

1. The thin-lens formula is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Define the symbols used in this equation. Comment briefly on any similarities with the mirror equation.

[3]

An object 100mm in front of a convex lens is imaged in a plane 50mm behind the lens. Calculate the focal length of the lens. Draw a ray diagram for this situation. If the object is moved a little further away from the lens, explain how the image moves with respect to the lens?

[5]

The refraction of light rays at the surfaces of a lens always follows Snell's law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

A convex lens is used to focus monochromatic light rays parallel to the optic axis of the lens.

(i) Draw a diagram to show how the rays are focused by the lens (indicate the focal length of the lens in your diagram). [2]

(ii) For a light ray that is not on the optic axis draw a diagram of its passage through the lens (indicating surface normals and angle of refraction). [4]

(iii) The refractive index of the lens is slightly greater for light with a shorter wavelength. Explain whether the focal length of the lens is longer or shorter for such light. [2]

2. The light beams from two light sources (in phase with each other) interfere constructively for path differences

$$\Delta = 0, \pm\lambda, \pm 2\lambda, \dots$$

Draw a sketch of two harmonic waves with a path difference $\Delta = \lambda$ and their sum. In your sketch, indicate the wavelength, λ . What is the phase difference of two light waves with a path difference $\Delta = \lambda$?

[4]

A pair of parallel slits are separated by 0.25mm and illuminated with light of wavelength $\lambda = 400\text{nm}$. Calculate the angle θ at which the first maximum is observed on a screen a long distance (e.g. 1m) behind the slit (you may assume the small-angle approximation).

[6]

A thin glass plate of refractive index $n=1.5$ is placed in front of one of the slits. The optical path length in the plate is half a wavelength longer than the thickness of the plate, so the plate delays the light passing through the slit it covers by $\lambda/2$. Calculate the thickness of the glass plate. Explain what difference you would see in the interference pattern with the plate present.

[6]

Solutions

1. Symbols used in thin-lens formula:

p : object distance (optional: i.e. distance of the object in front of lens)

q : image distance (optional: i.e. distance of the image behind lens)

f : focal length of the lens

[2 marks]

Does this formula differ from the mirror equation?

No, it's exactly the same (optional: although the image distance is now the distance *in front of* the mirror rather than *behind* the lens).

[1 mark]

An object 100mm in front of a convex lens is imaged into a plane 50mm behind the lens. What is the focal length of the lens?

Solve the thin-lens formula with $p = 100\text{mm}$ and $q = 50\text{mm}$ for f to find

$$f = 1 / (1/(100 \text{ mm}) + 1/(50 \text{ mm})) = 33.3 \text{ mm}$$

[3 marks]

If the object is moved a little further away from the lens, which way does the image move with respect to the lens?

Solve the thin-lens formula with p a little greater than 100mm, e.g.

$p=101\text{mm}$, and $f=33.3\text{mm}$ for q to find

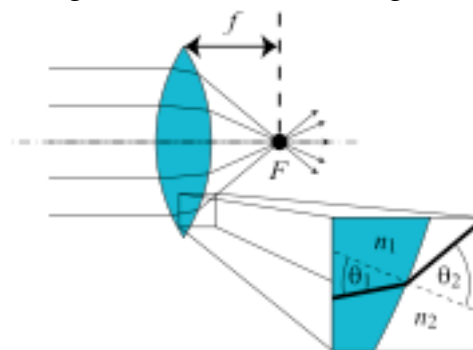
$$q = 1 / (1/(33.3 \text{ mm}) - 1/(101 \text{ mm})) = 49.7 \text{ mm}$$

This means the image moves closer to the lens.

[2 marks]

(i) Draw a diagram to show how the rays are focused by the lens (indicate the focal length of the lens in your diagram). [2]

(ii) For a light ray that is not on the optic axis draw a diagram of its passage through the lens (indicating surface normals and angle of refraction). [4]



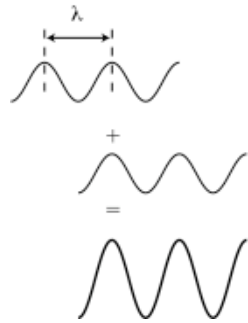
Due to dispersion, the refractive index of the lens material is slightly greater for light with a shorter wavelength. Is the focal length of the lens longer or shorter for such light?

The larger the angle θ_2 , the shorter the focal length, f .

According to Snell's law, a larger n_1 causes $\sin \theta_2$, and therefore θ_2 itself, to be larger. Therefore, f is shorter for light with shorter wavelengths.

[2 marks]

2. Draw a sketch of two harmonic waves with a path difference $\Delta = \lambda$ and their sum. In your sketch, indicate the wavelength, λ .



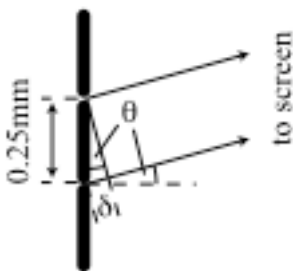
[3]

What is the phase difference of two light waves with a path difference $\Delta = \lambda$?

The phase difference is 2π .

[1]

A pair of slits separated by 0.25mm is illuminated with light of wavelength $\lambda = 400\text{nm}$. Calculate the angle θ under which the first maximum is observed on a screen far (e.g. 1m) behind the slit (use the small-angle approximation).



Insert the condition on the path difference for the first maximum, $\Delta = \pm \lambda$, [2]

into the equation for the path difference as a function of θ ,

$\Delta = 0.25\text{mm} \sin(\theta) \approx 0.25\text{mm} \theta$ (small-angle approximation!) [2]

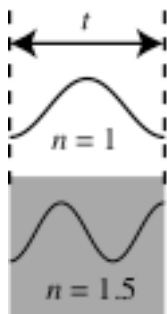
to find

$\pm \lambda = 0.25\text{mm} \theta$, i.e.

$\theta = \pm \lambda / (0.25\text{mm}) = \pm 400 \cdot 10^{-9}\text{m} / (0.25 \cdot 10^{-3}\text{m})$

$= \pm 1.6 \cdot 10^{-3} \text{ rad } (= \pm 0.092^\circ)$ [2]

A thin glass plate of refractive index $n=1.5$ is placed in front of one of the slits. The optical path length in the plate is half a wavelength longer than the thickness of the plate, so the plate delays the light passing through the slit it covers by $\lambda/2$. What is the thickness of the glass plate?



thickness of plate: t

optical path length without plate: t

optical path length with plate: $n t$

optical path difference introduced by plate: $\Delta = n t - t = t (n - 1)$

from question: $\Delta = \lambda/2$, so $t (n - 1) = \lambda/2$

solve for t : $t = \lambda / 2 (n - 1) = \lambda / 2 (1.5 - 1) = \lambda = 400\text{nm}$ [4]

At the position of a former maximum, is the interference now constructive or destructive?

Before the plate was inserted, the path difference was

$\Delta = i \lambda$, $i = 0, \pm 1, \pm 2, \dots$ (condition for maximum)

Then the glass plate added $\lambda/2$ to that, so

$\Delta = (i + 1/2) \lambda$,

which is the condition for *destructive interference*.

[2]