Investigating the EM signatures of ships due to corrosion and its countermeasures.

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Summary.

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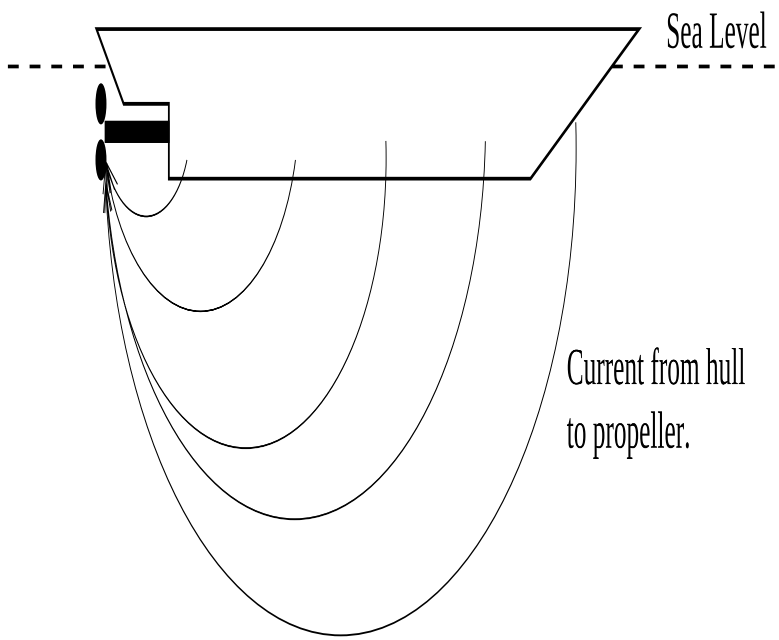
Introduction to the physical problem.

- Consider a corroding ship as a galvanic cell
- EM signatures of ships are important for detection and identification purposes.
- Problem: To calculate the electric potential about the ship due to the electrochemical reactions between the ship and the seawater, and the counter-corrosion devices.
- The potential about the ship allows evaluation of:-
  - corrosion protection
  - underwater **E** field signature
  - underwater **B** field signature

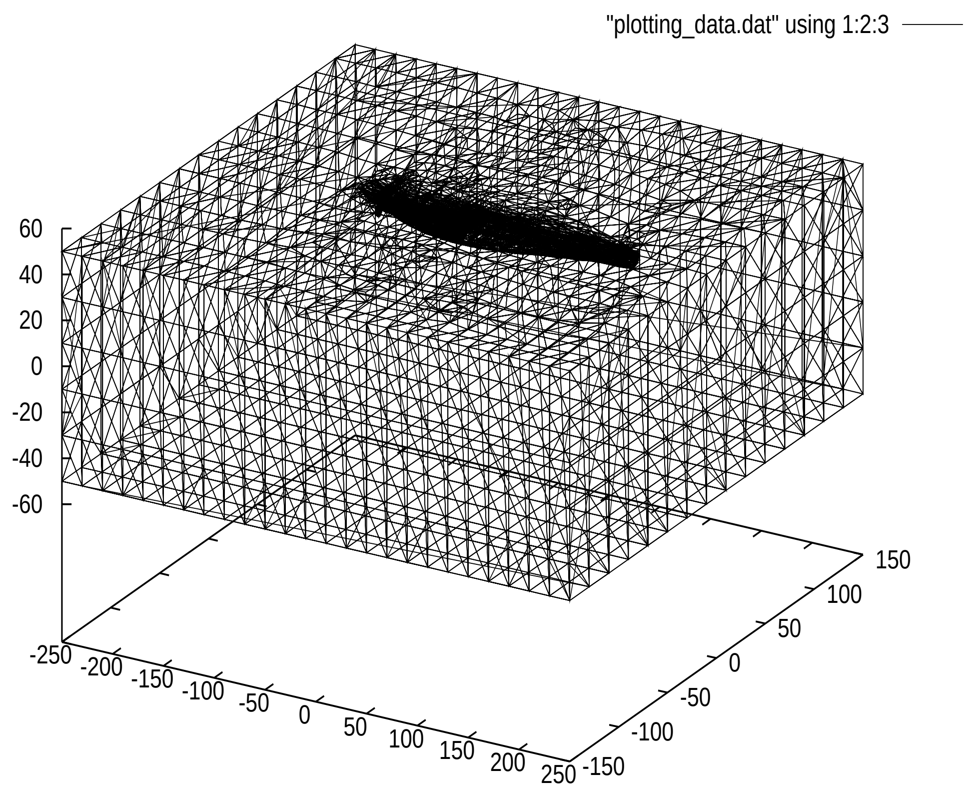
Currents flowing due to corrosion.



Outline of the computational problem.

- Sea is represented by a finite rectangular box with the ship floating centrally on the top surface.
- Sea is homogeneous and the non-ship boundaries are insulating.
- BEM most suitable numerical method $\Rightarrow$ Need surface mesh.
- Boundary meshed using constant, triangular elements
  - potentials evaluated at the centroids.

Mesh of the Model.



Forming the BEM equations.

- Use BEM to solve for electric potential
- If $\mathbf{u}$ denotes the potentials over the elements, and $\mathbf{q}$ the outward normal derivative of the potential

$$H\mathbf{u} = G\mathbf{q}.$$

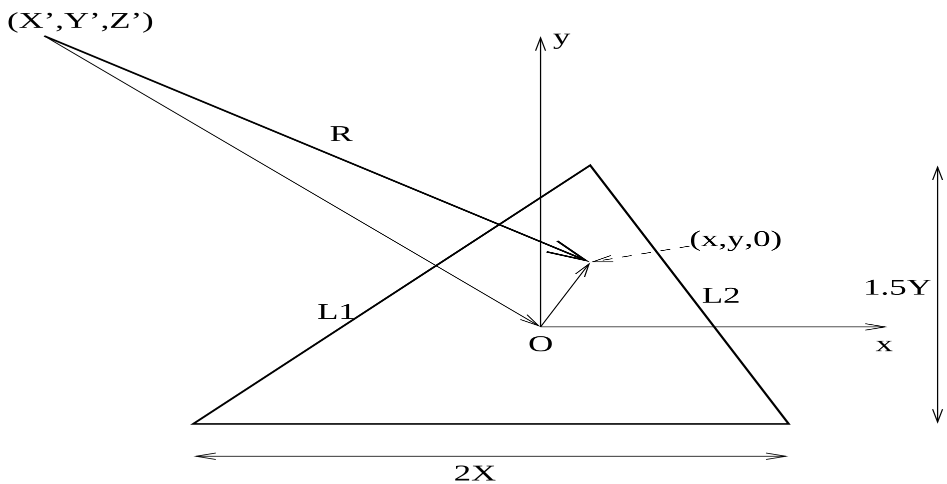
- $G_{ij} \propto \int_{\Gamma_j} \frac{1}{R_{ij}} d\Gamma$ and $H_{ij} \propto \int_{\Gamma_j} \frac{\partial}{\partial \mathbf{n}} \frac{1}{R_{ij}} d\Gamma$.
- Solve for the unknown $\mathbf{u}$ and $\mathbf{q}$ values (related via a polarization relation).
- Model has 5132 elements $\Rightarrow$ H and G have greater than 26 million elements!

Method of moments.

- Developed new technique for numerical integration based around the moments of the elements.
- Closed formulae for diagonal elements $\Rightarrow$ Focus on off-diagonal elements.
- Requires evaluation of integrals such as

$$\int_s \frac{1}{R} ds.$$

Method of moments.



- Power series expansion about the origin gives

$$\int_s \frac{1}{R} ds = \sum_{m,n=0}^{\infty} \left(\int_s x^m y^n ds \right) \Upsilon^{mn}.$$

- Υ^{mn} contains all signs, constants, etc. for ease of computation.
- $\int_s x^m y^n ds$ is the m, n^{th} moment of the triangle.

Method of moments - Normalised moments.

- Can remove the scaling of the moments

$$\int_s x^m y^n ds = X^{m+1} Y^{n+1} M_{mn}(\alpha)$$

- $M_{mn}(\alpha)$ is called the Normalised Moment of the element
  - finite polynomial of degree $m+1$ in $\alpha = p\frac{Y}{X}$ where p is the 'gradient' of $L1$.

- Most important property of normalised moments is

$$\frac{d^n}{d\alpha^n} M_{m+n0}(\alpha) = \frac{(m+n)!}{m!} M_{mn}(\alpha)$$

Method of moments - $M_{m0}(\alpha)$.

- $M_{m0}(\alpha) = \int_{-\frac{1}{2}}^{1} \sum_{r=0}^{m} C_r^{(m)}(\eta)\alpha^r d\eta = \sum_{r=0}^{m} d_r^{(m)}\alpha^r$
- Coefficients $d_r^{(m)}$ do not depend on size nor shape of the element.
  - calculate once and then used to obtain exact derivatives of any order for any α.
  - eliminates the requirement to perform time-consuming numerical integrations each time a matrix element is calculated.

Method of moments - Summary.

- Integration performed by forming a polynomial and then differentiating, e.g.

$$\int_s \frac{1}{R} ds = \sum_{m,n=0}^{20} \left\{ X^{m+1} Y^{n+1} \frac{m!}{(m+n)!} \frac{d^n}{d\alpha^n} \sum_{r=0}^{m+n} d_r^{(m+n)} \alpha^r \right\} \Upsilon^{m,n}.$$

- Localised coordinate system dictates that the outward normal direction is the positive z direction.
  - differentiate the above with respect to this direction to obtain the necessary integrals for the H matrix.
- Method tested for accuracy against Gauss-Legendre Quadrature and the Clenshaw-Curtis Method.
  - Comparable accuracy was obtained.

Solving the equations and field calculations.

- The Point Successive Over-relaxation Method is used to solve the BEM equation

$$\tilde{u}_i = u_i + \frac{r}{H_{ii}} \left((H\mathbf{u})_i - (G\mathbf{q})_i \right).$$

- Electric field at any point, p, within the domain can be calculated from

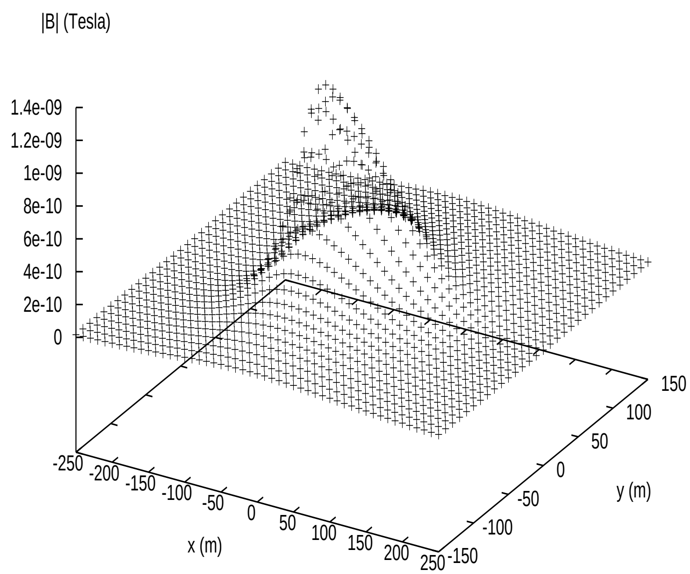
$$\mathbf{E}_p = -\nabla u_p.$$

- Magnetic field at any point within the domain can be calculated using

$$\mathbf{B}_p = -\frac{\mu_0}{4\pi}\sigma \int_s \frac{u}{R^3} d\mathbf{s} \times \mathbf{R}$$

- $\mathbf{R}$ points from the boundary element to the field point p.

|B|-Perfect Paint.



- Perfect Paint on hull $\Rightarrow$ Little or no hull corrosion
  - corrosion occurs near propeller.

Conclusions.

- BEM can be used to evaluate the electric potential (and its outward normal derivative) on the surface of a corroding ship.
- Complex geometry $\Rightarrow$ large coefficient matrices.
- New method developed for these integrations based round the moments of the triangular surface elements.
  - Method required formation of polynomials in a shape parameter α, followed by subsequent differentiation
  - Accuracy of method was comparable to established methods
  - New method is a viable alternative to existing ones.
- BEM solutions allow EM signatures calculations.
- Future work should include efficiency test of the proposed integration methods against existing ones.

Thanks.

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