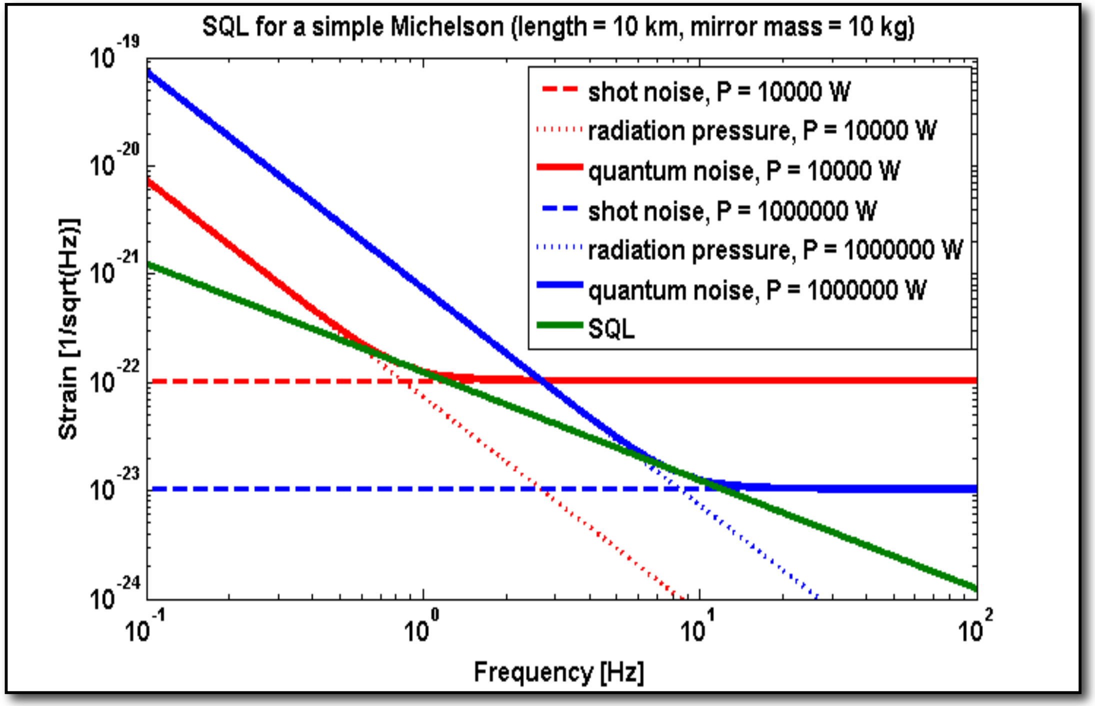
Radiation pressure introduces correlation



- Amplitude fluctuations act onto suspended mirror.
- Mirror is moved and gives contribution in the phase quadrature.
- This new contribution:
  - is correlated to E_1
  - depends on the mirror mass
  - Its magnitude goes with $1/f^2$

Where does Radiation pressure noise come from?





- At high frequencies radiation pressure is negilicable (due to $1/f^2$).
- At low frequencies radiation pressure is dominant

Overview

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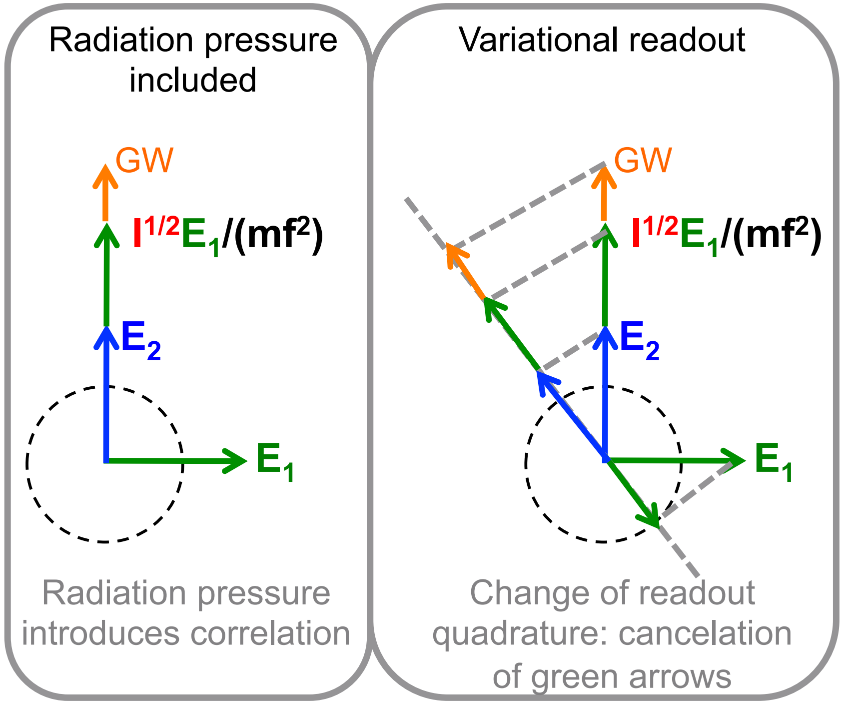
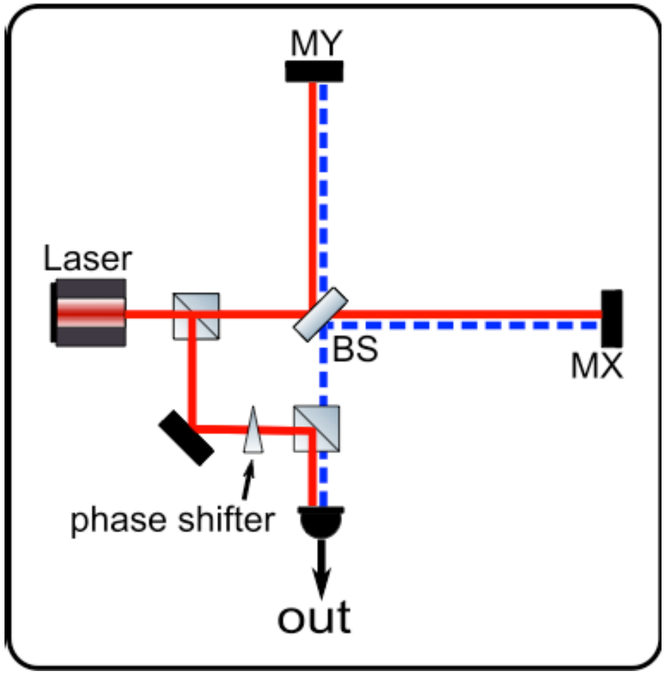
Surpassing the SQL at AEI-10m: Variational Readout

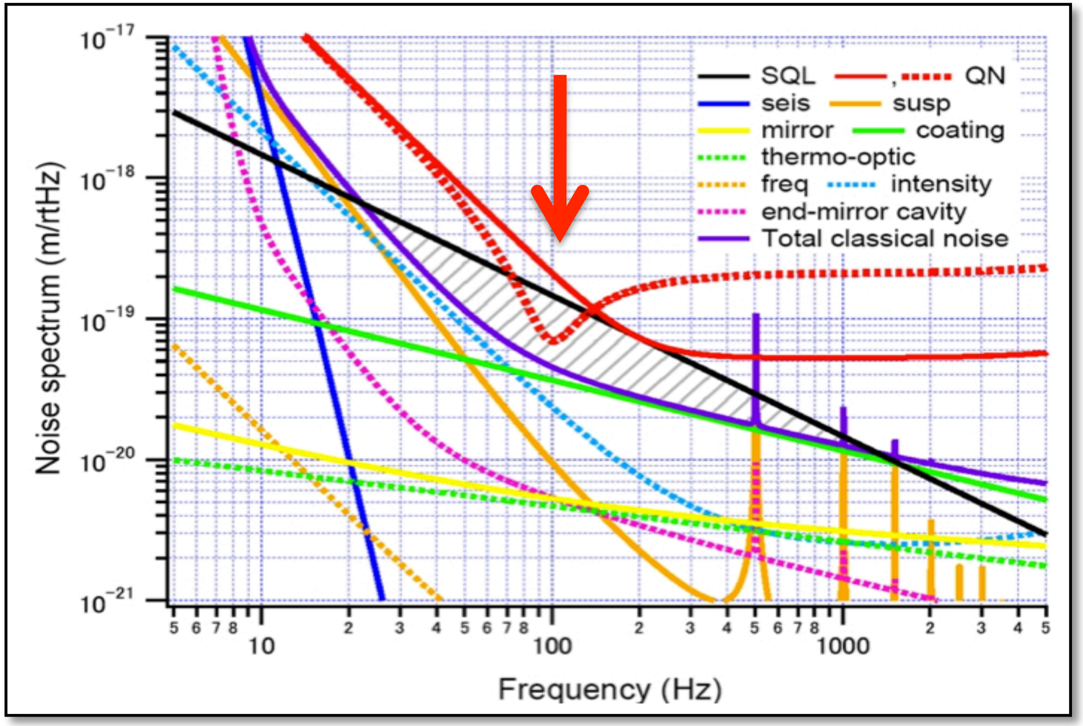


Homodyne

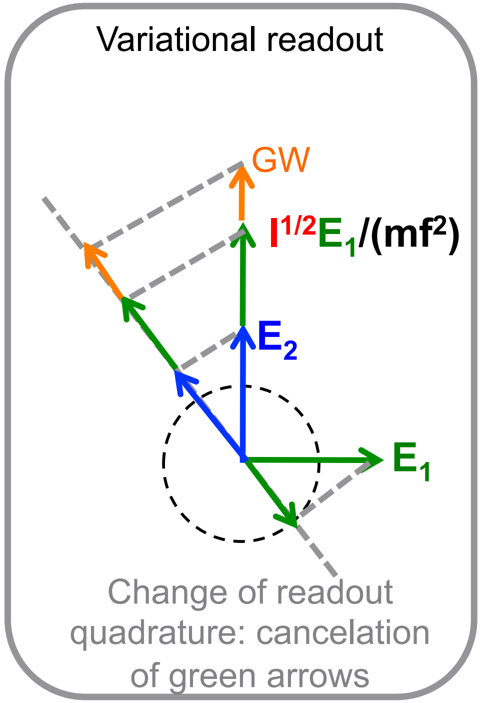


- Variational Readout enables us to partly cancel radiation pressure noise by selecting the appropriate readout quadrature.
- Readout quadarture (angle) can be chosen by phase shifter

Surpassing the SQL at AEI-10m-Interferometer



Variational readout



Change of readout quadrature: cancelation of green arrows

- Using Variational readout, we will be able to completely cancel the radiation pressure noise at **ONE** frequency (100Hz) and surpass the SQL a factor 2 to 3.
- To cancel the radiation pressure noise a all frequencies, we would need a frequency dependent angle of the readout quadrature

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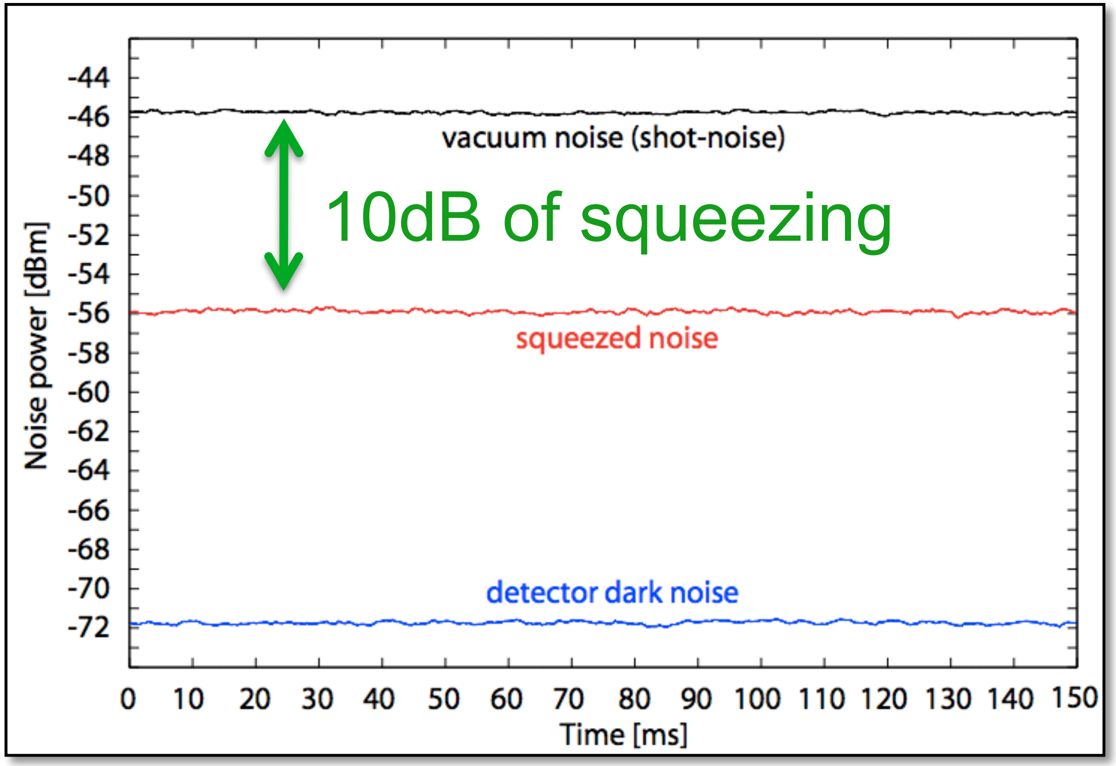
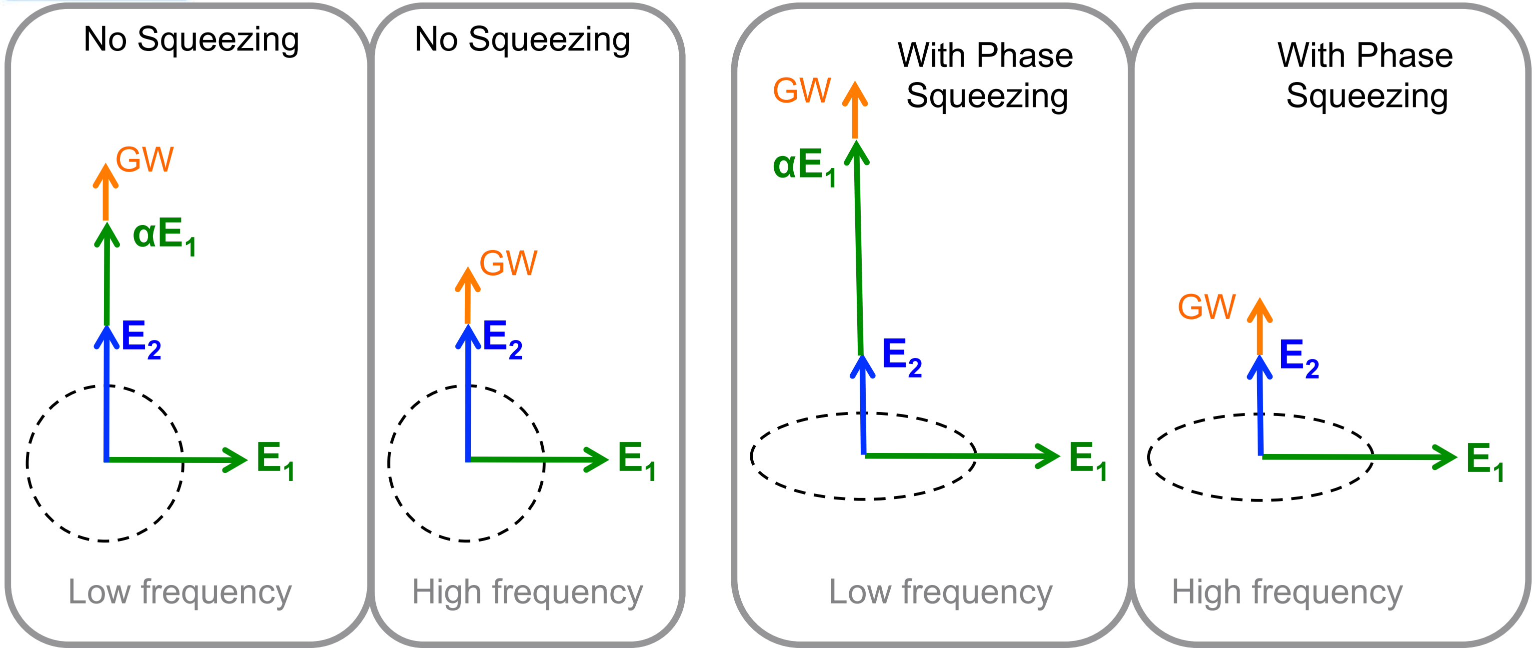
What squeezed light is available ?

- Over the past decade, squeezing made incredible progress.
- We have now all we need:
  - Squeezing at all frequencies of interest (as low as 1 Hz)
  - Squeezing factors > 10dB, improves the quantum noise by a factor 3 (or is equivalent to a power increase of 10)

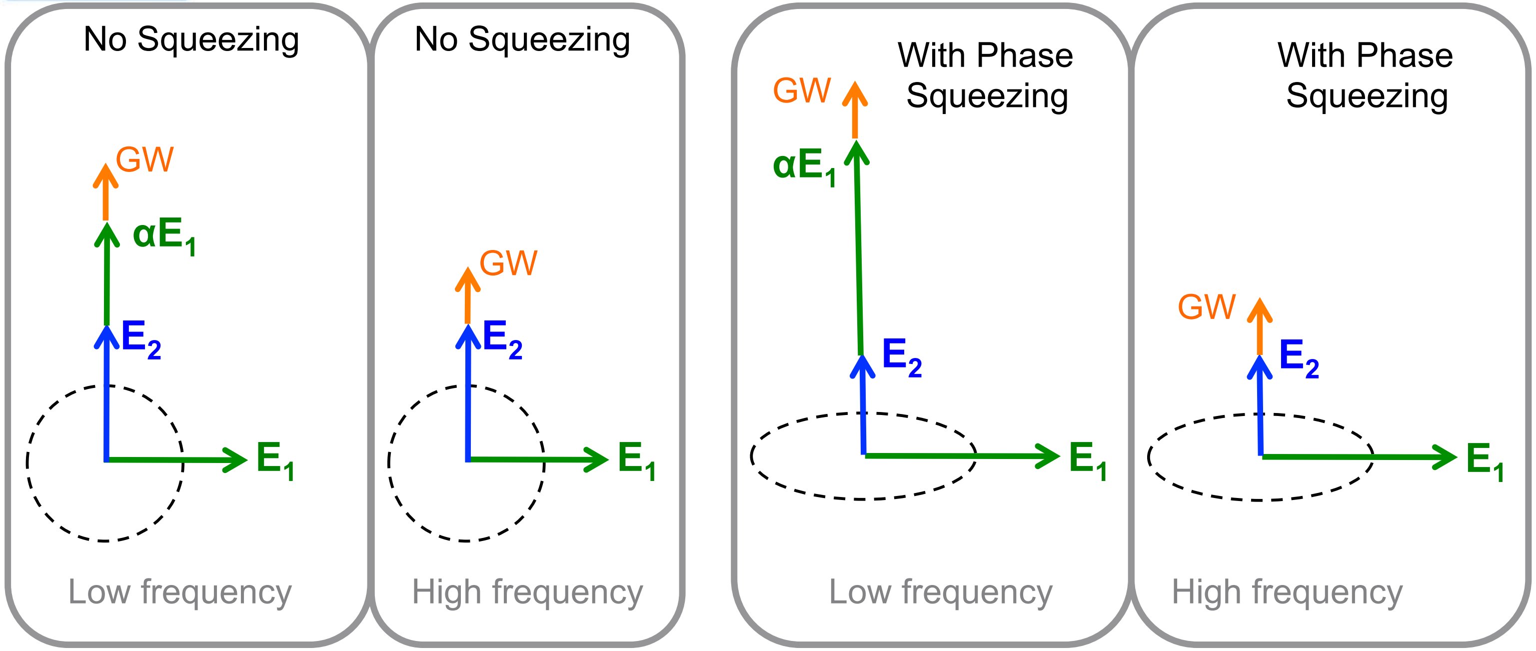


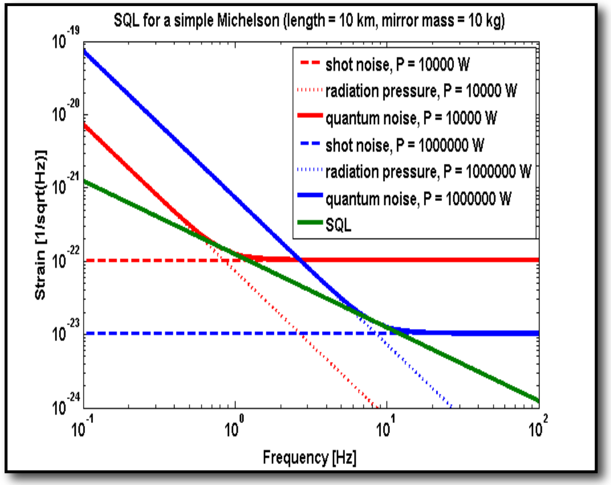
Vahlbruch *et al.*, PRL. 100, 033602

Injecting squeezed light into an interferometer



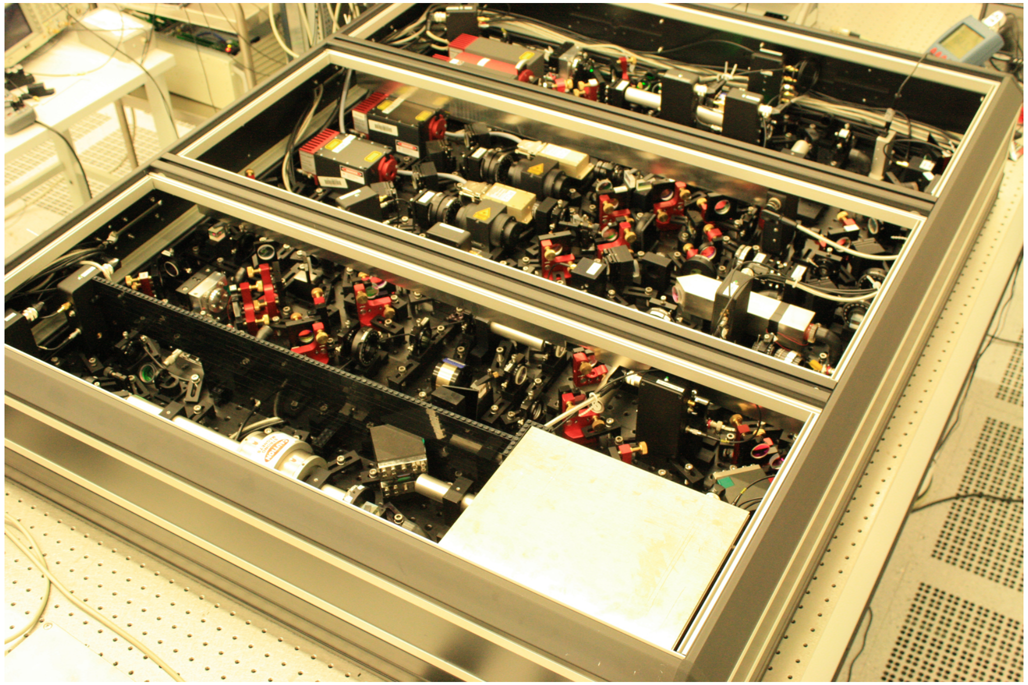
Injecting squeezed light into an interferometer



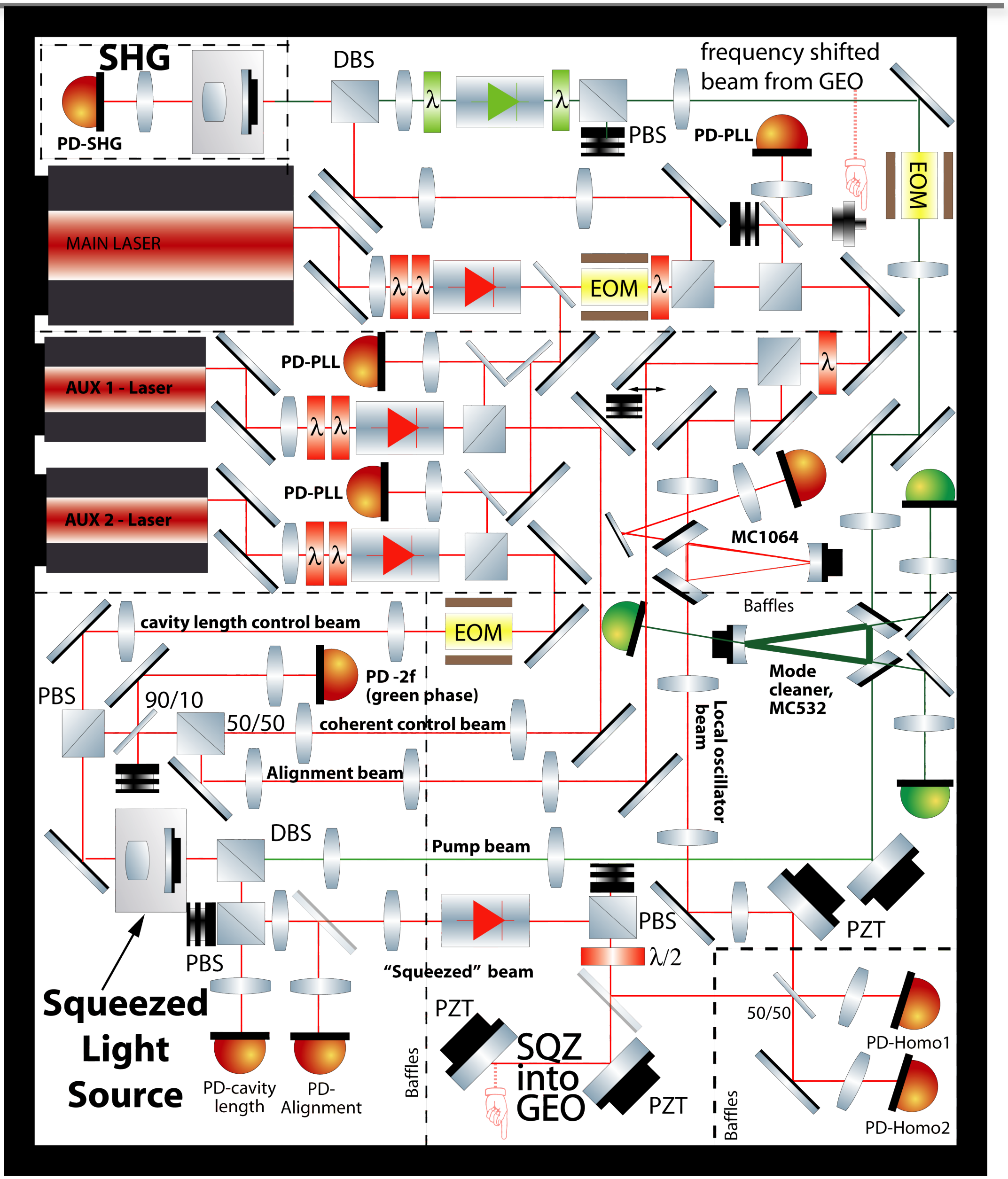


- Injecting phase squeezing into detector output:
  - High frequency sensitivity improved ☺
  - Low frequency sensitivity decreased ☹
- Phase squeezing gives in principle the same as a power increase.
- With pure phase squeezing you cannot beat SQL!

The GEO600 squeezer (schematic)

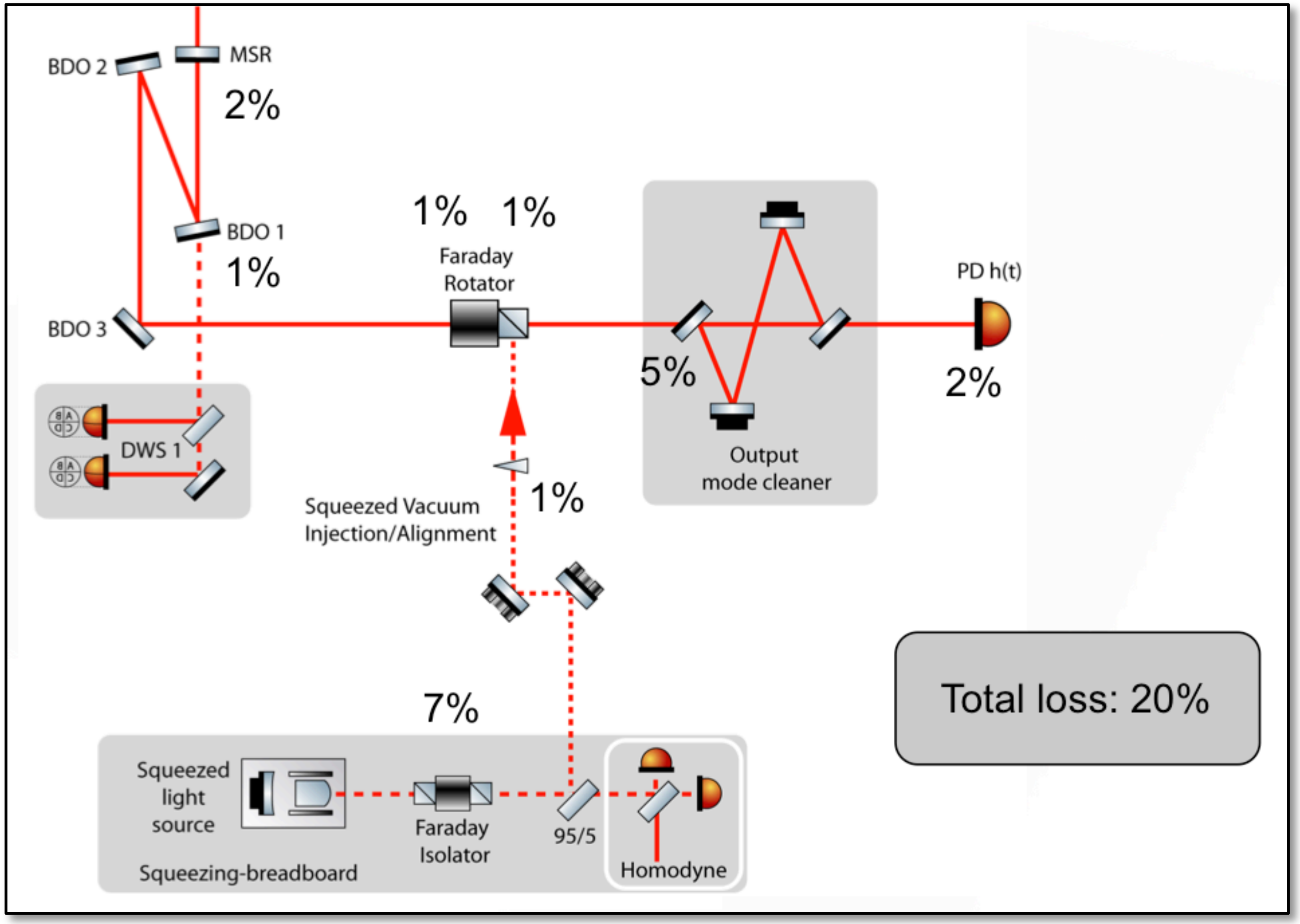


Images courtesy to GEO600 squeezing Group



How to inject squeezed light into GEO??

- Squeezed light is injected via a Faraday rotator into the back of the interferometer.
- It is then reflected from the signal recycling mirror (MSR) and detected at the main photodiode (PD).
- Squeezing requires low losses.



A. Khalaidovski: http://gw.icrr.u-tokyo.ac.jp/gwadw2010/program/2010_GWADW_Khalaidovski.ppt

Squeezing and losses

- If squeezed light is lost it is replaced again by vacuum fluctuations.
- For GEO-HF we start with 10dB. => 20% loss => effective quantum noise reduction of 6dB

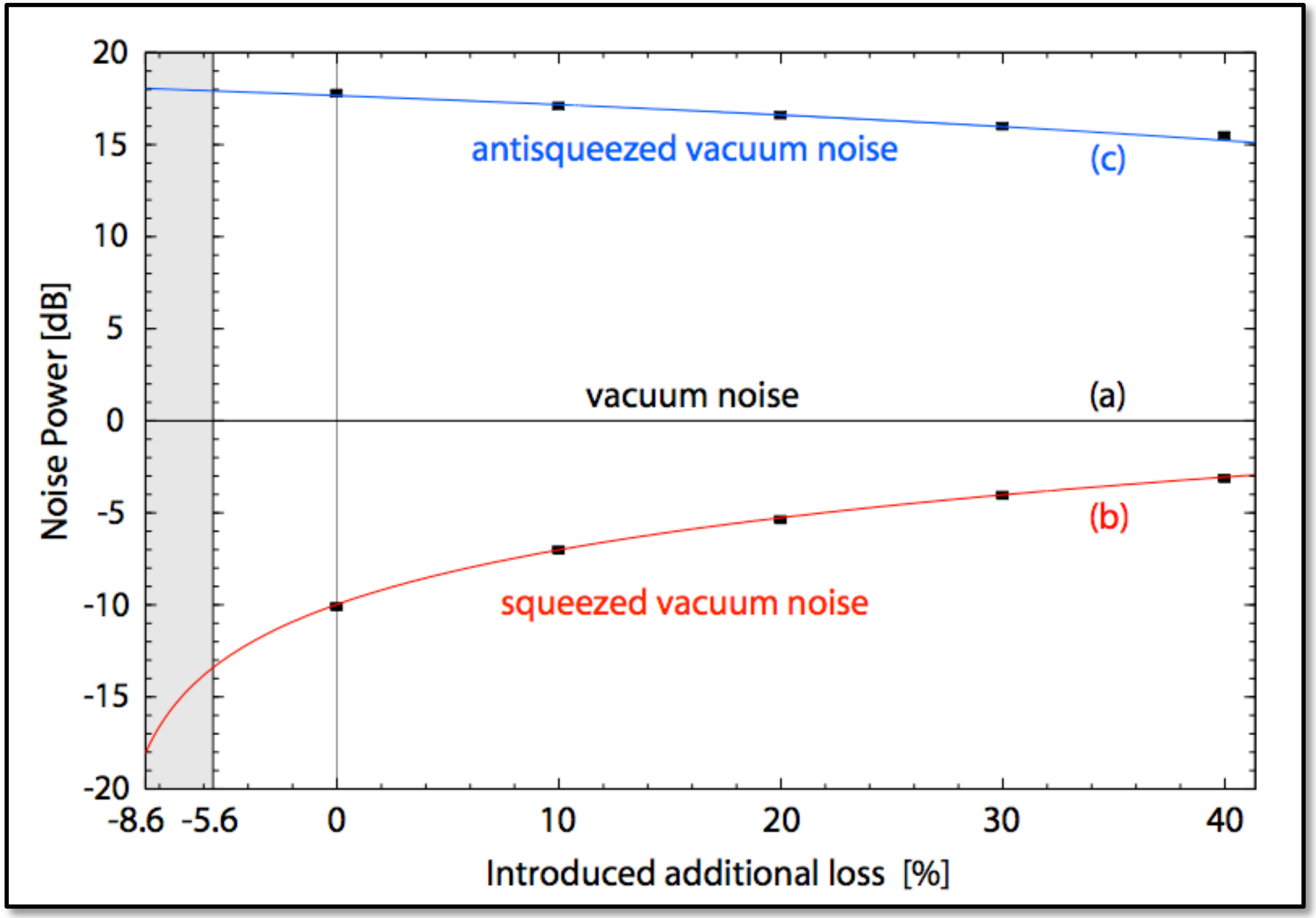
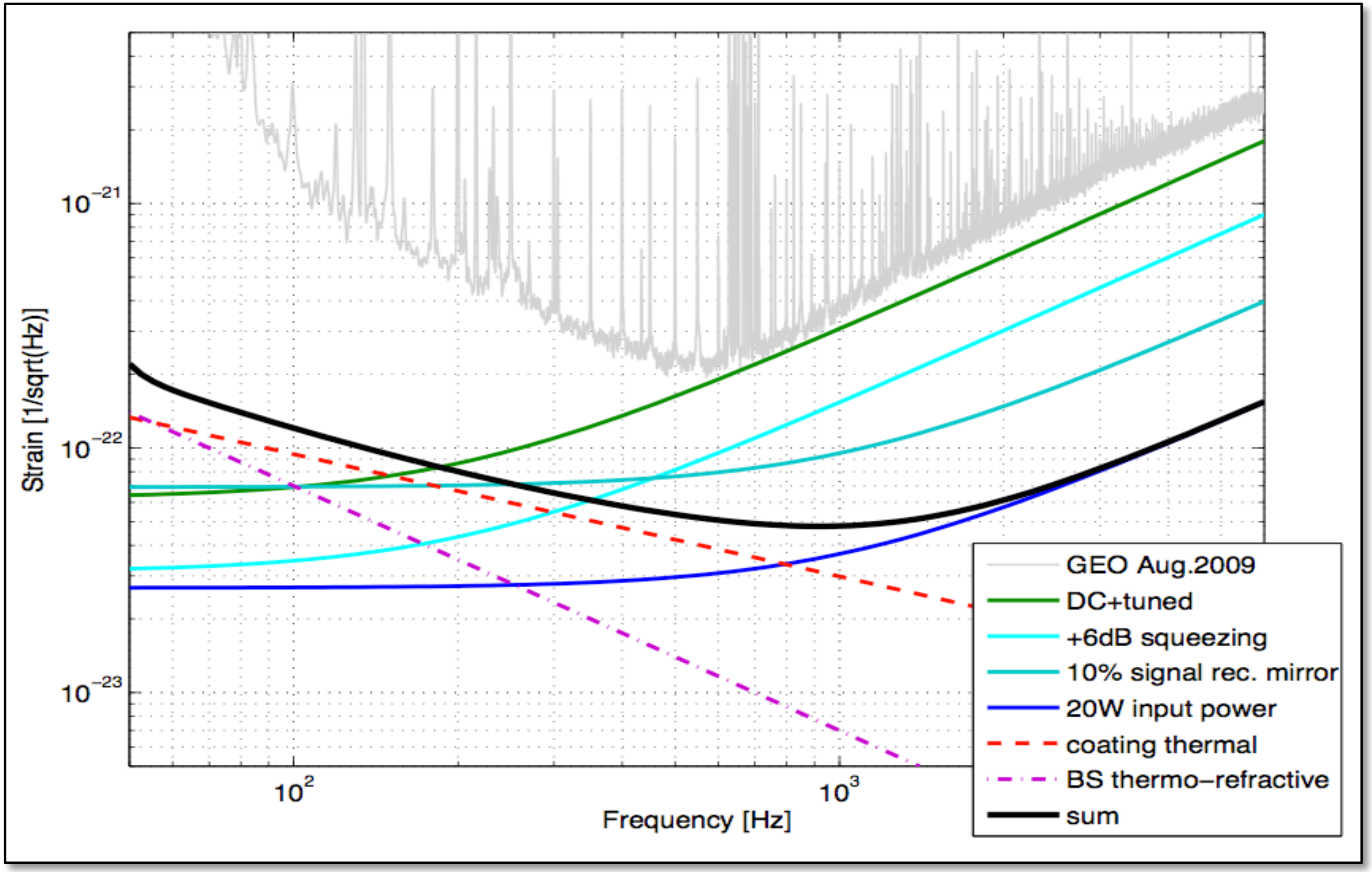


Image: H. Vahlbruch, PhD thesis.

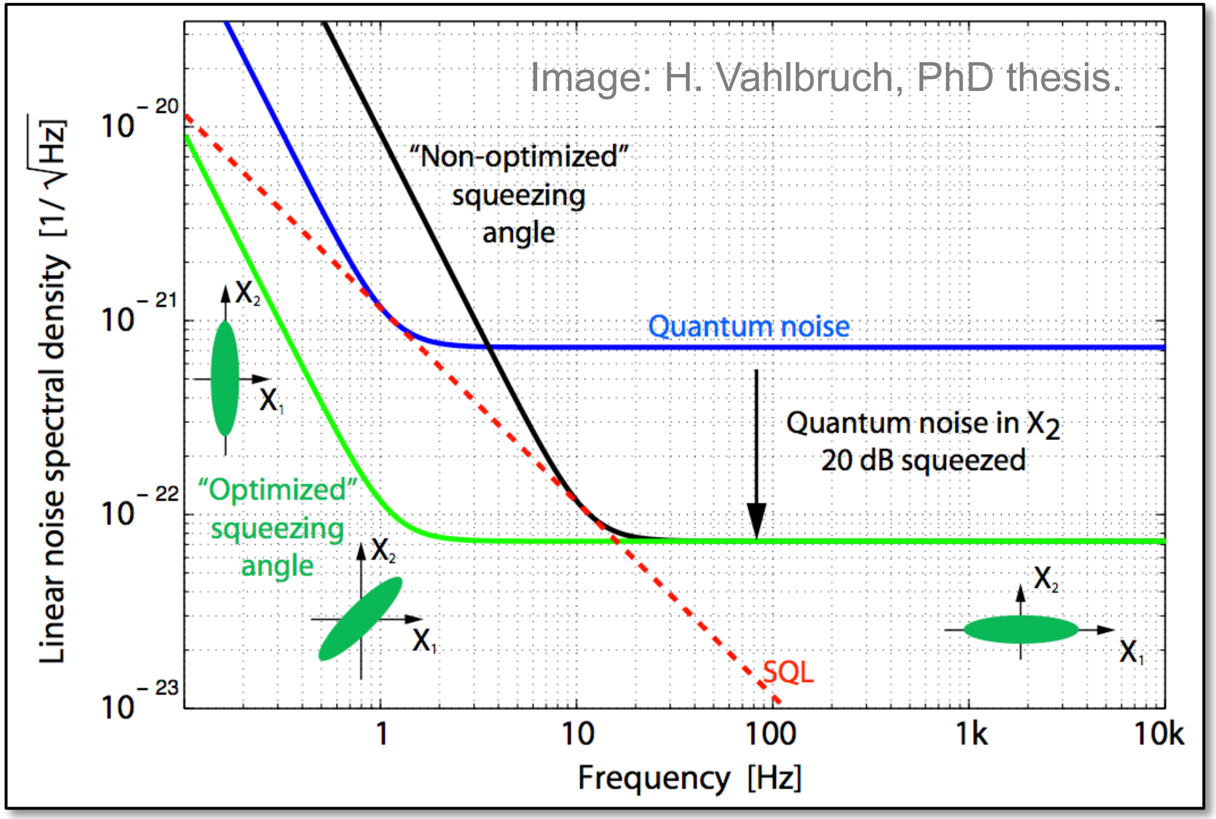
GEO-HF sensitivity



Frequency dependent Squeezing

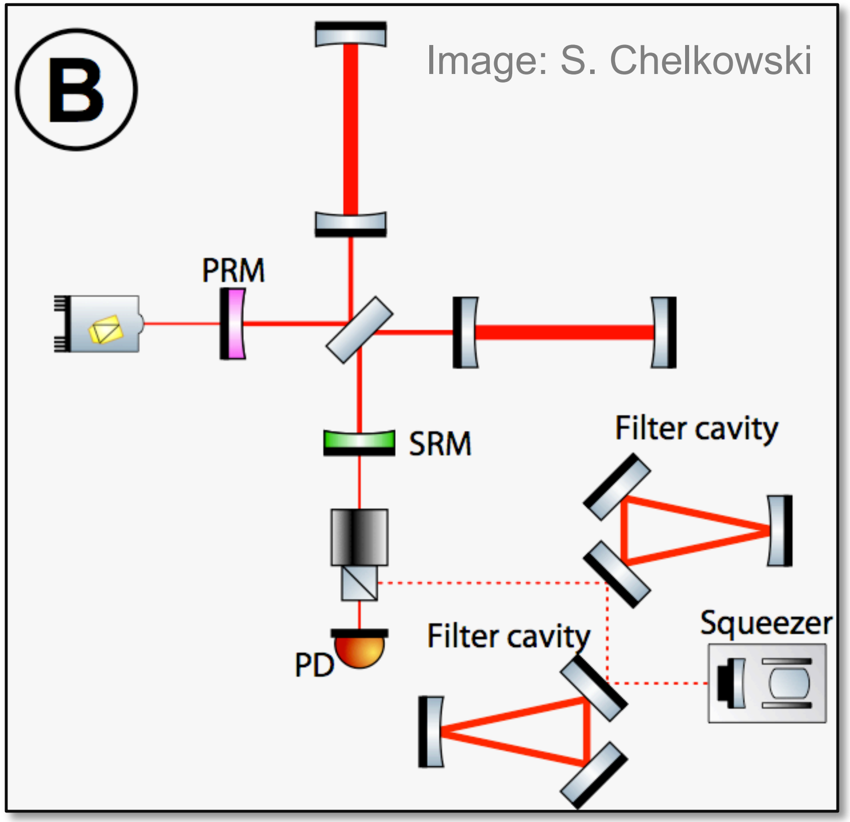
If squeezed light should reduce shot noise and radiation pressure noise, you need a frequency dependent squeezing angle:

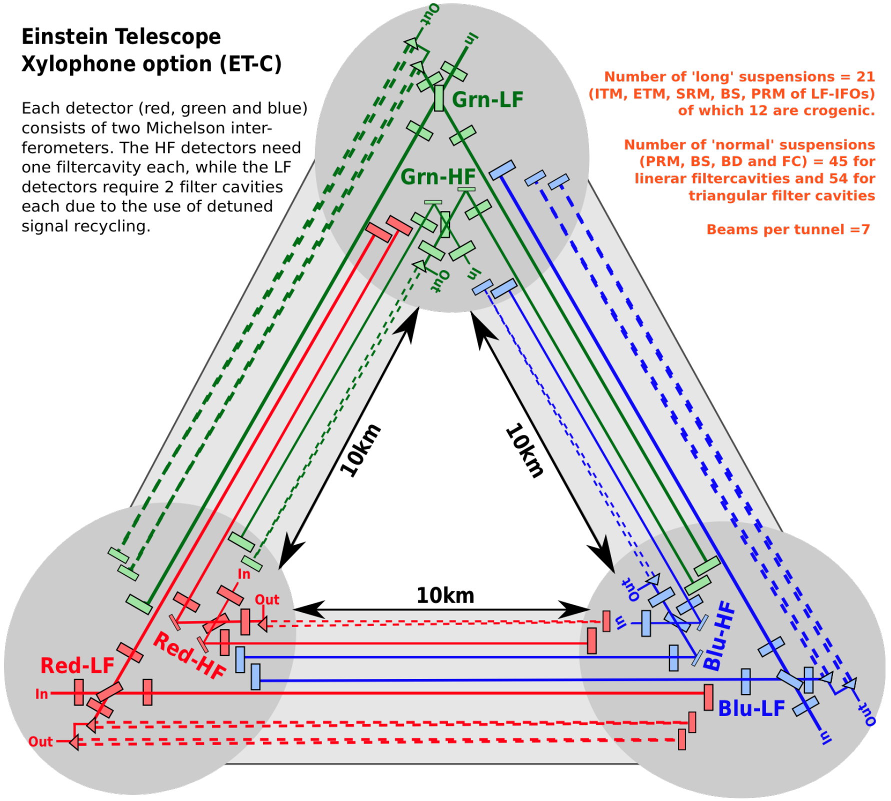
- Phase squeezing at high frequencies.
- Amplitude squeezing at low frequencies



With frequency dependent squeezing it is possible to surpass the SQL.

Creating frequency dependent Squeezing with filter cavities




Einstein Telescope Xylophone option (ET-C)

Each detector (red, green and blue) consists of two Michelson interferometers. The HF detectors need one filtercavity each, while the LF detectors require 2 filter cavities each due to the use of detuned signal recycling.

Number of 'long' suspensions = 21 (ITM, ETM, SRM, BS, PRM of LF-IFOs) of which 12 are crogenic.

Number of 'normal' suspensions (PRM, BS, BD and FC) = 45 for linerar filtercavities and 54 for triangular filter cavities

Beams per tunnel =7

- ➡ We can realise frequency dependent squeezing by reflecting it on a cavity, i.e. making use of the cavity's dispersion.

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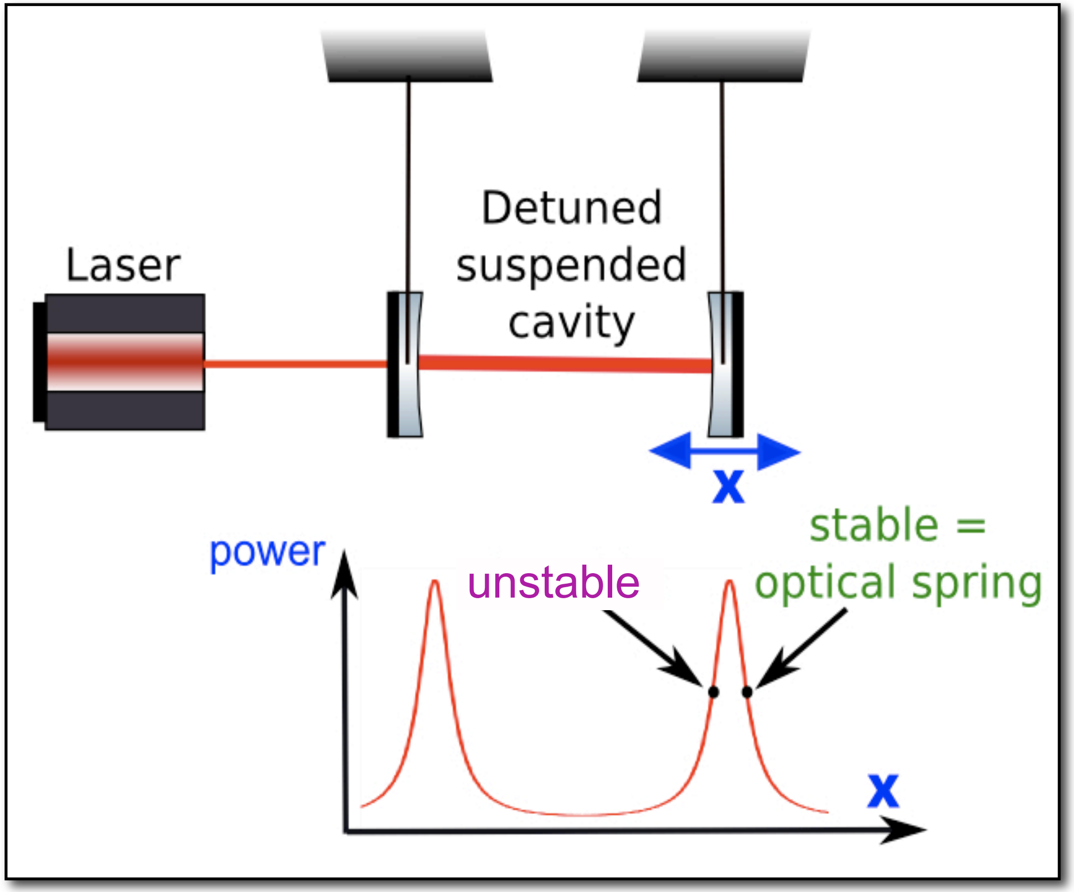
What is an Optical Spring?

- Detuned cavities can be used to create optical springs.
- Position change of the mirror => power changes => radiation pressure force changes.
- Optical springs couple the mirrors of a detuned cavity with a spring constant that can be as stiff as diamond.
- Can be used as low-noise transducer for GW signals to mirror movement in the local frame.



Beating the SQL with advanced detectors?

- Detuned Signal Recycling also creates a optical spring resonance => quantum noise shows two 'bumps', the optical spring (at low frequencies) and the pure optical resonance (at high frequencies).
- Actually advanced LIGO and advanced VIRGO could beat the SQL, if the quantum noise at low frequencies would not be coverd by other noise sources.

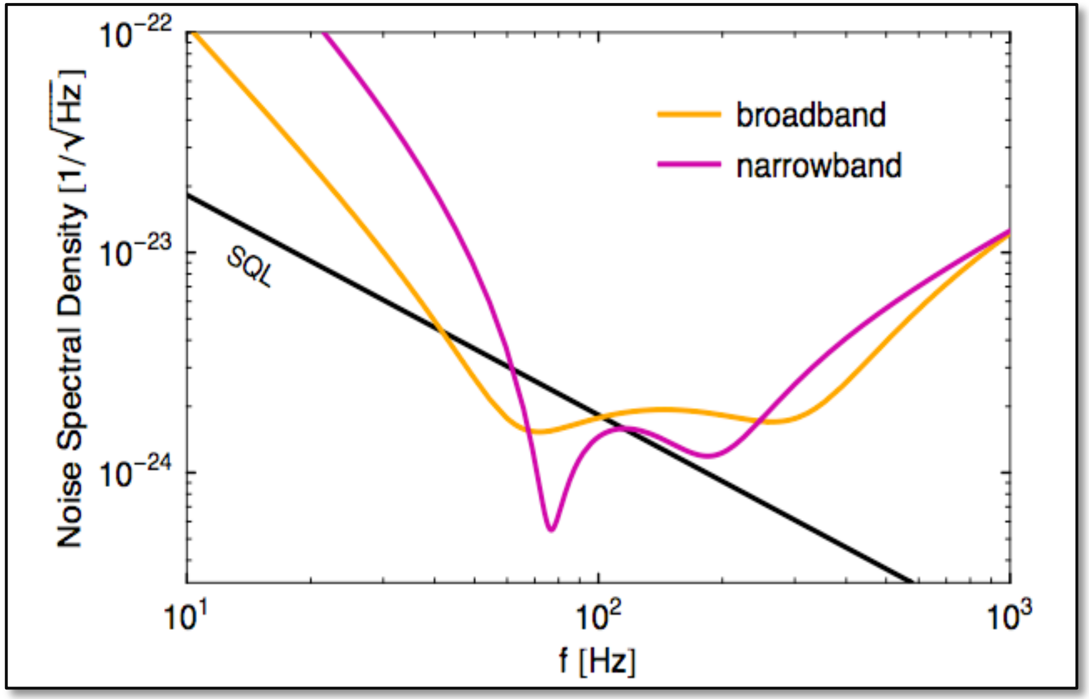
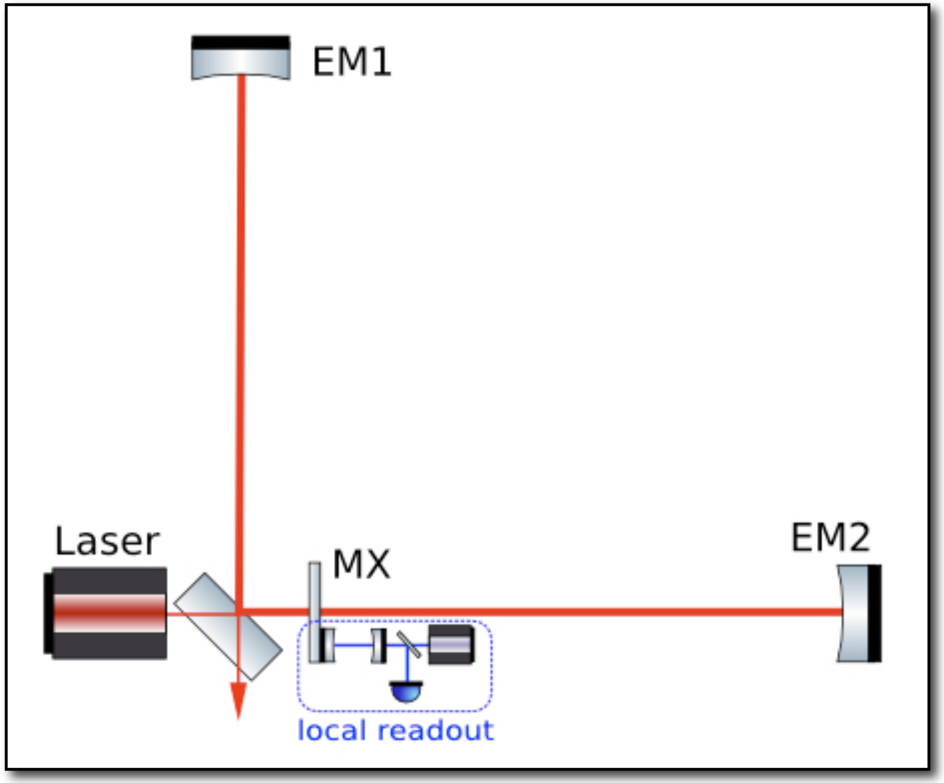


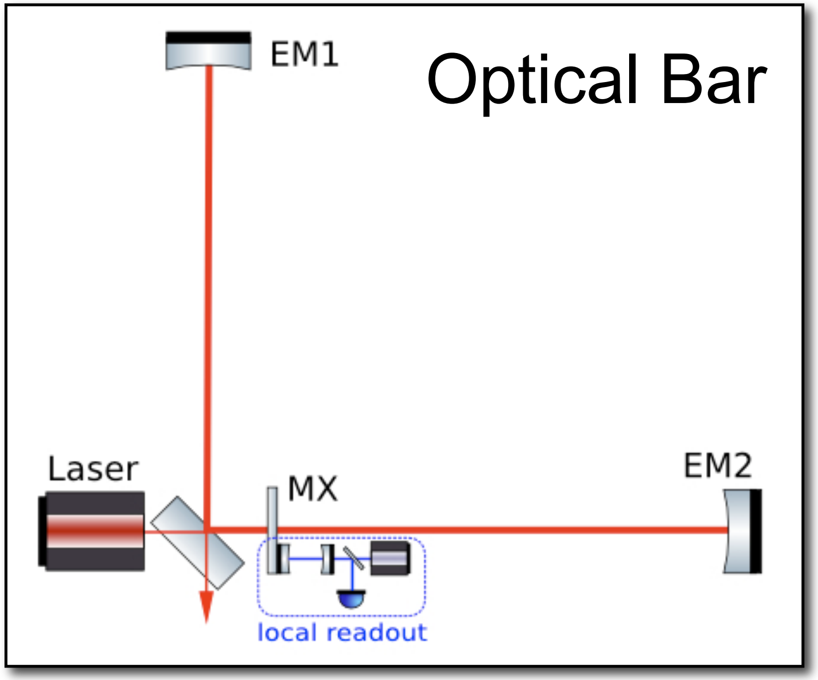
Image H. Mueller-Ebhardt, PhD thesis

Optical Bar configurations

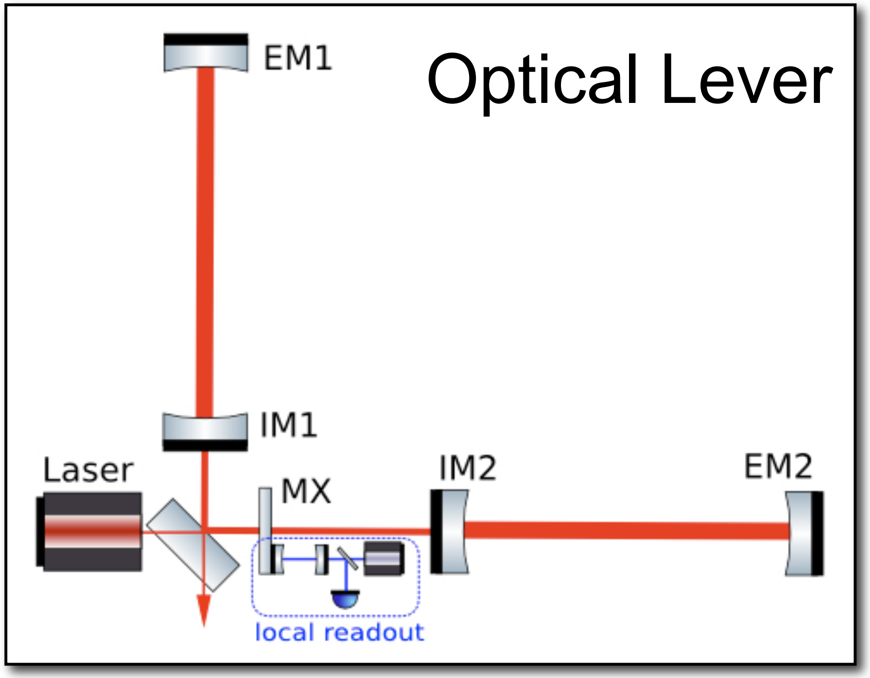
- Very light mirror (MX) is coupled to the movement of EM1 and EM2 via optical springs.
- MX can then locally read out by a small local meter without disturbing the quantum states in the main instrument (QND measurement).
- Split between GW transducer and readout allows separate optimisation of these two systems.



Optical Bars and Optical Levers



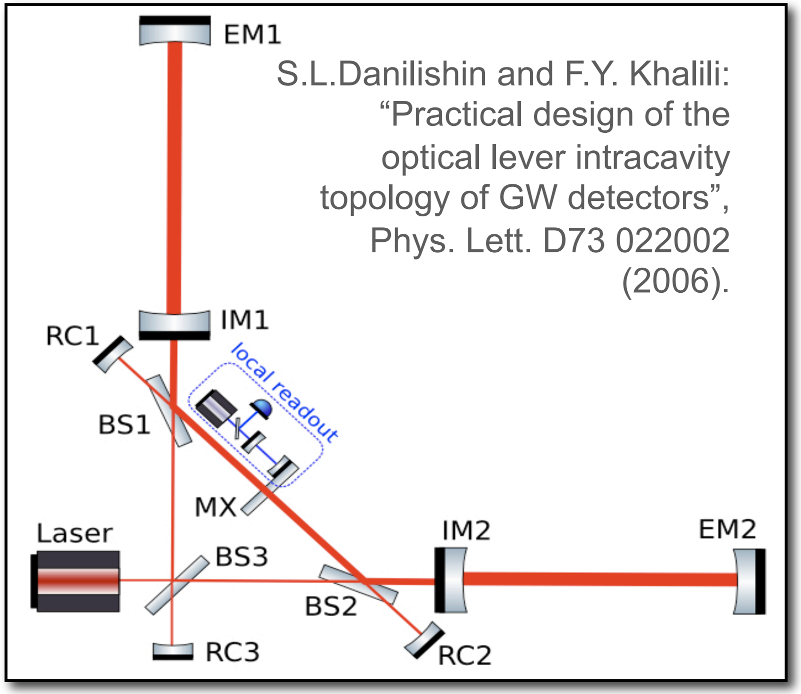
V.B. Braginsky and F.Y. Khalili: “Nonlinear meter for the gravitational wave antenna”,Phys. Lett. A 218 (1996).

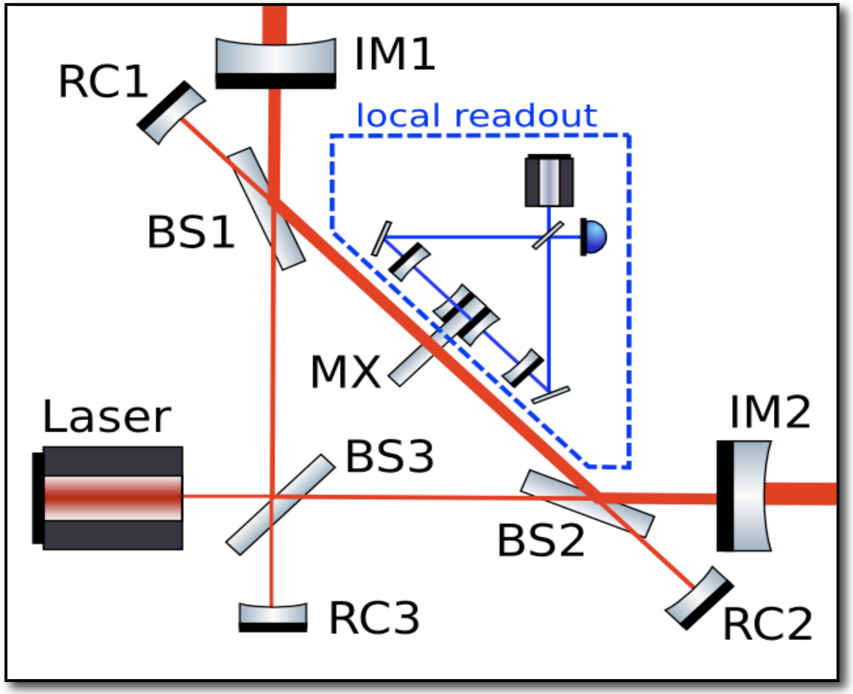


F.Y. Khalili: “The ‘ptical lever’ intracavity readout scheme for gravitational-wave antennae”,Phys. Lett. A 298 (2002).

- Optical lever: introducing arm cavities increases the movement of MX by the Finesse of the arm cavity.

More realistic Designs for an Optical Lever

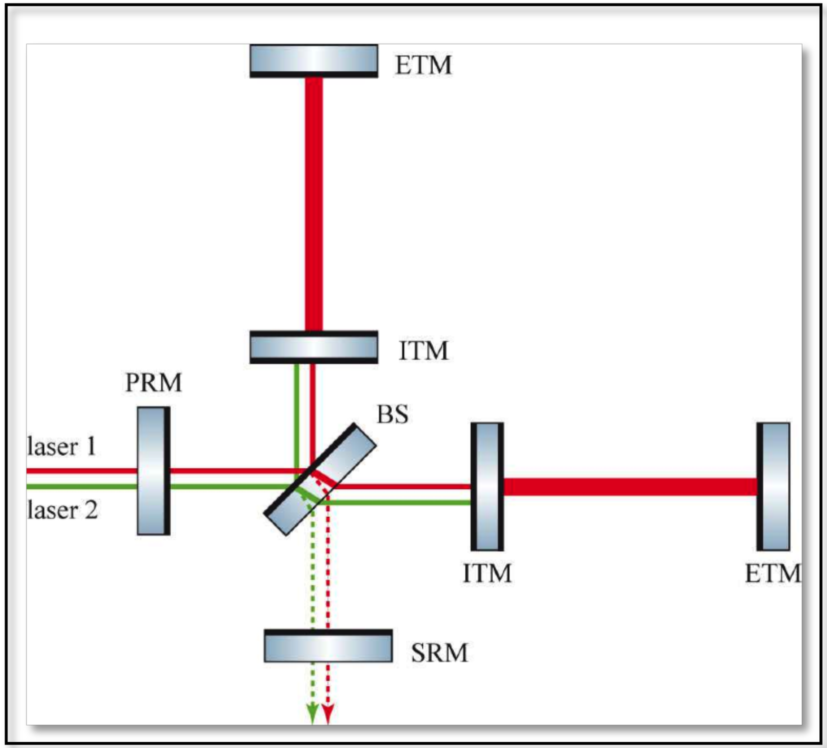

S.L.Danilishin and F.Y. Khalili: “Practical design of the optical lever intracavity topology of GW detectors”, Phys. Lett. D73 022002 (2006).



- Build symmetric optical levers => make use of common mode noise rejection.
- Use recycling techniques, such as power recycling.
- Increased number of components => more complex couplings => harder to control.
- KEYPOINT: Get the local readout to the required sensitivity.

Local Readout for Advanced LIGO

- While optical Bars and levers require a complete redesign of the interferometers, so-called ‘local readout’ is compatible within advanced LIGO infrastructure
- At low frequencies ITM and ETM are rigidly connected.
- At low frequencies GW signal is not in differential arm length, but in ITM movement (local frame).
- Use a separate laser system to read out the position of the ITM.



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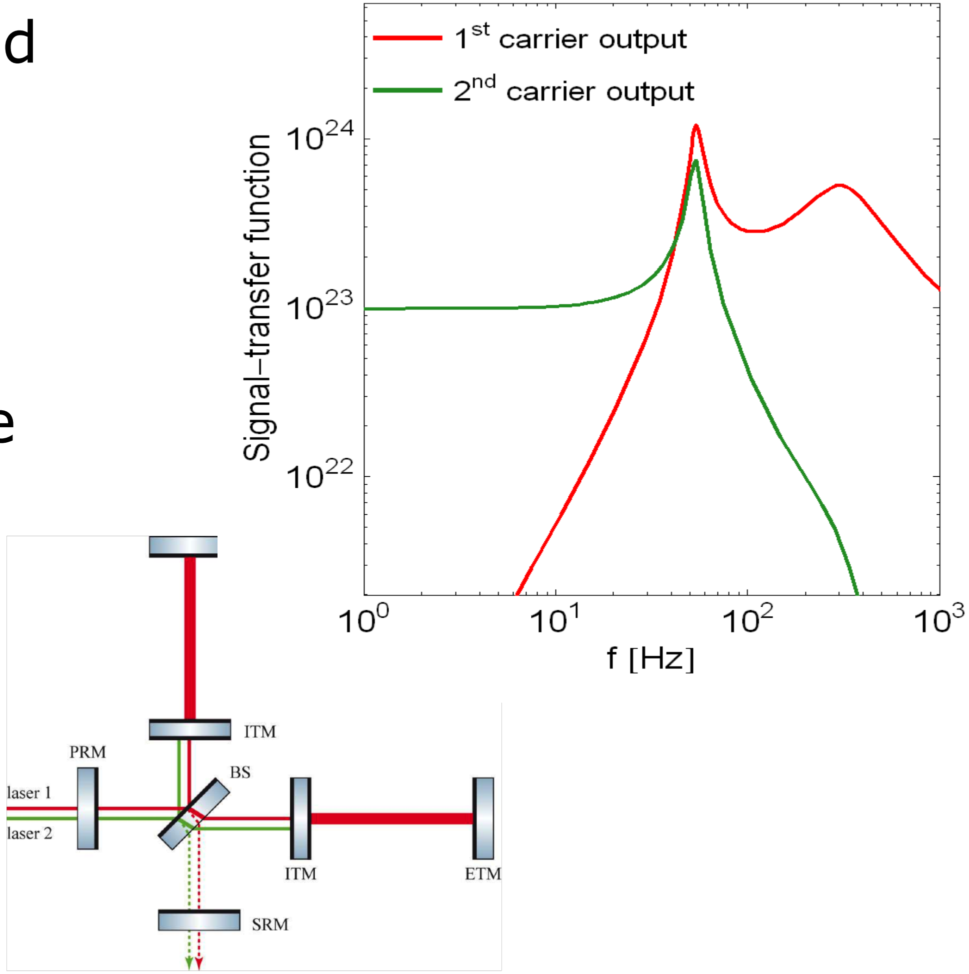
Local Readout for Advanced LIGO (2)

How does local readout for Advanced LIGO work?

- **At low frequencies:** the arm cavity mirrors are ‘rigidly’ connected by optical springs => GW does not change the distance between ITM and ETM. However, GW signal is imprinted on ITM movement (in respect to BS), which and can be read out by additional green laser.
- **At high frequencies:** no optical spring present => ITM and ETM can move independently.



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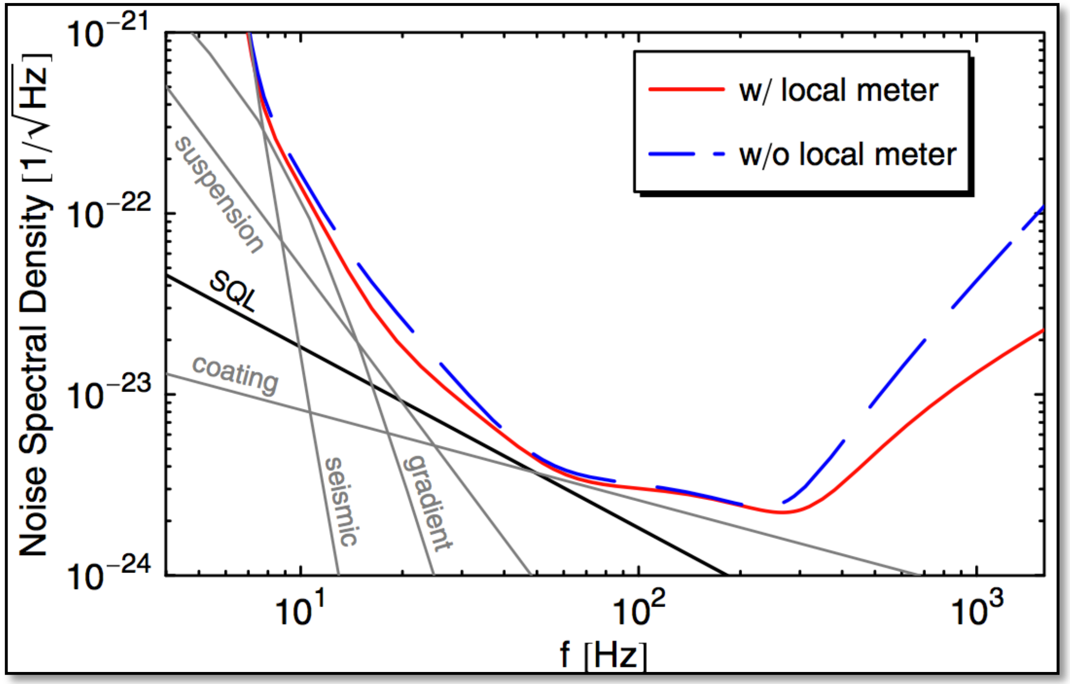
Local Readout for Advanced LIGO (3)

- Technique allows to increase low frequency sensitivity.
- In a second step the Signal-Recycling can then be re-tuned to slightly higher frequency.
- Win at low and high frequencies. =>



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Overview

- Introduction: Quantum noise, Standard Quantum Limit, Vacuum fluctuations, ‘Ball-on-a-stig’, Quadrature Picture.
- The easiest way to beat the SQL: Varaiational Readout
  - Example: AEI-10m interferometer
- The best quantum noise reduction technique for second generation GW detectors: Squeezed light injection
  - How to make squeezed light?
  - Example: GEO-HF and Einstein telescope
- Optical rigidity
  - What is an optical spring?
  - Optical Bar and Optical Lever schemes
  - Local Readout scheme for Advanced LIGO
- Speed meter configurations

Speedmeter 1

- So far we have only considered GW detectors that measure the position of the test masses.
- So-called speed meters have been suggested to be able to cancel radiation pressure noise.
- Idea: Measure the position difference after time delay.
- In principle this is then the same as measuring the speed of the testmass.
- How can we realise a speed-meter?

Speed meter 2

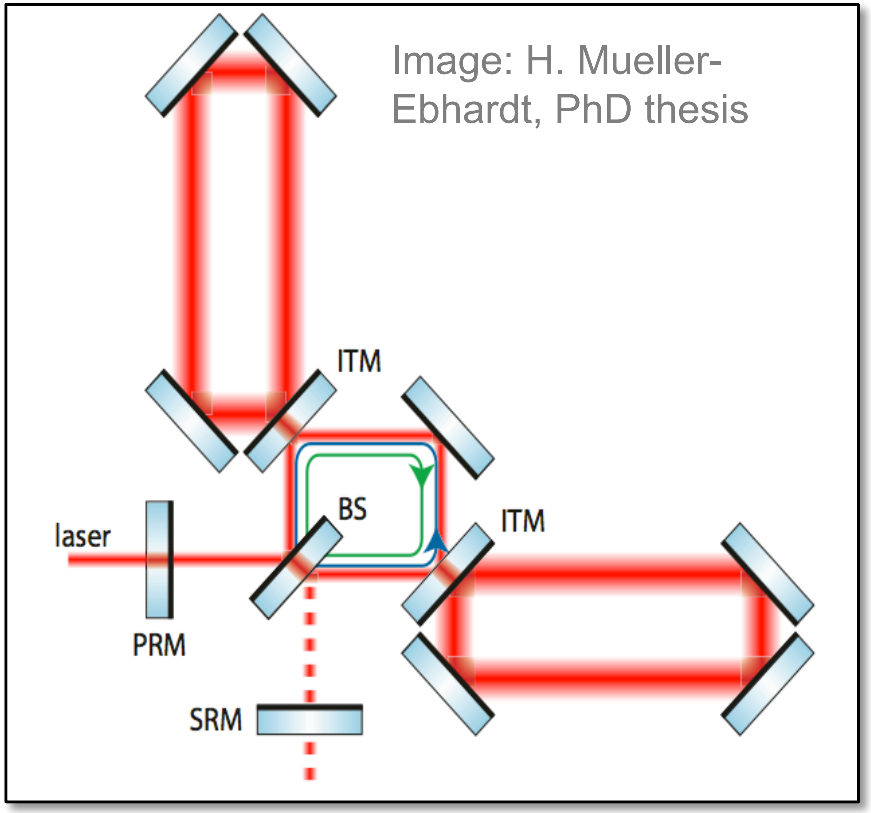

Image: H. Mueller-Ebhardt, PhD thesis

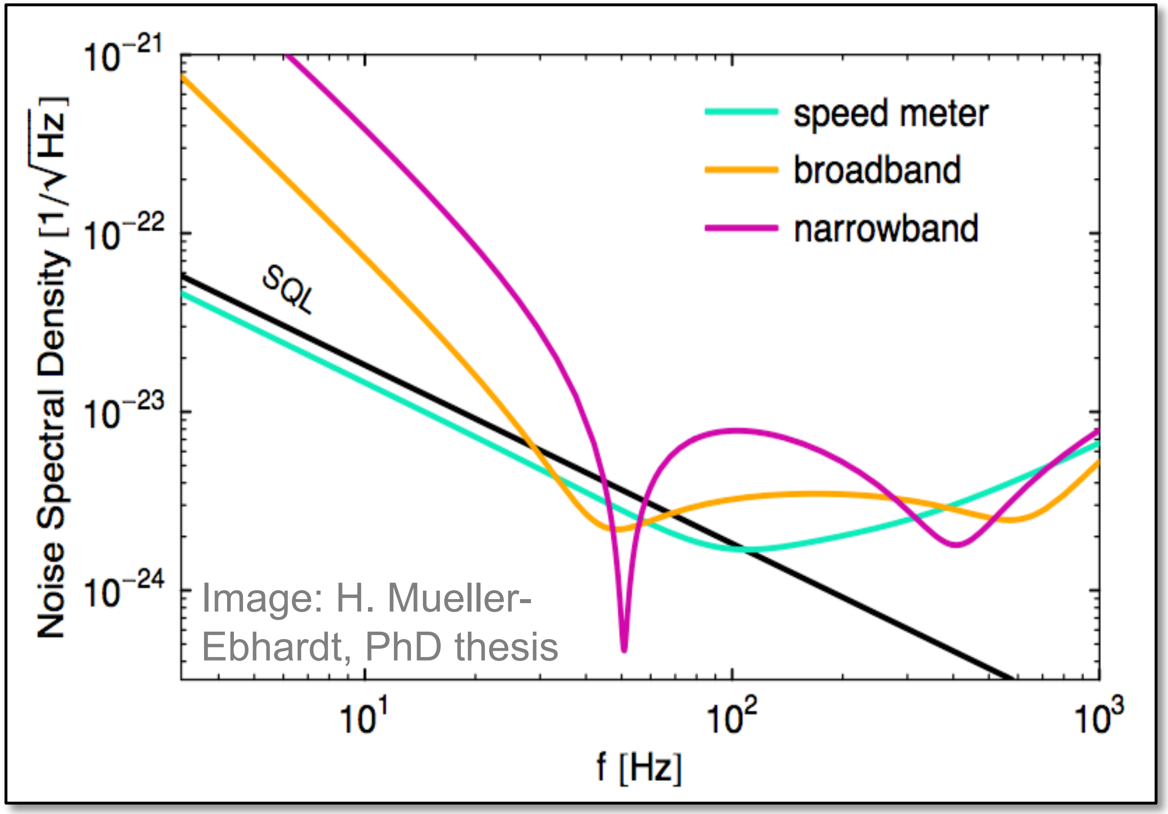

Image: H. Mueller-Ebhardt, PhD thesis

- Speedmeter can in principles cancel radiation pressure noise to a large extend and surpass the SQL over a wide frequency range

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