April 2004

Solutions

1. Both the thin-lens formula and the mirror equation are given by:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

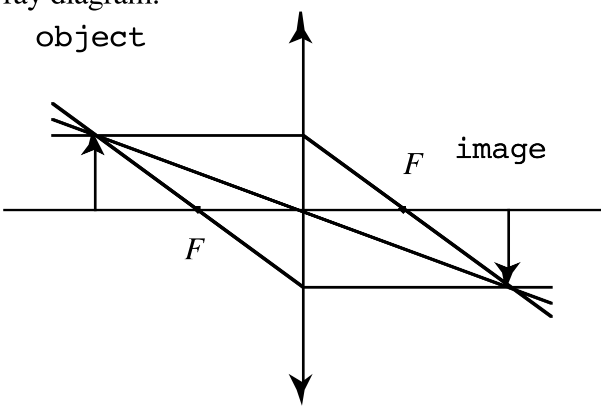
Define the symbols used in this equation. Where are *p* and *q* positive (a) in the case of a lens and (b) in the case of a mirror?

p: object distance; *q*: image distance; *f*: focal length **[2]**
(a) in the case of a lens, $p > 0$ in front of the lens, $q > 0$ behind the lens **[1]**
(b) in the case of a mirror, $p, q > 0$ in front of the mirror **[1]**

An object 100mm in front of a convex lens is imaged in a plane 100mm behind the lens. Calculate the focal length of the lens. Draw a ray diagram for this situation. If the object is moved a little closer to the lens, which way does the image move with respect to the lens?

Solve lens equation with p = 100mm and q = 100mm for f:
1/100mm + 1/100mm = $1/f$, so $1/f$ = 2/100mm = 1/50mm, i.e. f = 50mm **[2]**

ray diagram:



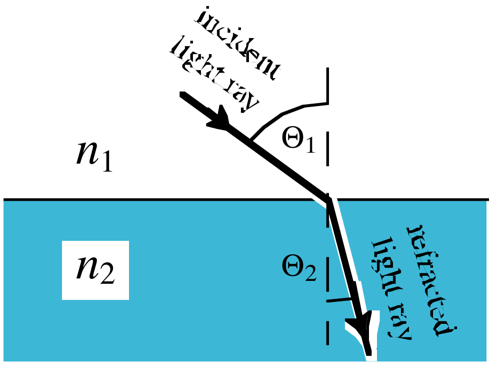
[2]

Solve the lens equation for slightly smaller value of p, e.g. p = 99mm:
1/99mm + $1/q$ = 1/50mm, so q = 1/(1/50mm – 1/99mm) = 101mm (approx)
This means the image moves away from the lens. **[2]**

2. **State Snell's law. Sketch the refraction of a light ray at the interface between two media with different refractive indices (with $n_1 < n_2$), indicating the angles θ_1 and θ_2.**

Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ **[2]**

Sketch:



[2]

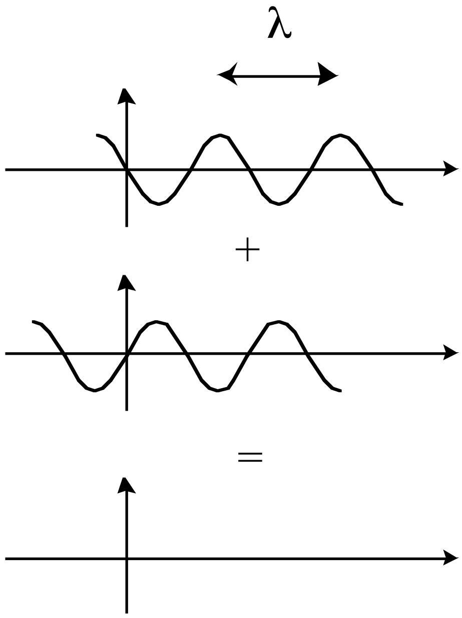
State the equation for the critical angle θ_c, which marks the onset of total internal reflection. Calculate the critical angle for the interface between water (n = 1.33) and air. Which of the following technologies/effects require total internal reflection? (a) optical fibres; (b) thin-film interference; (c) dispersion.

Equation for critical angle: $n_1 \sin \theta_c = n_2$ **[2]**
For water/air interface: $n_1 = 1.33$, $n_2 = 1$, so $\sin \theta_c = 1/1.33 = 0.752$, and therefore $\theta_c = 48.8°$ **[2]**
Only (a) uses total internal reflection, (b) and (c) do not. **[2]**

3. **The light beams from two light sources (in phase with each other) interfere destructively for path differences**

$$\delta = \pm\lambda/2, \pm 3\lambda/2, \pm 5\lambda/2, \ldots$$

Draw a sketch of two harmonic waves with a path difference δ = λ/2 and their sum. In your sketch, indicate the wavelength, λ.



[4]

A pair of slits separated by 0.02mm is illuminated with light of wavelength λ = 508nm. Calculate the angle θ at which the first minimum is observed on a screen a long distance (e.g. 1m) behind the slit (you may assume the small-angle approximation). At what angles are the first and second maxima observed?

Solve condition for first minimum, $d \sin\theta = \lambda/2$, **[2]**
with d = 0.2mm and λ = 508nm for θ **:** sin θ = 508nm/0.04mm = 0.0127, so θ = 0.728° **[2]**

The first maximum is observed at twice that angle, i.e. θ = 2*0.728° = 1.45°.
The second maximum is observed at twice the angle of the first maximum, i.e. θ = 2.90°. **[2]**