Precision charmonium and D physics from lattice QCD and determination of the charm quark mass

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University of Glasgow, HPQCD collaboration

ICHEP08
Philadelphia, July 08
QCD is key part of SM but quark confinement tricky

Lattice QCD = full QCD effects

**RECIPE**

• Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of sea quarks)
• Calculate averaged “hadron correlators” from valence q props.

\[ < 0 | M^+ (0) M(t) | 0 > \]

• Fit for masses and simple matrix elements
• Fix \( m_q \) and determine \( a \) to get physical results
HPQCD Priority PRECISION lattice QCD i.e. \( \sim 1\% \)

- Allows non-trivial tests of QCD i.e. better than models.
- Allows accurate determin of SM parameters (inc CKM)
- Provides the underpinning for other calcs.

Possible for ‘gold-plated quantities’ i.e. stable hadron masses and weak/em decay rates to single hadron states

Statistical errors must be very good to test systematics.

Systematics from:

- disc. errors (need several \( a \) values)
- extrapoln to physical u/d masses \( m_s/10 < m_u/d < m_s/2 \)
- finite volume
- errors in fixing QCD parameters. Use:

\[ \Upsilon(2S - 1S), m_\pi, m_K, m_{\eta_c}, m_\gamma \]
Recent highlight - very accurate charm physics
Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quarks:

- $E_{\text{sim}} = m$
- PCAC relation (if enough chiral symmetry) gives $Z = 1$
- same action as for u, d, s, so cancellation in ratios

Key issue is discretisation errors:

$m = m_{a=0}(1 + A(m_c a)^2 + B(m_c a)^4 + \ldots)\]

$m_c a \approx 0.4, (m_c a)^2 \approx 0.2, \alpha_s(m_c a)^2 \approx 0.06, (m_c a)^4 \approx 0.04$

for $a \approx 0.1 \text{fm}$

Need to remove all of these errors for precision results

This is done in the Highly Improved Staggered Quark formalism, further improving Improved Staggered Quarks
Very precise D/Ds masses obtained

NO free parameters

charmonium masses, HISQ on fine MILC

D/Ds masses vs expt.

Fix $m_c$

lattice errors 6 MeV - $a^2$ extrap /error in a and em corrs

A key test of disc. errors since charmonium and D have different dynamics → stringent test of QCD.

E.Follana et al, 0706.1726[hep-lat]
Decay constants of $D/D_s/K/\pi$ to 2%.

$Br(H \rightarrow \mu \nu) \propto V_{ab}^2 f_H^2$

$f_H m_H = <0|\bar{\psi} \gamma_0 \gamma_5 \psi|H>$

f is a property of the meson calculable in lattice QCD

Value can be extracted from expt if $V_{ab}$ known
Improved accuracy from CLEO-c

Leptonic rate $\rightarrow$ decay constant using $V_{cs} = V_{ud}$, $V_{cd} = V_{us}$

2008

Different lattice QCD methods

$\mathbf{f_D}$ $\mathbf{f_{Ds}}$

Agree

206(9) 268(9)

3 (exptl) $\sigma$ apart

207(4) 241(3)

3 different expts using different channels

200 225 250 275 300

206(9)

1st disagreement between lattice and expt. New physics?

HPQCD HISQ u,d,s sea
0706.1726[hep-lat]

FNAL/MILC u,d,s sea
LAT08 prelim.

CLEO-c, 0806.2112, ICHEP08

Belle
EPS2007

BaBar
hep-ex/0607094

ETMC u,d sea
LAT08 prelim.

no s in sea as yet

206(9)
Further checks of lattice QCD calcns important ...

1. Further masses of hadrons containing charm

Mass splitting V-PS accurately calculable

For staggered quarks there are different ‘tastes’

No dependence on $m_u/d$

Good agreement for all with expt. as $a \to 0$

New prelim. results on $a=0.06\text{fm}$ lattices

![Graph showing hyperfine splittings with different data points and error bars.](image)
2. Further decay constants of hadrons containing charm and strange

\[ \Gamma_{e^+e^-} = \frac{4\pi}{3} \alpha_{QED}^2 e^2 f_V^2 m_V \]

\[ f_V m_V = \langle 0 | J | V \rangle \]

Good agreement with expt for all tastes as \( a \to 0 \)

Need to complete with conserved vector current
3. Compare charmonium correlators to perturbation theory - allows accurate determn $m_c, \alpha_s$

J. Kühn talk QCD/Lattice

Small t correlators perturbative - take t moments

$$G_n = \sum_t (t/a)^n G(t)$$

$$\to \frac{\partial^n}{\partial E^n} \Pi(E = 0)$$

$$G_n = \frac{g_n(\alpha_{MS}(\mu), \mu/m_c)}{(am_c(\mu))^{n-4}}$$

lattice calcn. extrapolated to $a=0$

continuum pert. th. (4-loop for low n)

I. Allison et al, 0805.2999[hep-lat]
HPQCD + Karlsruhe/Brookhaven
+ new results here
Gives 1% accurate value for $m_c$

<table>
<thead>
<tr>
<th>$m_c(\mu)/\text{GeV}$</th>
<th>$j_5^{(5)}$</th>
<th>$j_5^{(5\mu)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c(\mu)/\text{GeV}$</td>
<td>$j_\mu^{(\mu)}$</td>
<td>$j_\mu^{(1)}$</td>
</tr>
</tbody>
</table>

$\mu = 3\text{GeV}$

Best lattice result from pseudoscalar

$m_c(3\text{GeV}) = 0.986(10)\text{GeV} \quad m_c(m_c) = 1.267(9)\text{GeV}$

Contnm uses vector, $R(e^+e^-) = 0.986(13)\text{ GeV}$

4 different currents agree

Full error budget – biggest is determin of a

Published by Kühn et al
\( \alpha_s \) determination

\[ \alpha_{MS}(M_Z) = 0.1174(12) \]

Reduced moments have less dependence

New superfine results

agrees with determn from Wilson loops (2008)

\[ \alpha_{MS}(M_Z) = 0.1183(7) \]

Davies et al, 0807.1687

Give \( m_c \)
Conclusions

• We now have lattice results in charm physics with accuracy (2%) similar to that for light hadrons.
• $D_s$ decay constant is the only result (from ~15 quantities) that disagrees with experiment.
• Further tests this year confirm confidence in the lattice calculation must take this seriously.

Future:

• Need significantly improved experimental error on $f_{D_s}$ - currently 3x lattice error.
• Further lattice calculations in other formalisms needed.
• Similarly accurate semileptonic form factors for $D/D_s/K$ need to be calculated.
## Error budgets

E. Follana et al,
HPQCD
0706.1726[hep-lat]

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_K/f_\pi$</th>
<th>$f_K$</th>
<th>$f_\pi$</th>
<th>$f_{Ds}/f_D$</th>
<th>$f_{Ds}$</th>
<th>$f_D$</th>
<th>$\Delta_s/\Delta_d$</th>
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<tbody>
<tr>
<td>$r_1$ uncertainty</td>
<td>0.3</td>
<td>1.1</td>
<td>1.4</td>
<td>0.4</td>
<td>1.0</td>
<td>1.4</td>
<td>0.7</td>
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<tr>
<td>$a^2$ extrapol.</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
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<tr>
<td>finite vol.</td>
<td>0.4</td>
<td>0.4</td>
<td>0.8</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
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<tr>
<td>$m_{u/d}$ extrapol.</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>stat. errors</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.6</td>
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<td>$m_s$ evoln.</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$m_d$, QED etc</td>
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<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
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<tr>
<td><strong>Total %</strong></td>
<td><strong>0.6</strong></td>
<td><strong>1.3</strong></td>
<td><strong>1.7</strong></td>
<td><strong>0.9</strong></td>
<td><strong>1.3</strong></td>
<td><strong>1.8</strong></td>
<td><strong>1.2</strong></td>
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</tbody>
</table>

### $m_c(\mu)$

<table>
<thead>
<tr>
<th>Source</th>
<th>$R_6$</th>
<th>$R_8$</th>
<th>$R_4$</th>
<th>$R_6/R_8$</th>
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</thead>
<tbody>
<tr>
<td>$a^2$ extrapolation</td>
<td>0.3%</td>
<td>0.3%</td>
<td>0.4%</td>
<td>0.2%</td>
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<tr>
<td>perturbation theory</td>
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<tr>
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<tr>
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<td>0.1</td>
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<td>0.5</td>
<td>0.3</td>
<td>0.4</td>
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<tr>
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<td>0.1</td>
<td>0.1</td>
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<td>0.1</td>
<td>0.2</td>
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<tr>
<td>finite volume</td>
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<td>0.1</td>
<td>0.0</td>
<td>0.3</td>
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<tr>
<td>$\mu \rightarrow M_Z$ evolution</td>
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</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.0%</strong></td>
<td><strong>1.0%</strong></td>
<td><strong>1.0%</strong></td>
<td><strong>1.1%</strong></td>
</tr>
</tbody>
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Update of:
I. Allison et al,
0805.2999[hep-lat]
HPQCD + Karlsruhe/
Brookhaven