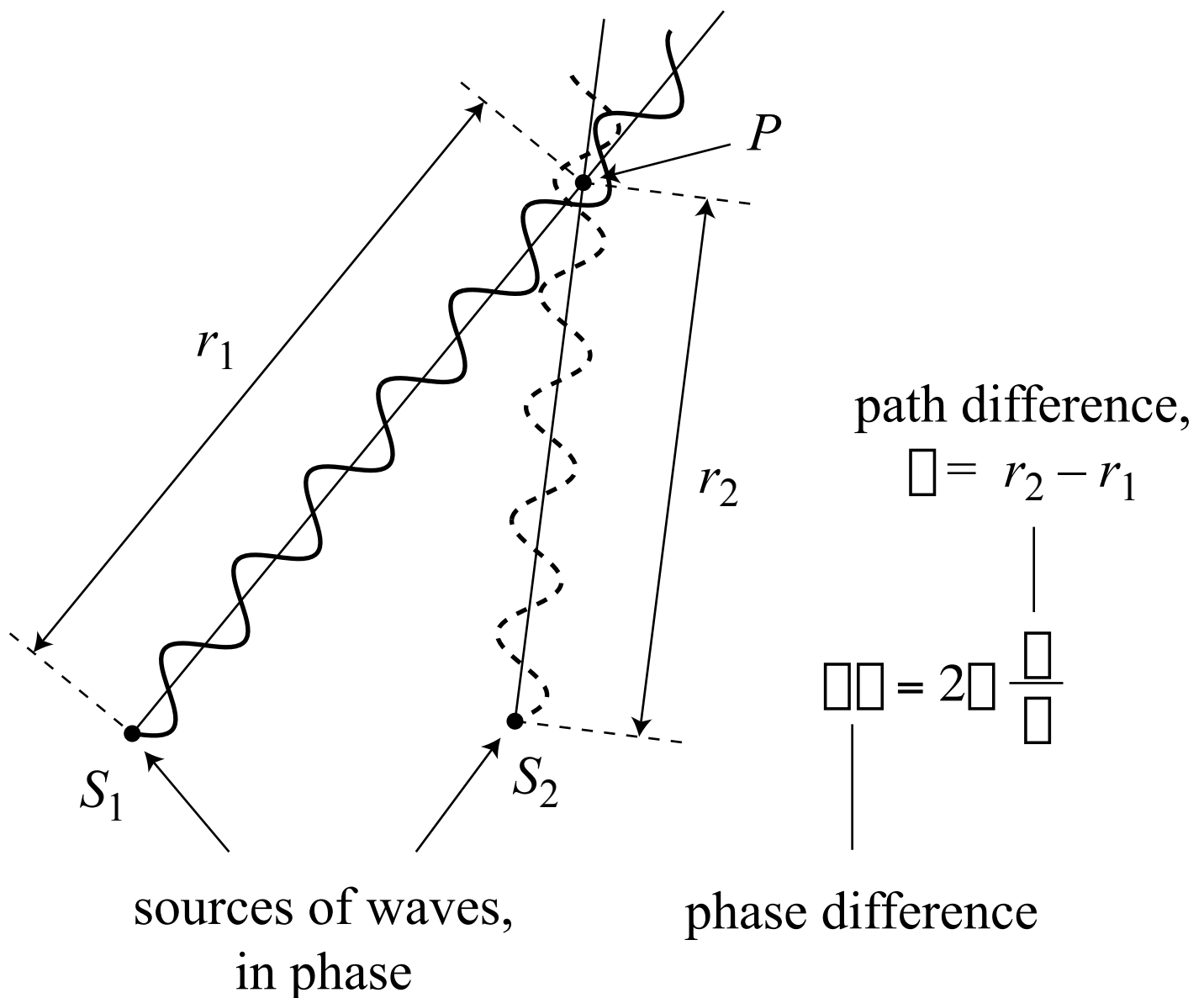
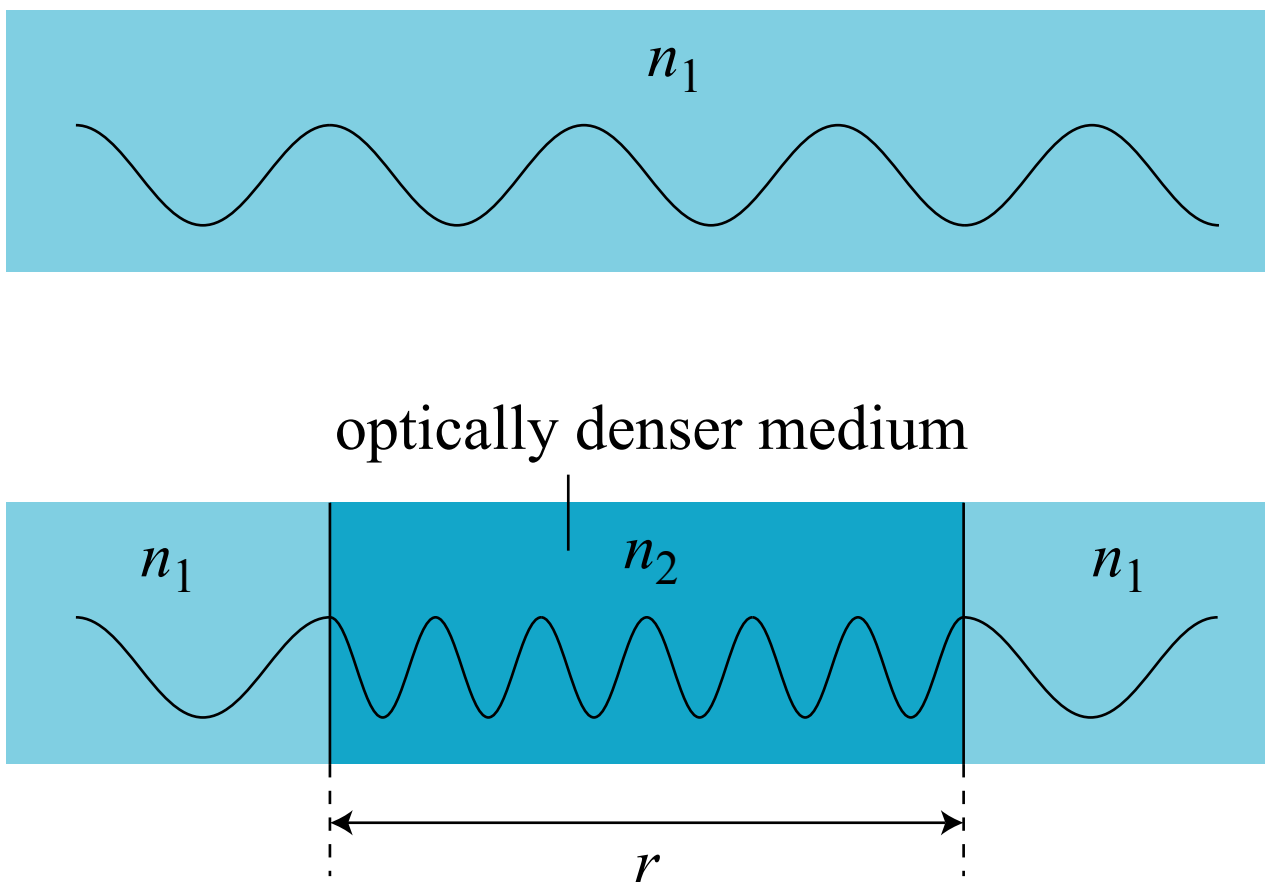


2.2.6 Optical path length

In section 2.2.2 (Path difference & phase difference) it was stated that the phase difference between two waves that were initially in phase is simply proportional to the difference in the respective path lengths traversed by the two waves between their origin and the point at which they interfere.

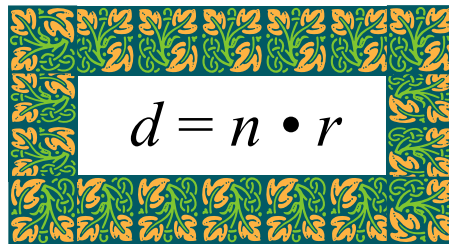


However, the following example shows that two waves that were originally in phase and that have travelled the same distance *can* be out of phase:



In the optically denser medium the wave oscillates with the same frequency but travels at a slower speed. This implies that its wavelength is shorter in the medium.

What effect does this have on the phase of the wave?
Define the **optical path length** traversed by a wave travelling a distance r in a medium of refractive index n as


$$d = n \cdot r$$

Then the phase difference $\Delta\phi$ of two waves that, after having been in phase initially, have traversed optical path lengths d_1 and d_2 , respectively, is

$$\Delta\phi = 2\pi \frac{\Delta d}{\lambda}$$

where Δd is now the optical path difference

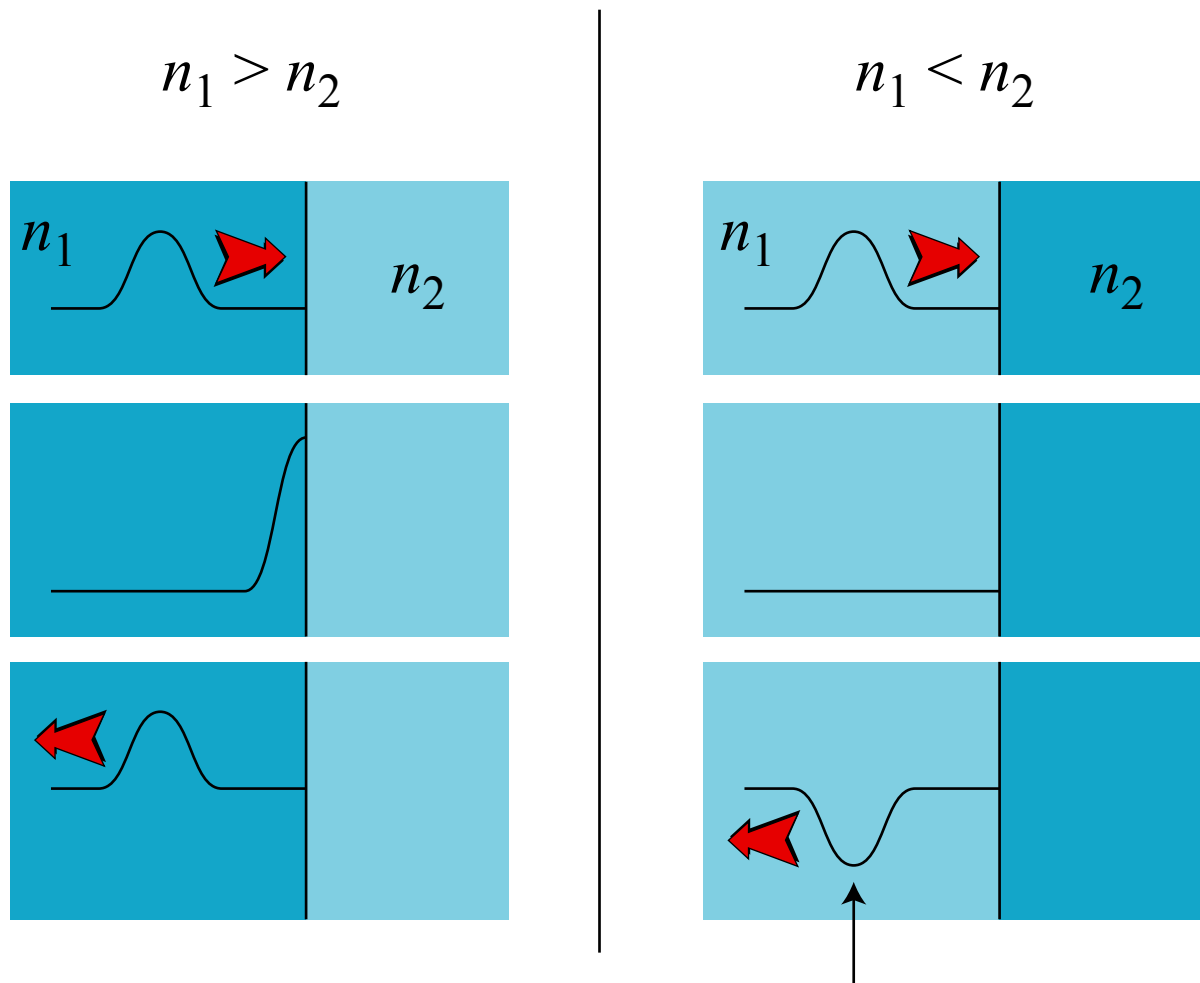
$$\Delta d = d_2 - d_1$$

We'll apply this in a moment when we look at thin-film interference (which is what produces the 'rainbows' in soap bubbles and oil patches on the road).

2.2.7 Phase change on reflection

Benson 37.5

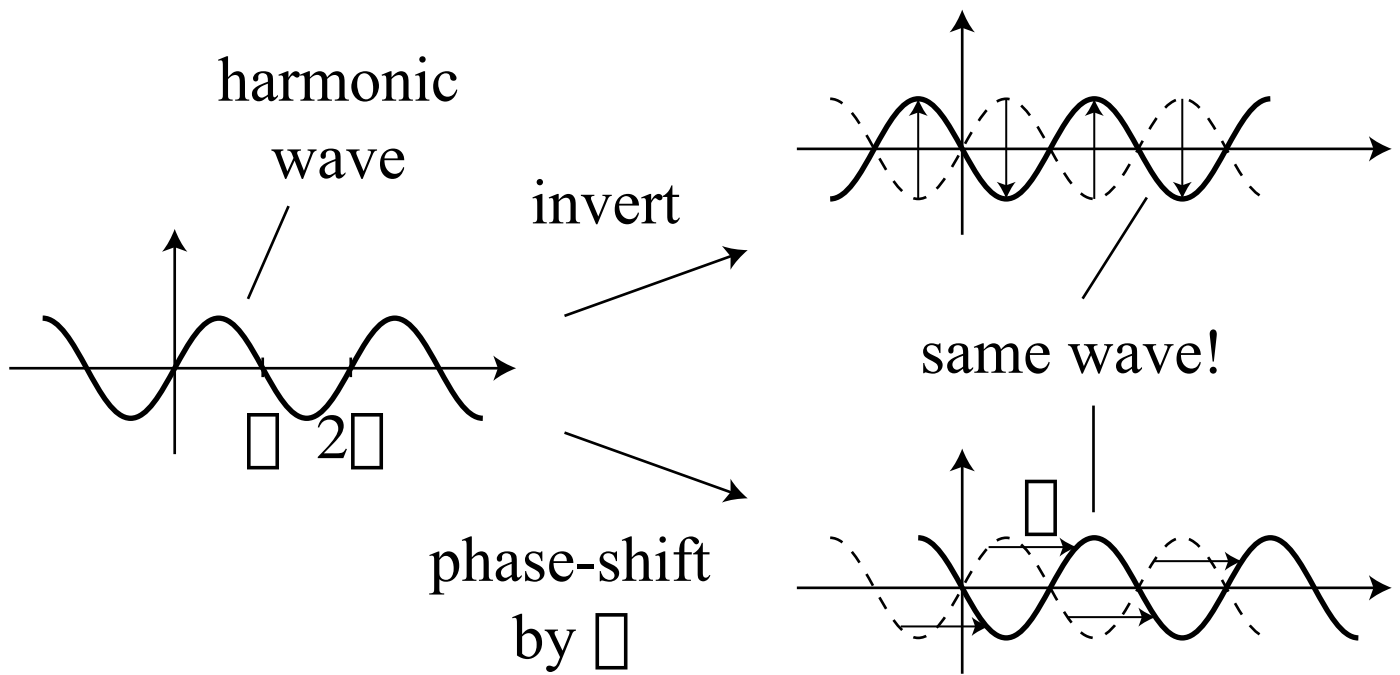
reflection of a pulse at the interface between two media with different refractive indices, n_1 and n_2



"phase-changed by π ", i.e. inverted

When light encounters a medium of higher refractive index, the reflected wave suffers a phase change of π

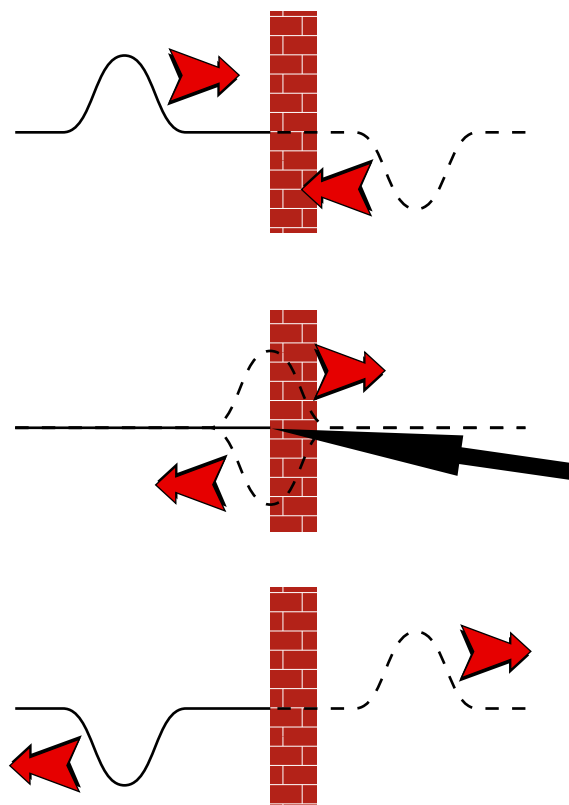
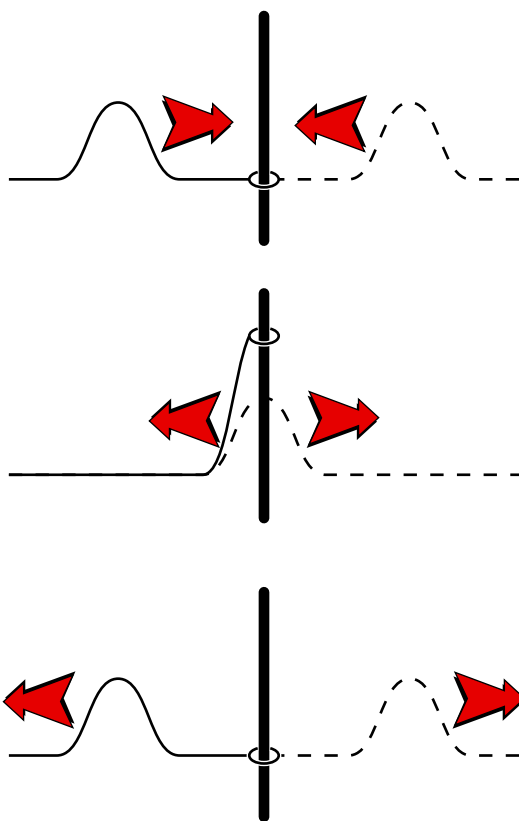
Why would anybody in their right mind describe an inversion as a "phase change by π "?



Perhaps we can understand this a little better by considering this phase change on reflection in mechanics:

rope with loose end

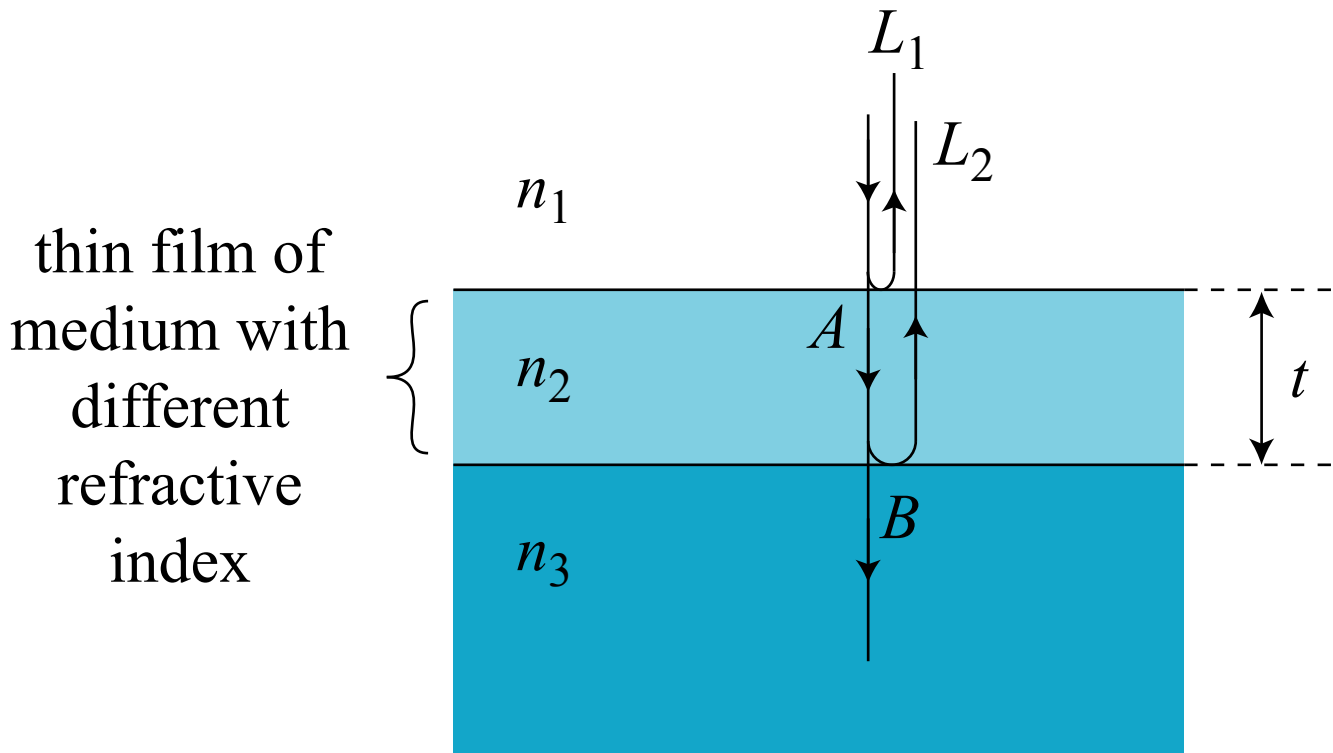
rope with fixed end



anchor point can't move, so displacement has to be zero!

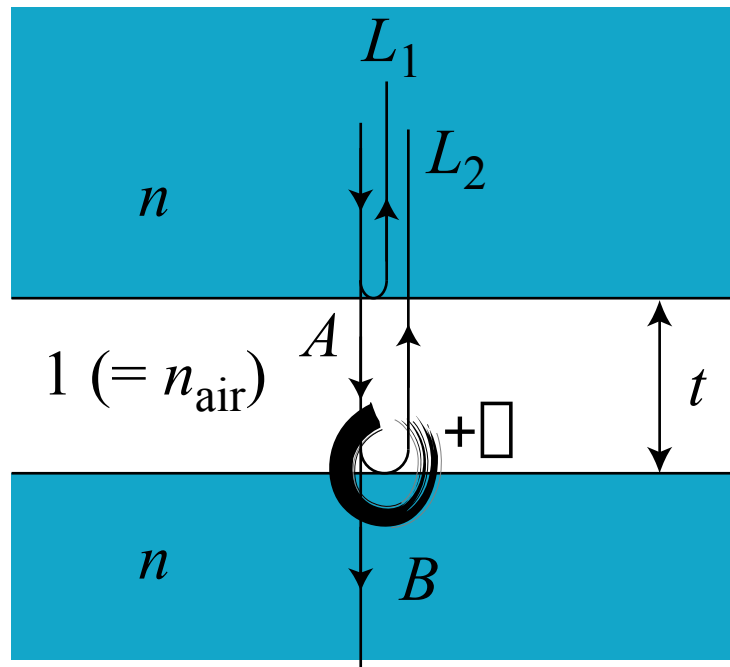
2.2.8 Thin-film interference

Benson 37.5



What is the phase difference between the two reflected light beams L_1 and L_2 ? It comprises a phase difference due to the optical path difference (L_2 has to travel a distance t and back through the thin film with refractive index n_2 , which corresponds to an optical path length $d = n_2 t$ and consequently a phase difference of $\frac{2\pi}{\lambda} n_2 2t$) and any phase changes of π due to reflections at an optically denser medium.

Example: air gap between glass plates



phase difference between L_1 and L_2 :

$$\Delta\phi = \underbrace{2\pi \cdot 2t / \lambda}_{\text{due to additional optical path length}} + \underbrace{\pi}_{\text{phase change during reflection at point } B}$$

$\Delta\phi_1 = 2\pi n_2 2t / \lambda$
 due to additional
 optical path length
 ($2t$) traversed by L_2
 in air ($n_2 = 1$)

π phase change
 during reflection
 at point B

constructive interference occurs whenever

$\Delta\phi = 0, \pm 2\pi, \pm 4\pi, \dots$, i.e. when

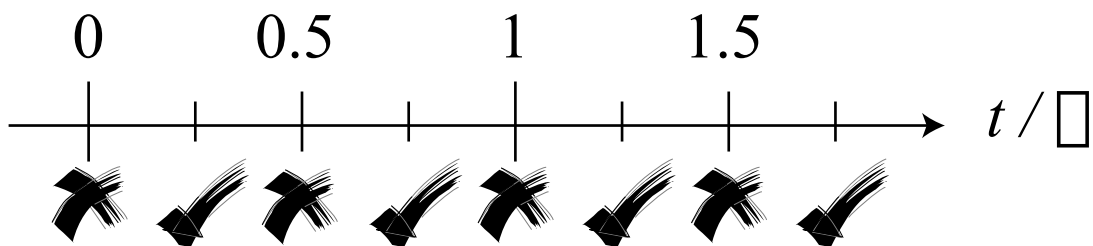
$$4\pi t / \lambda + \pi = n 2\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

$$4 t / \lambda = 2n - 1$$

$$t = (n - 1/2) \lambda/2$$

i.e. for thicknesses $t = 0.25 \lambda, 0.75 \lambda, 1.25 \lambda, \dots$

destructive interference presumably happens in between, i.e.



 : positions of constructive interference

 : positions of destructive interference?

let's check: the condition for destructive

interference for the phase is $\Delta\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \dots$

Substitute expression for phase difference between L_1 and L_2 :

$$4\pi t / \lambda + \pi = (2n + 1) \pi, \quad n = 0, \pm 1, \pm 2, \dots$$

$$t = n \lambda/2$$

so destructive interference does indeed occur for thicknesses $t = 0, 0.5 \lambda, \lambda, 1.5 \lambda, \dots$