The Spin and CP of the Higgs

David J Miller
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What do we need to know?

To complete the **Standard Model** we need to verify the Higgs mechanism as the **mechanism of electroweak symmetry breaking** and as the **mechanism for giving fermions mass**.

(Though I personally hope that in doing so we will find something new!)

To do this we need:

- To show the Higgs is spin 0 and CP even
- The HVV couplings
- The Hff couplings
- The Higgs self couplings → the Mexican hat potential are proportion to the masses
What do we already know?

Both ATLAS and CMS have seen a resonance at around 125-126 GeV.
The Landau-Yang Theorem

The most (only?) convincing channel so far is $H \rightarrow \gamma\gamma$ which tells us that the new state is **not spin 1**.

Imagine a vector decaying to two photons:

There are 3 possible vertex structures,

$$
\epsilon_1 \times \epsilon_2 \cdot \epsilon_v \quad F_1(k^2)
$$

$$
\epsilon_1 \times \epsilon_2 \cdot k \quad \epsilon_v \cdot k \quad F_2(k^2)
$$

$$
\epsilon_1 \cdot \epsilon_2 \quad \epsilon_v \cdot k \quad F_3(k^2)
$$

But **Bose** (Go India!) tells us that $\mathcal{M}(\epsilon_1, \epsilon_2, k) = \mathcal{M}(\epsilon_2, \epsilon_1, -k)$ so these terms are all zero.
What are we allowed to assume?

We have an underlying philosophical dilemma:

**How much theory should we assume** while “proving” the new particle is the Higgs boson?

For example, there is **no pseudoscalar-ZZ coupling** at tree-level in the general **two Higgs doublet model**.

Seeing \(H \to ZZ\) at SM levels, we can be confident that the new resonance is **mainly CP even**.

But what if the new state is a **singlet**?

In my opinion we should **measure everything we can**, even when we **think** we know the answer. Maybe we will find some **surprises**!
Where should we look?

In principle, any Higgs coupling contains information on the Higgs spin and CP.

The only two decay channels with significant events so far are:

\[
\begin{align*}
\text{• } & H \to \gamma\gamma \\
\text{• } & H \to ZZ \to 4 \text{ leptons}
\end{align*}
\]

But we could also look at

\[
\begin{align*}
\text{• } & H \to WW \to l \bar{\nu} l \nu \text{ (or } l \nu jj) \\
\end{align*}
\]

Or production mechanisms such as

\[
\begin{align*}
\text{• } & WW \text{ fusion} \\
\text{• } & pp \to Htt
\end{align*}
\]
The golden channel: $H \rightarrow ZZ \rightarrow$ 4 leptons

The most general vertex for a spin 0 object decaying to two Z's has the form:

$$V_{HZZ}^{\mu \nu} = \frac{igmZ}{\cos \theta_W} \left[ a g^{\mu \nu} + b \frac{p^{\mu} p^{\nu}}{m_Z^2} + c \epsilon^{\mu \nu \rho \sigma} \frac{p_{\rho} k_{\sigma}}{m_Z^2} \right]$$

Here $p = q_1 + q_2$ and $k = q_1 - q_2$ with $q_1$ and $q_2$ the Z momenta.

Generally $a$, $b$, and $c$ may be momentum dependent. $b$ and $c$ may be complex.

For the SM, $a = 1$, $b = c = 0$.

Alternatively, one may consider higher dimensional operators in addition to the SM:

$$\mathcal{L}_{H.D.O.} = \frac{1}{\Lambda_1^2} \Phi^\dagger \Phi V_{\mu \nu} V^{\mu \nu} + \frac{1}{\Lambda_2^2} \Phi^\dagger \Phi V_{\mu \nu} \tilde{V}^{\mu \nu}$$

$$\tilde{V}^{\mu \nu} = \epsilon^{\mu \nu \rho \sigma} V_{\rho \sigma}$$
The threshold distribution

The Higgs boson width for this general coupling is

\[
\Gamma_{H \rightarrow ZZ} = \frac{G_F m_H^3}{16\sqrt{2}\pi} \beta \left\{ a^2 \left[ \beta^2 + \frac{12m_1^2m_2^2}{m_H^4} \right] + |b|^2 \frac{m_H^4 \beta^4}{m_Z^4} + \left| c \right|^2 x^2 8\beta^2 \right. \\
\left. + a\text{Re}(b) \frac{m_H^2}{m_Z^2} \beta^2 \sqrt{\beta^2 + 4m_1^2m_2^2/m_H^4} \right\}
\]

Only the SM term proportional to \( a^2 \) has a contribution linear in \( \beta \).

This is due to the momentum dependence in all the other terms.

All non-standard contributions give a faster rise.

[This was originally pointed out by Choi, DJM, Mühlleitner, Zerwas, 2002]
The H→ZZ rate

Adding additional vertices will change the Higgs boson width. In principle, this should allow one to put limits on new vertices.

Here we assume that only the HZZ vertex is modified. Obviously if HWW were also modified, then the BR for H→ZZ would also change.

One can’t tell in this way if the change is due to an enhanced CP even coefficient or an additional CP odd term.

Godbole, DJM, Mühlleitner 2007
Angular distributions

The effect of additional vertices should also show up in the angular distributions.

The angles $\theta_1$, $\theta_2$ are in the parent $Z$ rest frame. $\phi$ is in the Higgs rest frame.
The azimuthal angle dependence in the SM is

$$\frac{d\Gamma}{d\phi} = 1 - \frac{9\pi^2}{32}\eta_1\eta_2\frac{\gamma}{2 + \gamma^2}\cos\phi + \frac{1}{2}\frac{1}{2 + \gamma^2}\cos 2\phi$$

$$\eta_i = \frac{v_{f_i}a_{f_i}}{v_{f_i}^2 + a_{f_i}^2}$$

For a pure pseudoscalar it is

$$\frac{d\Gamma}{d\phi} = 1 - \frac{1}{4}\cos 2\phi$$

If we mix them we get lots of terms...

$$\frac{d\Gamma}{d\phi} = 1 + d_1\cos\phi + d_2\sin\phi + d_3\cos 2\phi + d_4\sin 2\phi$$
This angle is directly analogous to the angle between the tagging jets in WW scattering.

[Diagram: Azimuthal angle between jets]

[Plehn, Rainwater, Zeppenfeld 2001]
The polar angle

The polar angle dependence in the SM is

\[
\frac{d\Gamma}{d\cos\theta_1} \sim \frac{2}{\gamma^2 - 1} + \sin^2\theta_1
\]

For a pure pseudoscalar it is

\[
\frac{d\Gamma}{d\cos\theta_1} \sim 1 + \cos^2\theta_1
\]

If we mix them we also get a term proportional to Im(c)

\[
\frac{d\Gamma}{d\cos\theta_1} \sim a^2 \left[ 2 + (\gamma^2 - 1) \sin^2\theta_1 \right]
\]

\[
+ |c|^2 (...) (1 + \cos\theta_1) + a \text{Im}(c) \eta_1 (...) \cos\theta_1
\]
CP and $\tilde{T}$

The coefficients map under CP and $\tilde{T}$ (pseudo-time reversal) according to

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>Re b</th>
<th>Im b</th>
<th>Re c</th>
<th>Im c</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\tilde{T}$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Now $\cos \theta_1 = \frac{(p_2^{(Z)} - p_1^{(Z)}) \cdot (p_3^{(H)} + p_4^{(H)})}{|p_2^{(Z)} - p_1^{(Z)}||p_3^{(H)} + p_4^{(H)}|}$ which is **CP odd** and $\tilde{T}$ **even**

探查 Im(c) 和 CP 违背

实际上，对应的不对称性

$$A = \frac{\Gamma_{c\theta>0} - \Gamma_{c\theta<0}}{\Gamma_{c\theta>0} + \Gamma_{c\theta<0}}$$

**不是很理想** 因为 $\eta_1$
Asymmetries

In a similar way, one can think up lots of asymmetries to probe the real and imaginary parts of each coupling.

\[
\begin{align*}
\cos \theta_1 & \quad \mathcal{A} \propto a \Re(c) \eta_1 \\
\sin \phi \sin \theta_1 & \quad \mathcal{A} \propto \eta_1 \eta_2 \left[ a \Re(c) \gamma + \Re(b^* c) \gamma^2 x \right] \\
\cos \theta_1 \sin \theta_2 \cos \theta_2 \sin \phi & \quad \mathcal{A} \propto \left[ a \Re(c) \gamma + \Re(b^* c) \gamma^2 x \right] \\
\sin 2\phi & \quad \mathcal{A} \propto a \Re(c)
\end{align*}
\]

.... and many more.

Unfortunately most of these asymmetries are very small and to me it seems unlikely that one will be able to really determine the phase in couplings.

We will probably need to rely on threshold and angular distributions to determine or place limits on the magnitude of the couplings.
Adding a little spin

Can we distinguish between a scalar and a spin 2 boson?

The tensor structure of a $H\beta_1\beta_2 Z_\mu Z_\nu$ vertex is in general very complicated.

**CP even**

\[
a_1 g^{\mu\beta_1} g^{\nu\beta_2} \\
+ a_2 g^{\mu\nu} k^{\beta_1} k^{\beta_2} \\
+ a_3 (g^{\mu\beta_1} p^{\nu} - g^{\nu\beta_1} p^{\mu}) k^{\beta_2} \\
+ a_4 p^{\mu} p^{\nu} k^{\beta_1} k^{\beta_2} \\
+ \beta_1 \leftrightarrow \beta_2
\]

**CP odd**

\[
b_1 \epsilon^{\mu\nu\beta_1\rho} p_\rho k^{\beta}_2 \\
+ b_2 (\epsilon^{\mu\beta_1\rho\sigma} p^{\nu} + \epsilon^{\nu\beta_1\rho\sigma} p^{\mu}) k^{\beta_1}_p k^{\rho}_k \\
+ b_3 \epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma k^{\beta_1} k^{\beta_2} \\
+ \beta_1 \leftrightarrow \beta_2
\]

However, specific models, such as a **KK graviton**, may have subsets of these structures.
The threshold distribution (again)

The **threshold rise** again depends on the **number of momenta** in the leading term.

For spin 2, only the term $g^{\mu\beta_1} g_{\nu\beta_2} + g^{\nu\beta_1} g_{\mu\beta_2}$ contains **no momentum**, so will rise $\sim \beta$.

Any model that **misses this term** will rise at least $\sim \beta^3$ and possibly a higher power.

We can test the threshold behaviour first. If it rises $\sim \beta$, we know it is either:

- a scalar
- a spin 2 with this tensor structure.
The angular dependence of spin 2

For any (CP conserving) spin, the most general polar angle dependences are

\[
\frac{d \Gamma}{d \cos \theta_1 \cos \theta_2} \sim \sin^2 \theta_1 \sin^2 \theta_2 |T_{00}|^2 \\
+ \frac{1}{2} \left( 1 + \cos^2 \theta_1 \right) \left( 1 + \cos^2 \theta_2 \right) \left( |T_{11}|^2 + |T_{1-1}|^2 \right) \\
+ \left( 1 + \cos^2 \theta_1 \right) \sin^2 \theta_2 |T_{10}|^2 + \sin^2 \theta_1 \left( 1 + \cos^2 \theta_2 \right) |T_{01}|^2 \\
+ 2 \eta_1 \eta_2 \cos \theta_1 \cos \theta_2 \left( |T_{11}|^2 - |T_{1-1}|^2 \right)
\]

These $T_{ij}$ are reduced vertices in the sense

\[
\langle Z(\lambda_1)Z(\lambda_2)|H_J \rangle = \frac{ig_W m_Z}{\cos \theta_w} T_{\lambda_1 \lambda_2} d^J_{m, \lambda_1 - \lambda_2} (\Theta) e^{-i(m-\lambda_1-\lambda_2)\Phi}
\]

The Standard Model has $T_{00} = \frac{m_H^2 - m_Z^2 - m_*^2}{2m_Z m_*}$, $T_{11} = -1$, $T_{01} = T_{10} = T_{1-1} = 0$

On the other hand, $g^{\mu \beta_1} g^{\nu \beta_2} + g^{\nu \beta_1} g^{\mu \beta_2}$ contributes to all of them.
Graviton-like polar distribution

Integrating over $\cos \theta_2$ gives

$$\frac{d \Gamma}{d \cos \theta_1} \sim |\mathcal{T}_{00}|^2 + 2|\mathcal{T}_{11}|^2 + 2|\mathcal{T}_{1-1}|^2 + |\mathcal{T}_{10}|^2 + \frac{6}{5}|\mathcal{T}_{01}|^2$$

$$+ |\mathcal{T}_{10}|^2 \cos \theta_1$$

$$+ \left[ -|\mathcal{T}_{00}|^2 + 2|\mathcal{T}_{11}|^2 + 2|\mathcal{T}_{1-1}|^2 + |\mathcal{T}_{10}|^2 \right] \cos^2 \theta_1$$

The graviton-like coupling presents a **very different angular dependence** from the SM.

(This plot isn’t very realistic since I have set $m_* = 30$ GeV. In principle it will vary. However, the difference becomes bigger as we move away from threshold.)
A paper this week!

This week Boughezal, LeCompte and Petriello (arXiv:1208.4311) analysed the threshold rise of 10 LHC $H \rightarrow ZZ \rightarrow 4$ leptons events.

They set up an asymmetry $\mathcal{A}_{M_c} = \frac{N_{M^* > M_c} - N_{M^* < M_c}}{N_{M^* > M_c} + N_{M^* < M_c}}$ varying $M_c$ for best results.

They examined a spin-2 operator designed to have a $\beta^5$ threshold and found a predicted $\mathcal{A}_{26} = -0.31$ compared to data’s $\mathcal{A}_{26} = 0.0 \pm 0.28$ (statistical error only).

This was the most significant result.
Conclusions and Summary

• To determine whether the resonance at 125 GeV is indeed the Higgs boson, we need to measure its spin and CP quantum numbers.

• Spin 1 is ruled out by the Landau-Yang Theorem, but a pseudoscalar or spin 2 “Higgs” are still possible.

• A pseudoscalar Higgs can be ruled out by looking at the threshold dependence of $H \rightarrow ZZ \rightarrow 4$ leptons (that is, the dependence on the off-shell Z virtuality).

• Investigating CP violation will be difficult but limits may be set using asymmetries.

• Most spin 2 tensor structures may also be eliminated using the threshold behaviour, but this is not the case for a graviton-like (e.g. KK) tensor structure.

• For the graviton like structure, the dependence on the polar angle in the Z decay looks very different from the SM.