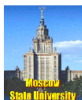


# Khalili Etalons for the Einstein GW Telescope

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Moscow State University, University of Jena, University of Glasgow and Caltech

2.3.2010



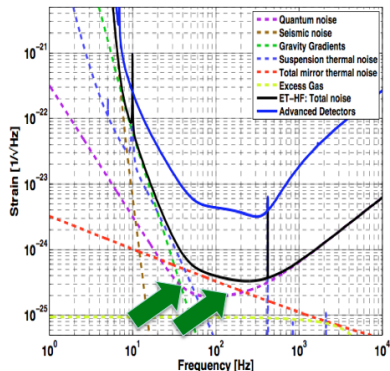
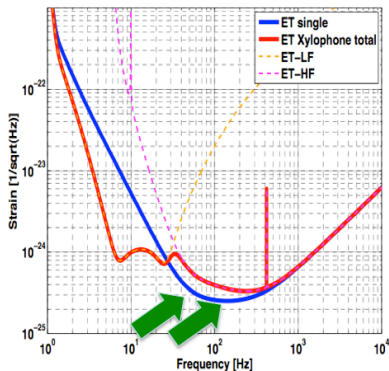
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of GLASGOW



# Overview of this presentation

- 1 **Motivation for Khalili Etalons (Stefan)**
- 2 Noise contributions for a Khalili cavity, i.e. no coupling through the substrate considered (Ronny)
- 3 How to calculate the noise couplings inside an etalon.? Example: Coating Brownian noise for finite size etalon (Kentarō)
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# Motivation: Coating Brownian Noise in ET-C



- ET-C is limited by Coating Brownian noise around 100Hz.
- ET-C-HF detector: Room temperature, maximised beam radius (12cm) and LG33

# How can we further decrease coating noise w/o cooling?



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## Reducing the mirrors coating noise in laser gravitational-wave antennae by means of double mirrors

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### Abstract

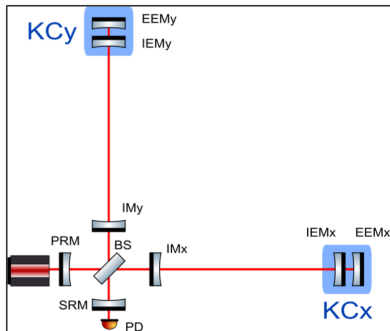
Recent researches show that the fluctuations of the dielectric mirrors coating thickness can introduce a substantial part of the future laser gravitational-wave antennae total noise budget. These fluctuations are especially large in the high-reflectivity end mirrors of the Fabry–Pérot cavities which are being used in the laser gravitational-wave antennae.

We show here that the influence of these fluctuations can be substantially decreased by using additional short Fabry–Pérot cavities, tuned in antiresonance instead of the end mirrors.

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### 1. Introduction

One of the basis components of laser gravitational-wave antennae [1–3] are high-reflectivity mirrors with multilayer dielectric coating. Recent researches [4–13] have shown that fluctuations of the coating thickness produced

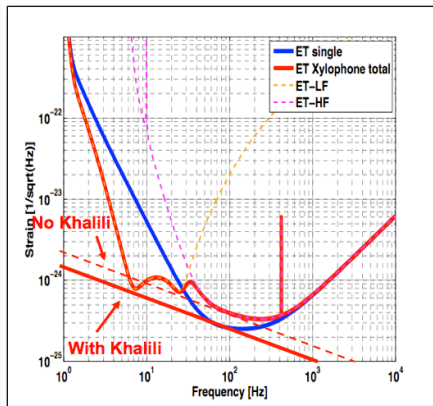
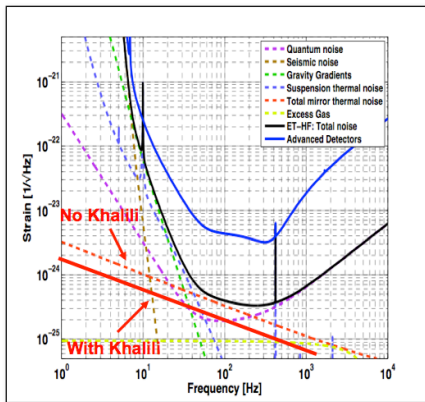


## Crude estimate of Coating Brownian Reduction from Khalili cavities

From talk at WP3 meeting in Glasgow, December 2009:

- Assuming a reflectivity of about 0.7 for the first Khalili mirror gives:
- Reduce the coating noise of the end mirrors by a factor 2.7
- Reduce the overall coating Brownian contribution (full IFO) by a factor 1.5

# How much can we win with Khalili cavities?



From S.Hild: "Khalili etalons for ET", presentation at ET-WP3 meeting in Glasgow, December 2009.

# Motivation of using Khalili Etalons instead of Khalili cavities?

- One potential disadvantage of Khalili cavities for ET is that **it might be hard to control them** in case the Khalili cavities are short (a few meters) compared to the arm cavities (10 km).
- This disadvantage might potentially be overcome by using an Khalili etalon instead of two independent mirrors.
- However, nothing comes for free! If we make use of an etalon, then the coating noise benefit is reduced, because there is a mechanical coupling of noise components from the back surface to the front surface and vice versa.

## The questions we want to answer in this presentation

- Which are the dominant noise contributions of a Khalili etalon?
- How can we actually take the coupling through the substrate into account?
- How much worse is a Khalili etalon compared to a Khalili cavity?

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# Dummy set of parameters used for our analysis

We decided to do our analysis with a specific set of parameters: The first end mirror surface carries 3 coating doublets leading to a reflectivity of about 0.7 and optical power inside the Khalili cavity/etalon of 300 kW for ET-C HF.

parameter	bulk	SiO <sub>2</sub> layer	Ta <sub>2</sub> O <sub>5</sub> layer
thickness	30 cm	7.34×10 <sup>-7</sup> m front 3.12×10 <sup>-6</sup> m back	3.92×10 <sup>-7</sup> m front 2.09×10 <sup>-6</sup> m back
$n$	1.45	1.45	2.035
$\beta = \frac{dn}{dT}$	8×10 <sup>-6</sup> 1/K	8×10 <sup>-6</sup> 1/K	1.4×10 <sup>-5</sup> 1/K
$\rho$	2202 kg/m <sup>3</sup>	2202 kg/m <sup>3</sup>	6850 kg/m <sup>3</sup>
$Y$	72 GPa	72 GPa	140 GPa
$\nu$	0.17	0.17	0.23
$\alpha$	5.5×10 <sup>-7</sup> 1/K	5.5×10 <sup>-7</sup> 1/K	3.6×10 <sup>-6</sup> 1/K
$\kappa$	1.38 W/m K	1.38 W/m K	0.46 W/m K
$C$	746 J/kg K	746 J/kg K	306 J/kg K
$\phi$	4×10 <sup>-10</sup>	4×10 <sup>-5</sup>	2×10 <sup>-4</sup>

**Table:** Material parameters. The layer thicknesses are total thicknesses summing over all  $\lambda/4$  layers.



- **Brownian bulk thermal noise**

$$S_{x,b}(f, T) = \frac{2k_B T(1 - \nu_b^2)}{\pi^{3/2} Y_b f w} \phi_b, \quad (1)$$

- **Thermo-elastic bulk thermal noise**

$$S_{TE,b}(f, T) = \frac{4k_B T^2 \alpha_b^2 (1 + \nu_b)^2 \kappa_b}{\pi^{5/2} (\rho_b C_b)^2 w^2 f^2} \quad (2)$$

- **Thermo-refractive bulk thermal noise**

$$S_{TR,b}(f, T) = \frac{4k_B T^2 \beta_b^2 h \kappa_b}{\pi^3 (\rho_b C_b)^2 w^4 f^2} \quad (3)$$

- **Brownian coating thermal noise**

$$S_{x,c}(f, T) = \frac{2k_B T(1 - \nu_b^2)}{\pi^{3/2} Y_b f w} \phi_c, \quad (4)$$

# Formulas for the noise contributions (continued)

- **Thermo-elastic coating thermal noise**

$$S_{\text{TE,c}}(f, T) = \frac{8k_{\text{B}} T^2}{\pi^2 f} \frac{t_{\text{l}} + t_{\text{h}}}{w^2} \frac{\alpha_{\text{b}} \rho_{\text{c}} C_{\text{c}}}{(\rho_{\text{b}} C_{\text{b}})^2} (1 + \nu_{\text{b}})^2 \Delta^2 g(\omega) \quad (5)$$

- **Thermo-refractive coating thermal noise**

$$S_{\text{TR,coating}}(f, T) = \frac{2k_{\text{B}} T^2 \beta_{\text{eff}}^2 \lambda^2}{\pi^{3/2} \sqrt{\kappa_{\text{b}} \rho_{\text{b}} C_{\text{b}}} w^2 \sqrt{f}}, \quad (6)$$

## Summation of the different noise contributions

$$S_{\text{Khalili}}(f, T) = S_{\text{front}}(f, T) + TF^2 \times S_{\text{back}}(f, T). \quad (7)$$

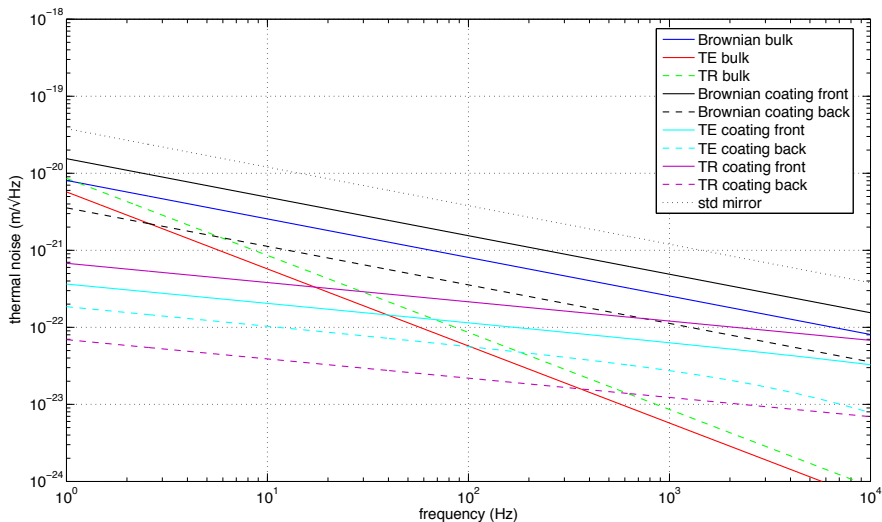
with

$$S_{\text{front}} = S_{\text{x,b}} + S_{\text{TE,b}} + S_{\text{x,c,front}} + S_{\text{TE,c,front}} + S_{\text{TR,c,front}} \quad (8)$$

$$S_{\text{back}} = S_{\text{x,b}} + S_{\text{TE,b}} + S_{\text{x,c,back}} + S_{\text{TE,c,back}} + S_{\text{TR,c,back}} + S_{\text{TR,b}}. \quad (9)$$

and  $TF$  being the optical coupling transferfunction ( $=0.102$ ) for the second mirror of the Khalili cavity, determined by a numerical simulation (Finesse).

# Summary of the different noise sources in the Khalili-etalon under investigation.



noise	standard mirror (absolut)	standard mirror (relativ)	Khalili-etalon (absolut)	Khalili-etalon (normalized)
bulk				
Brownian	$8.04 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.21	$8.08 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.52
TE	$5.70 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.15	$5.73 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$	0.04
TR	—	—	$8.51 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$	
front coating				
Brownian	$3.90 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$	1.00	$1.55 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$	1.00
TE	$6.78 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.17	$1.14 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.07
TR	$2.15 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.06	$2.15 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.14
back coating				
Brownian	—	—	$3.56 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	0.23
TE	—	—	$5.62 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$	0.04
TR	—	—	$2.19 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$	0.01

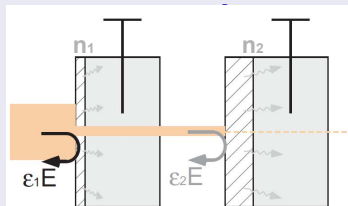
**Table:** Numerical summary of the thermal noise contributions. All contributions are extracted from the total thermal noise. The effect of transfer function is included. The relative numbers are normalised to the largest noise source which is in both cases the Brownian thermal noise of the front surface.

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# Mechanical separation

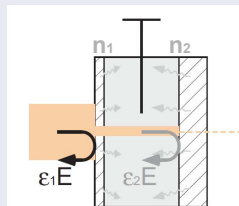
## Khalili-Cavity



$$TN = \varepsilon_1 \cdot n_1 + \varepsilon_2 \cdot n_2 \quad (10)$$

complete mechanical separation

## Khalili-Etalon



$$TN = \varepsilon_1 \cdot (n_1 + TF_{BtoF} \cdot n_2) \quad (11)$$

$$+ \varepsilon_2 \cdot (n_2 + TF_{FtoB} \cdot n_1) \quad (12)$$

partial mechanical separation

**Terms with same  $n_j$  should be coherently added.**

Derivation of TN is different from that with a simple mirror.

# Review of TN calculation

## 1. Fluctuation-dissipation Theorem

$$\frac{\text{thermal energy}}{\text{kinetic energy}} = \frac{\text{work by external force}}{\text{dissipated energy}} \quad (13)$$

Left-hand side = **real world**, right-hand side = **imaginary world**

- thermal energy  $\sim k_B T$
- kinetic energy  $\sim m x_{TN}^2 \Omega^2 / 2$
- work by external force  $\sim F_0 x_0$  (with  $x_0 = F_0 / (m \Omega^2)$ )
- dissipated energy  $\sim W / \omega$ .

$$\implies S_x = \frac{8 k_B T W}{\Omega^2 F_0^2} \quad (14)$$

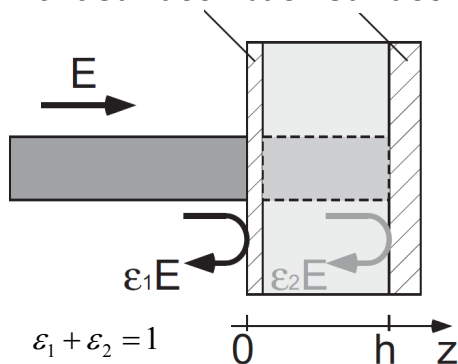
## 2. Levin's method

Gaussian force at  $z = 0$  (boundary condition)

$\implies$  Elastic equation gives dissipation  $W$ .

# Levin's method with double pump

front surface    back surface



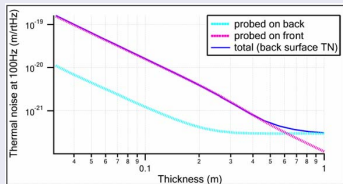
- Finite-size-mirror analysis
- New boundary condition at  $z = h$
- Integrate  $W$  in each surface

Motion of a surface appearing on each surface is coherently added

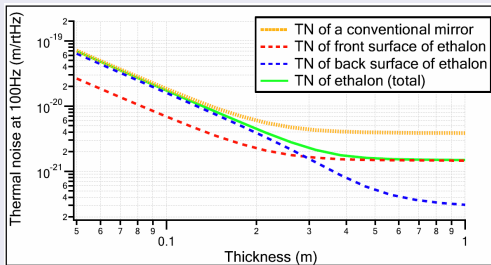


# Calculation results for Coating Brownian noise

## Back-surface coating Brownian noise



## Total coating Brownian noise of Khalili Etalon



- The thicker the substrate, the more mechanical separation
- Default  $h = 30 \text{ cm} \Rightarrow$  Etalon should better be  $40 \text{ cm}+$

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# Numerical noise spectra calculation

Energy dissipated through Brownian and TE mechanism:

$$U_{B,i} = \pi \phi_i \int_{x_i}^{x_{i+1}} dz \int_0^R (E_{rr} T_{rr} + E_{\phi\phi} T_{\phi\phi} + E_{zz} T_{zz} + 2E_{rz} T_{rz}) r dr$$

$$U_{TE,i} = 2\pi\kappa_0 T \left( \frac{Y_0 \alpha_0}{(1 - 2\nu_0) C_0 \rho_0} \right)^2 \int_{x_i}^{x_{i+1}} dz \int_0^R \left( \vec{\nabla} (E_{rr} + E_{\phi\phi} + E_{zz}) \right)^2 r dr$$

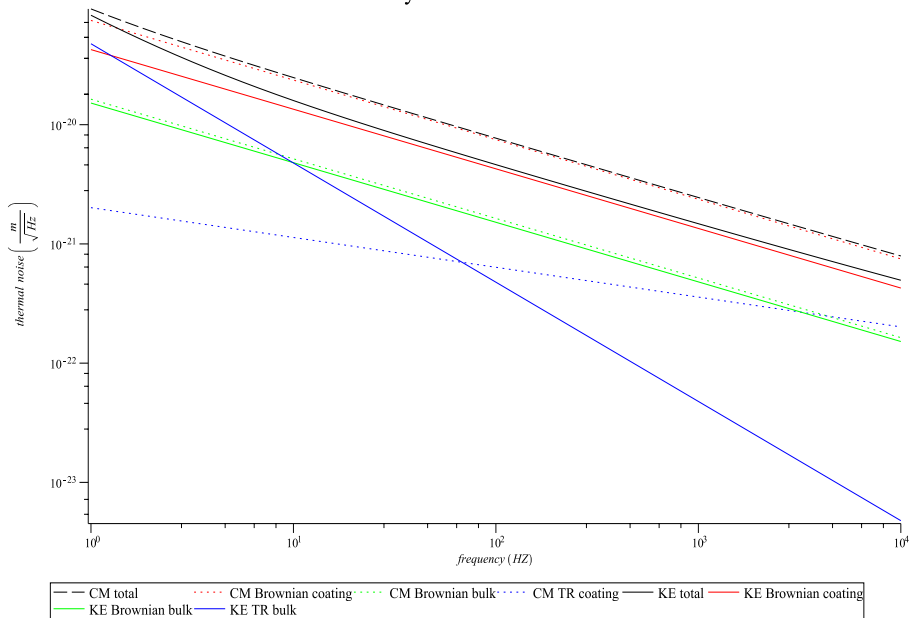
Brownian, TE and TR noise spectra

$$S_{B,i} = \frac{8kT}{\omega} U_{B,i} \quad S_{TE,i} = \frac{8kT}{\omega^2} U_{TE,i}$$

$$S_{TR,s} = \epsilon_2^2 \frac{16kT^2 \beta_0^2 h \kappa_0}{\pi \rho_0^2 C_0^2 \omega^4 \omega^2} \quad S_{TR,c,CM} = \frac{2\sqrt{2}kT^2 \beta_{\text{eff}}^2 \lambda^2}{\pi \sqrt{\kappa_0 \rho_0} C_0 \omega^2 \sqrt{\omega}}$$

$$S_{TR,c,KE} = (1 + \epsilon_2^2) \frac{2\sqrt{2}kT^2 \beta_{\text{eff}}^2 \lambda^2}{\pi \sqrt{\kappa_0 \rho_0} C_0 \omega^2 \sqrt{\omega}} \quad \beta_{\text{eff}} = \frac{1}{4} \frac{\beta_1 n_2^2 + \beta_2 n_1^2}{n_1^2 - n_2^2} + \frac{\beta_2^2}{4n_2^2}$$

## Summary of different noises



# Noises at 100 Hz

Noise Source	CM	KE
substrate		
Brownian	$6.529 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	$6.059 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$
TE	$8.512 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$	$7.983 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$
TR	-	$7.599 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$
total coating		
Brownian	$2.987 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$	$1.701 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$
TE	$4.515 \times 10^{-24} \text{ m}/\sqrt{\text{Hz}}$	$1.794 \times 10^{-24} \text{ m}/\sqrt{\text{Hz}}$
TR	$4.023 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	$4.039 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$
front coating		
Brownian	$2.987 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$	$1.111 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$
TE	$4.515 \times 10^{-24} \text{ m}/\sqrt{\text{Hz}}$	$1.615 \times 10^{-24} \text{ m}/\sqrt{\text{Hz}}$
TR	$4.023 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$	$4.023 \times 10^{-22} \text{ m}/\sqrt{\text{Hz}}$
rear coating		
Brownian	-	$1.288 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}}$
TE	-	$7.815 \times 10^{-25} \text{ m}/\sqrt{\text{Hz}}$
TR	-	$3.630 \times 10^{-23} \text{ m}/\sqrt{\text{Hz}}$

# Influence of coupling and mirror finiteness

## Noises at 100 Hz

Here we provide comparison of numerical results with account of coupling and without it.

noise source	with coupling		no coupling	
	CM	KE	CM	KE
substrate				
Brownian	$6.53 \times 10^{-22}$	$6.06 \times 10^{-22}$	$8.04 \times 10^{-22}$	$8.08 \times 10^{-22}$
TE	$8.51 \times 10^{-23}$	$7.98 \times 10^{-23}$	$5.70 \times 10^{-23}$	$5.73 \times 10^{-23}$
TR	-	$7.60 \times 10^{-23}$	-	$8.51 \times 10^{-23}$
total coating				
Brownian	$2.99 \times 10^{-21}$	$1.70 \times 10^{-21}$	$3.90 \times 10^{-21}$	$1.59 \times 10^{-21}$
TE	$4.52 \times 10^{-24}$	$1.79 \times 10^{-24}$	$6.78 \times 10^{-22}$	$1.27 \times 10^{-22}$
TR	$4.02 \times 10^{-22}$	$4.04 \times 10^{-22}$	$2.15 \times 10^{-22}$	$2.16 \times 10^{-22}$

The biggest difference is in coating TE noise: coupling lessens it 150 times for CM and 71 times for KE.

# Total KE noise gain compared to CM

## Total noise gain at 100 Hz

$$\sqrt{\frac{S_{\text{tot,CM}}}{S_{\text{tot,KE}}}} = 1.664$$

It comes mainly from coating Brownian noise which is the main part of noise budget.

## Brownian coating noise gain at 100 Hz

$$\sqrt{\frac{S_{\text{B,c,CM}}}{S_{\text{B,c,KE}}}} = 1.756$$

Other noises are reduced more slightly.

## Maximum total noise gain

Interesting that maximum total noise reduction is:

$$\sqrt{S_{\text{tot,CM}}/S_{\text{tot,KE}}} = 1.671 \quad \text{at 201 Hz.}$$

## Quick gain estimation

Omit  $\text{SiO}_2$  layers taking into account only  $\text{Ta}_2\text{O}_5$  layers.

CM: number of  $\text{Ta}_2\text{O}_5$  layers is  $N_0 = 20$

$$S_{B,c,CM} \sim N_0 = 20$$

KE: numbers of  $\text{Ta}_2\text{O}_5$  layers are  $N_1 = 3$  and  $N_2 = 17$  in front and rear coatings

Energy coupling of rear and front surface is evaluated to be 0.2, i.e. energy from rear coating layer comes with coefficient 0.2 while from front – with 1:

$$S_{B,c,KE} \sim N_1 + 0.2 \times N_2 = 3 + 0.2 \times 17 = 6.4$$

Ratio CM to KE means gain

$$\sqrt{\frac{S_{B,c,CM}}{S_{B,c,KE}}} \cong \sqrt{\frac{20}{6.4}} \cong 1.768$$

That is a good coincidence with exact ratio 1.756.



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# FE calculation of Brownian noise in a Khalili-cavity

idea: finite element calculation as check for analytical model

## Levin's approach

- virtual pressure on surface  $p(r) = F_0 f(r)$
- FE computation of total elastic energy  $E_{tot}$
- structural damping  $E_{diss} = \phi E_{tot}$
- spectral noise energy density  $S_x(\omega) = \frac{kT}{\pi^2 f} \frac{E_{diss}}{F_0^2}$

⇒ comparison of elastic energy is enough

## Symmetry

- radiallysymmetric virtual pressure
- radiallysymmetric geometry
- isotropic materials

⇒ simplification from 3D to radiallysymmetric 2D-analysis

# test on substrate Brownian noise

crucial for FE calculations: test problems with known analytical solutions

## substrate Brownian noise of finite sized test masses

- analytical solution by Liu and Thorne 2000

$$E_{tot}^{ITM} = \frac{2}{\sqrt{\pi}} \frac{1 - \sigma^2}{Y w_0}$$

$$E_{tot}^{FTM} = C_{FTM}^2 E_{tot}^{ITM}$$

diameter	50 cm
height	30 cm
laser beam radius $w_0$	90 mm
frequency	100 Hz
silicon (111)	
Young's modulus $Y$	188 GPa
Poisson's ratio $\sigma$	0.23
density	2331 kg/m <sup>3</sup>

Table: properties of the test substrate

results

Liu and Thorne

$$E_{tot} = 1.043 \times 10^{-11} \text{ J}$$

COMSOL

$$E_{tot} = 1.05 \times 10^{-11} \text{ J}$$

⇒ agreement of better than 1%

## etalon geometry

- cylindrical substrate with height of some 10 cm
- two coating stacks with height of some  $\mu\text{m}$

⇒ coating is negligible for elastic response

## numerical receipt

- simulate elastic response of the substrate to virtual pressure
- estimate coating response via boundary conditions

$$\sigma'_{xz} = \sigma_{xz} , \sigma'_{yz} = \sigma_{yz} , \sigma'_{zz} = \sigma_{zz}$$

$$u'_{xx} = u_{xx} , u'_{yy} = u_{yy} , u'_{xy} = u_{xy}$$

- compute coating's elastic energy  $w = \frac{1}{2} \sigma_{ij} u_{ij}$

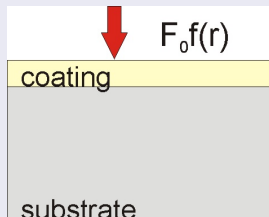
# test on coating Brownian noise

comparison of

- FE simulation of coatings with finite thickness
- model of infinitely small coating

## finite coating FE results

coating thickness [m]	elastic energy [J]
$1 \times 10^{-3}$	$1.5091 \times 10^{-13}$
$1 \times 10^{-4}$	$1.5211 \times 10^{-14}$
$1 \times 10^{-5}$	$1.5224 \times 10^{-15}$
$1 \times 10^{-6}$	$1.5225 \times 10^{-16}$



## infinite small coating

$$w_{inf} = 1.5227 \times 10^{-10} \text{ J/m}$$

⇒ deviations of under 1% for coating thickness  $< 1 \text{ mm}$

⇒ suited for Khalili-etalon

# elastic energies in a Khalili-etalon

calculation for a given mirror geometry

## parameters of the cavity

diameter	62 cm
height	30 cm
laser beam radius	120 mm
frequency	100 Hz
material	fused silica
Young's modulus	72 GPa
Poisson's ratio	0.17
density	2202 kg/m <sup>3</sup>
silica coating	
Young's modulus	72 GPa
Poisson's ratio	0.17
tantala coating	
Young's modulus	140 GPa
Poisson's ratio	0.23

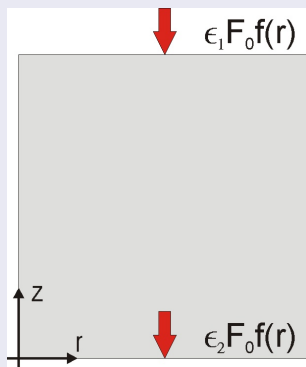


Figure: sketch of simulated geometry,  $\epsilon_1=0.91$ ,  $\epsilon_2=0.09$

## FE results for the Khalili cavity

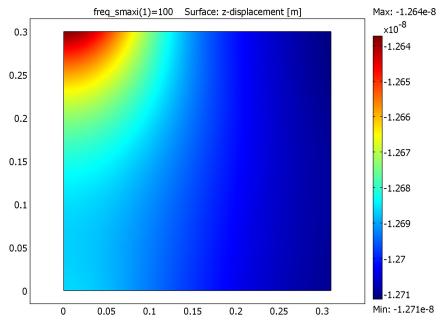


Figure: z-displacement

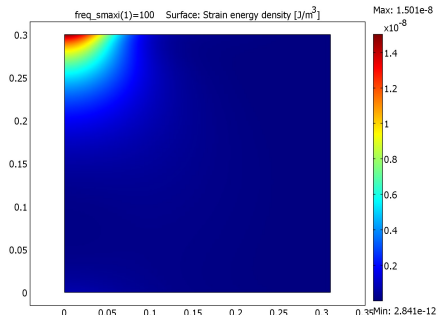


Figure: elastic strain energy

# results for FE calculation

comparison with analytical calculation yield

## front coatings

material	$w_{analytical}$ [J/m]	$w_{FE}$ [J/m]
silica	$2.06 \times 10^{-10}$	$2.05 \times 10^{-10}$
tantala	$2.32 \times 10^{-10}$	$2.30 \times 10^{-10}$

## back coatings

material	$w_{analytical}$ [J/m]	$w_{FE}$ [J/m]
silica	$3.31 \times 10^{-11}$	$3.01 \times 10^{-11}$
tantala	$6.49 \times 10^{-11}$	$6.40 \times 10^{-11}$

⇒ successful confirmation of the analytical model



# Overview of this presentation

- 1 Motivation for Khalili Etalons (Stefan)
- 2 Noise contributions for a Khalili cavity, i.e. no coupling through the substrate considered (Ronny)
- 3 How to calculate the noise couplings inside an etalon? Example: Coating Brownian noise for finite size etalon (Kentaro)
- 4 Detailed calculations of all noise sources inside a Khalili etalon with mechanical coupling of front and back surface (Sergey and Alexey)
- 5 Confirmation of calculations by FEM results
- 6 **Summary and outlook (Stefan)**

- Khalili-etalons might be an alternative to Khalili-cavities!
- A Khalili-etalon has *more noise* due to the mechanical coupling of front and back surface.
- A method based on Levin was developed to calculate the coating Brownian noise contributions of the front and back surface, taking the mechanical coupling into account.
- Finally we extended this method all relevant thermal noise contributions of the etalon.
- We compared the analytical derivation with FEM and found good agreement.
- For our dummy example the Khalili cavity gives an overall noise reduction of a factor **2.52**, while the Khalili etalon would give a reduction factor of **1.66**.

Our next steps are:

- continue our team effort ...
- to properly write this up ... :)
- to perform an optimisation of the etalon parameters (coating distribution and substrate thickness), i.e. trade-off between thermal lensing and thermal noise reduction.
- to start investigating control issues of Khalili cavities and Khalili etalons