

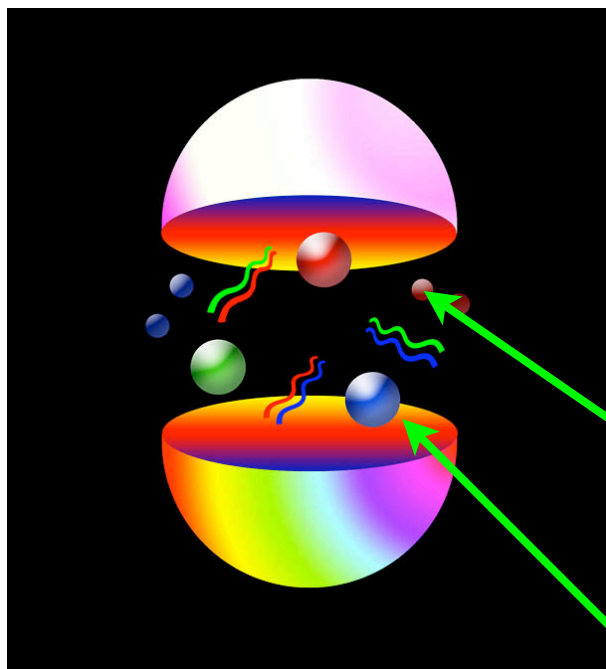
Precision charmonium  
and D physics from  
lattice QCD  
and determination of the  
charm quark mass

Christine Davies  
University of Glasgow,  
HPQCD collaboration

ICHEP08

Philadelphia, July 08

# QCD is key part of SM but quark confinement tricky



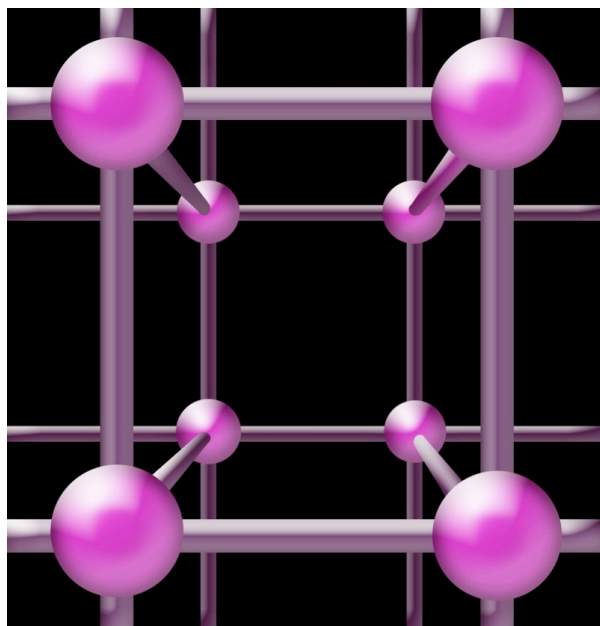
Lattice QCD = full QCD effects

## RECIPE

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of sea quarks)
- Calculate averaged “hadron correlators” from valence  $q$  props.

$$\langle 0 | M^\dagger(0) M(t) | 0 \rangle$$

- Fit for masses and simple matrix elements
- Fix  $m_q$  and determine  $a$  to get physical results



$a$

HPQCD Priority **PRECISION** lattice QCD i.e.  $\sim 1\%$

- Allows non-trivial tests of QCD i.e. better than models.
- Allows accurate determin of SM parameters (inc CKM)
- Provides the underpinning for other calcs.

Possible for ‘gold-plated quantities’ i.e. stable hadron masses and weak/em decay rates to single hadron states

Statistical errors must be very good to test systematics.

Systematics from:

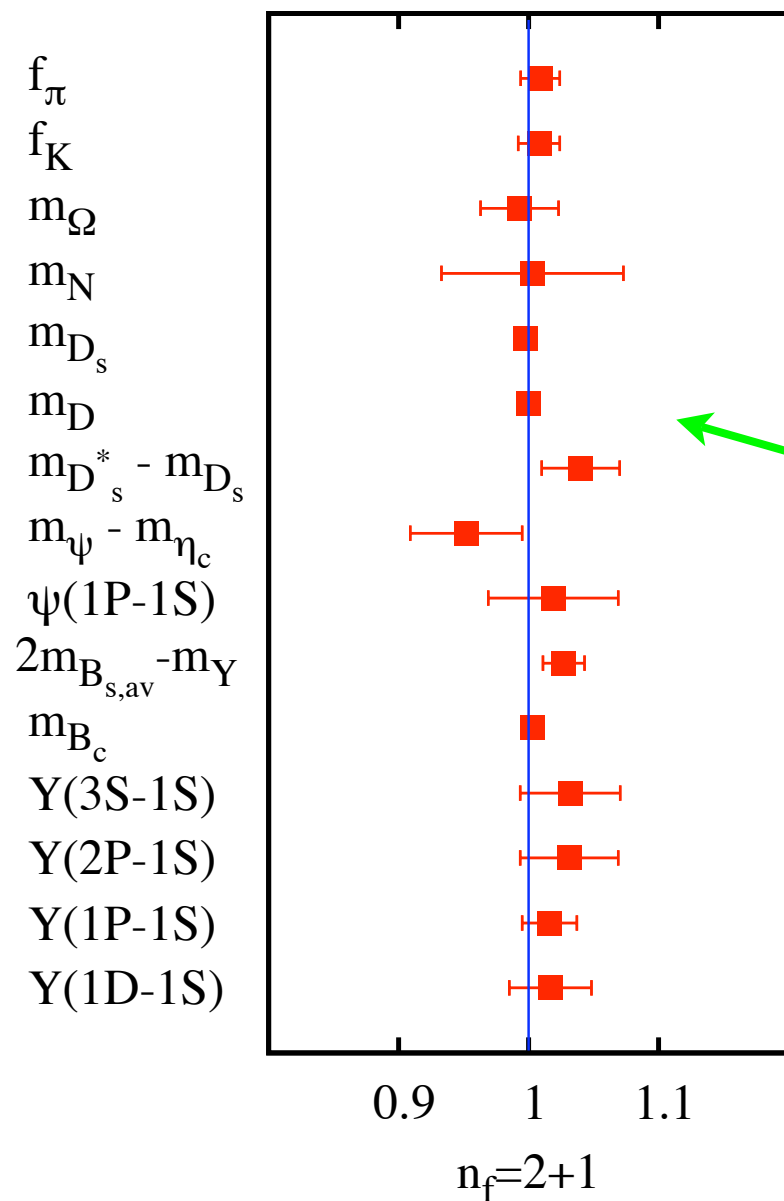
Expect an error budget ....

- disc. errors (need several  $a$  values)
- extrapln to physical u/d masses  $m_s/10 < m_{u/d} < m_s/2$
- finite volume
- errors in fixing QCD parameters. Use:

$$Y(2S - 1S), m_\pi, m_K, m_{\eta_c}, m_\Upsilon$$

# 2007 HPQCD/MILC/FNAL results

*latt / expt*



Analysis on MILC  
configs that include u,d, s  
improved staggered sea  
quarks - numerically fast

Recent highlight -  
very accurate  
charm physics

# Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quarks:

- $E_{sim} = m$
- PCAC relation (if enough chiral symmetry) gives  $Z = 1$
- same action as for u, d, s, so cancellation in ratios

Key issue is discretisation errors:

$$m = m_{a=0}(1 + A(m_c a)^2 + B(m_c a)^4 + \dots)$$

$$m_c a \approx 0.4, (m_c a)^2 \approx 0.2, \alpha_s(m_c a)^2 \approx 0.06, (m_c a)^4 \approx 0.04$$

for  $a \approx 0.1 \text{ fm}$

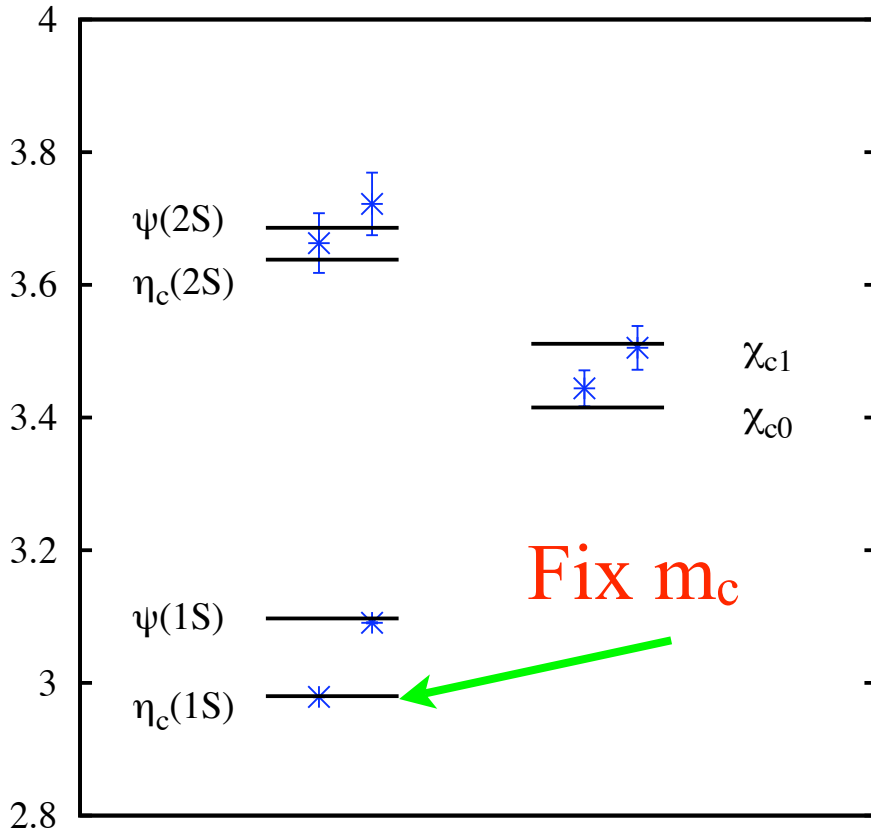
Need to remove *all* of these errors for precision results

This is done in the Highly Improved Staggered Quark formalism, further improving Improved Staggered Quarks

# Very precise D/Ds masses obtained

NO free parameters

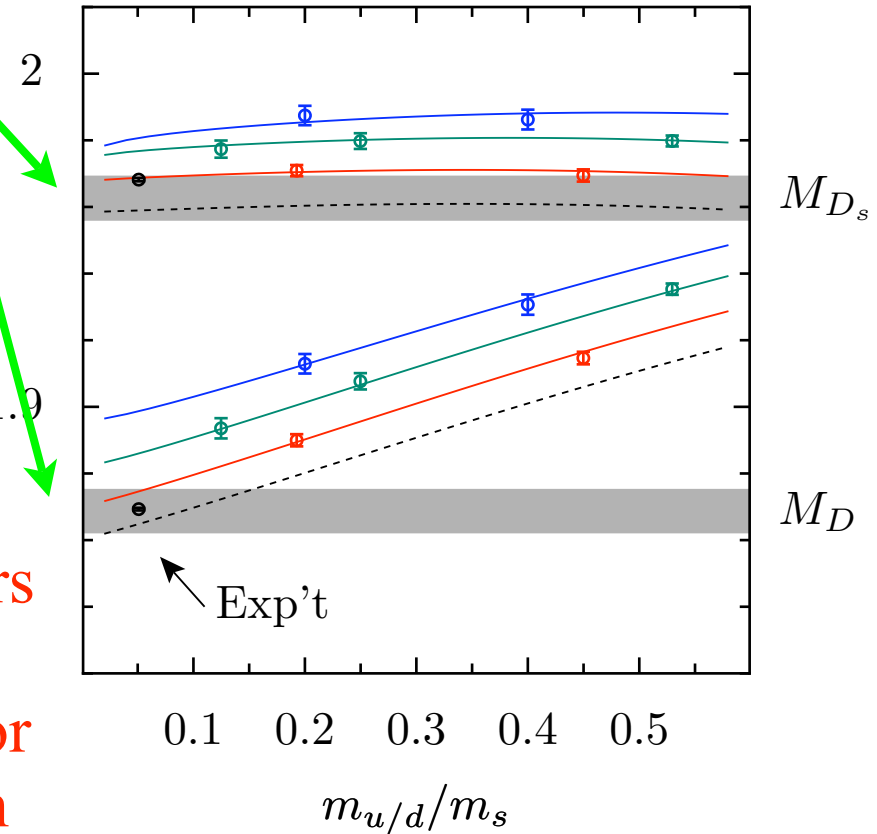
charmonium masses, HISQ on fine MILC



D/Ds masses vs expt.

Mass (GeV)

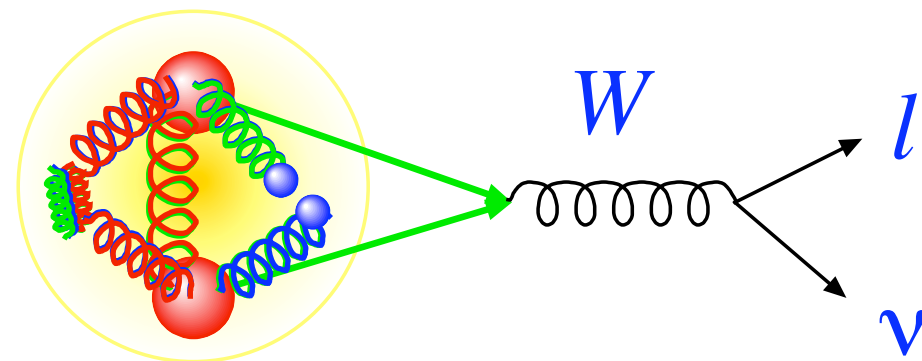
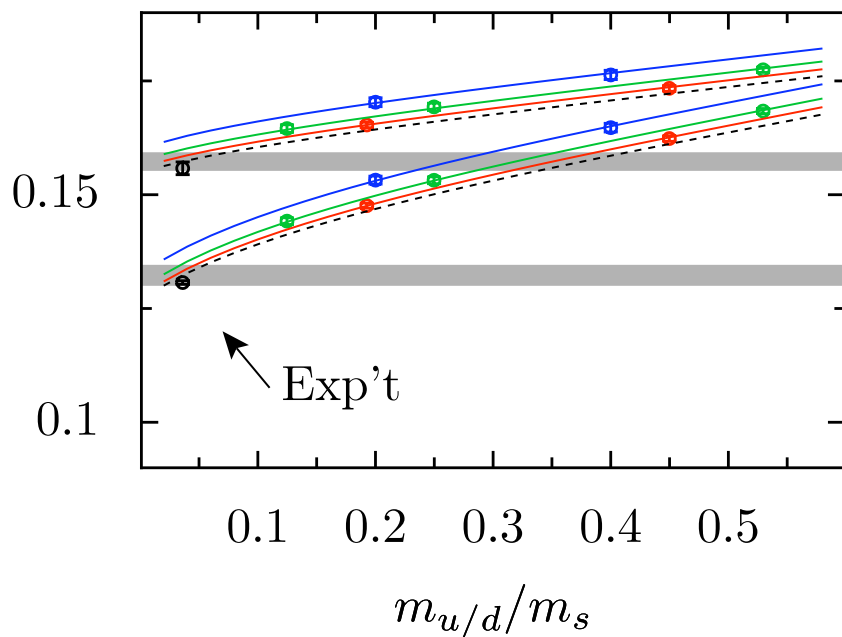
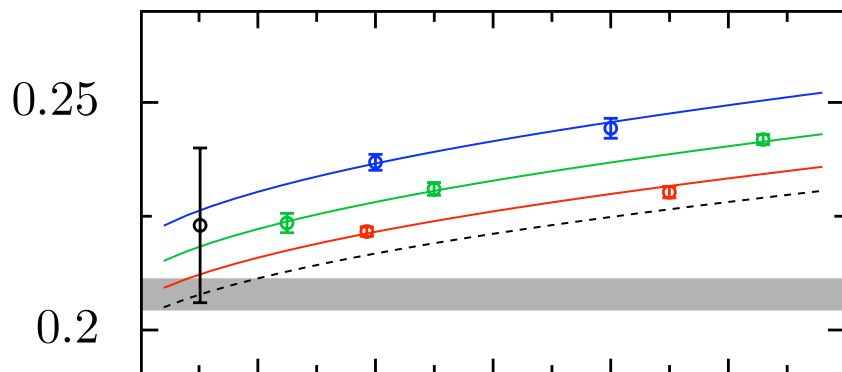
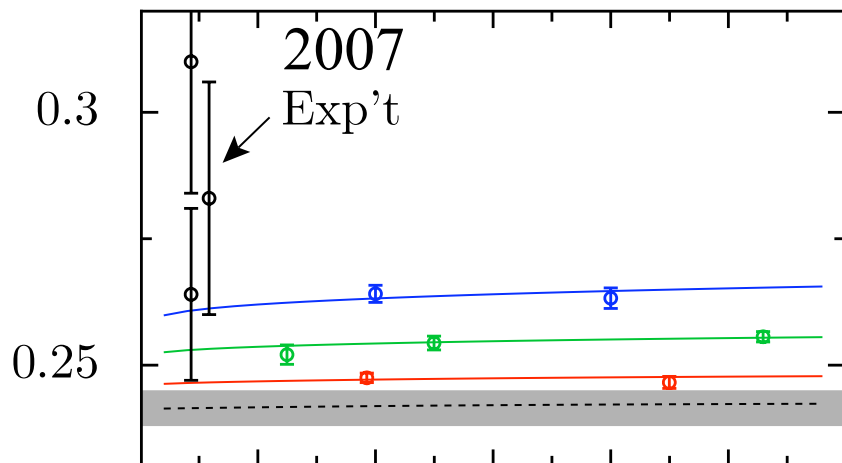
lattice errors  
6 MeV -  $a^2$   
extrap /error  
in  $a$  and em  
corrs



A key test of disc. errors since charmonium and D have different dynamics  $\longrightarrow$  stringent test of QCD.

# Decay constants of $D/D_s/K/\pi$ to 2%.

Decay Const. (GeV)



$$Br(H \rightarrow \mu\nu) \propto V_{ab}^2 f_H^2$$

$$f_H m_H = \langle 0 | \bar{\Psi} \gamma_0 \gamma_5 \Psi | H \rangle$$

$f$  is a property of the meson  
calculable in lattice QCD

Value can be extracted from  
expt if  $V_{ab}$  known

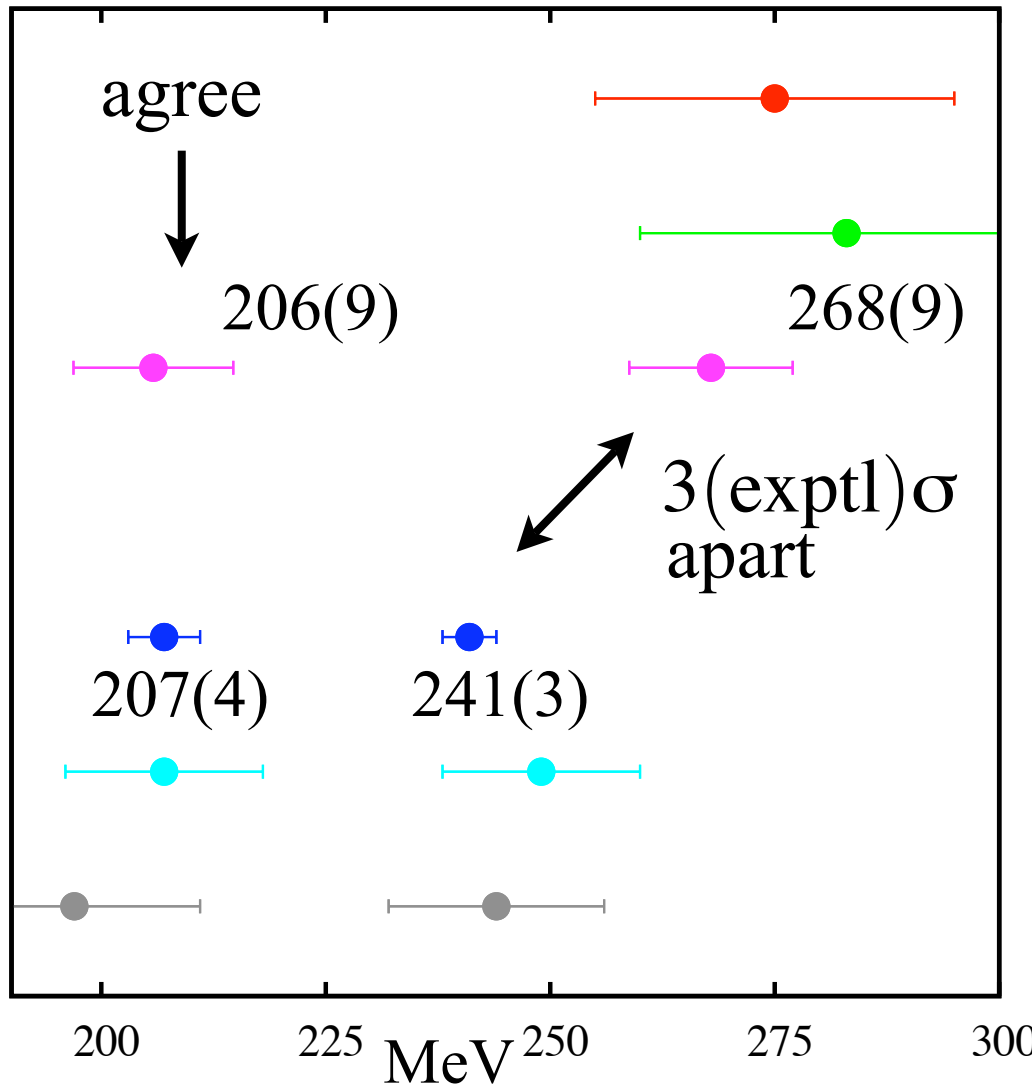
# 2008 Improved accuracy from CLEO-c

Leptonic rate  $\longrightarrow$  decay constant using  $V_{cs}=V_{ud}$ ,  $V_{cd}=V_{us}$

$f_D$

$f_{D_s}$

Zhang talk QCD/Lattice



Belle  
EPS2007

BaBar  
hep-ex/0607094

CLEO-c, 0806.2112,  
ICHEP08

HPQCD HISQ u,d,s sea  
0706.1726[hep-lat]

FNAL/MILC u,d,s sea  
LAT08 prelim.

ETMC u,d sea  
LAT08 prelim.

no s in sea as yet

3 different  
expts using  
different  
channels

3  
different  
lattice  
QCD  
methods

*First disagreement between lattice and expt. New physics?*

# Further checks of lattice QCD calcns important ...

## 1. Further masses of hadrons containing charm

Mass splitting V-PS  
accurately calculable

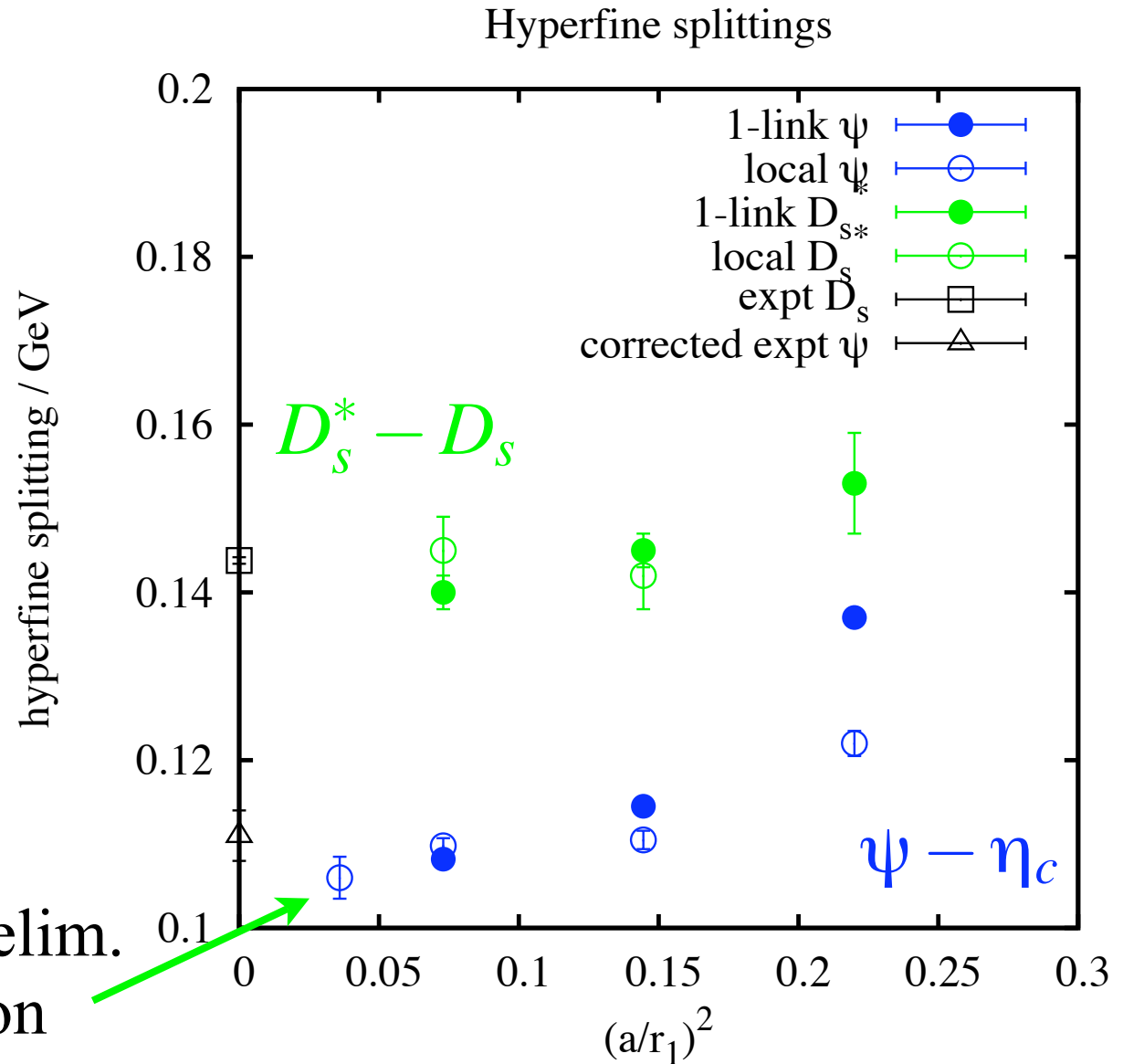
For staggered quarks  
there are different  
'tastes'

No dependence on  
 $m_{u/d}$

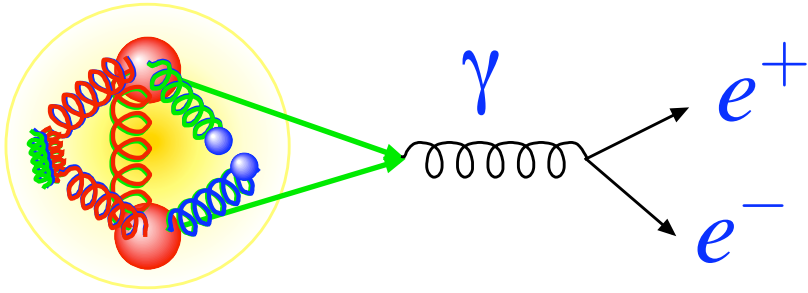
Good agreement  
for all with expt. as

$a \rightarrow 0$

New prelim.  
results on  
 $a=0.06\text{fm}$  lattices



## 2. Further decay constants of hadrons containing charm and strange



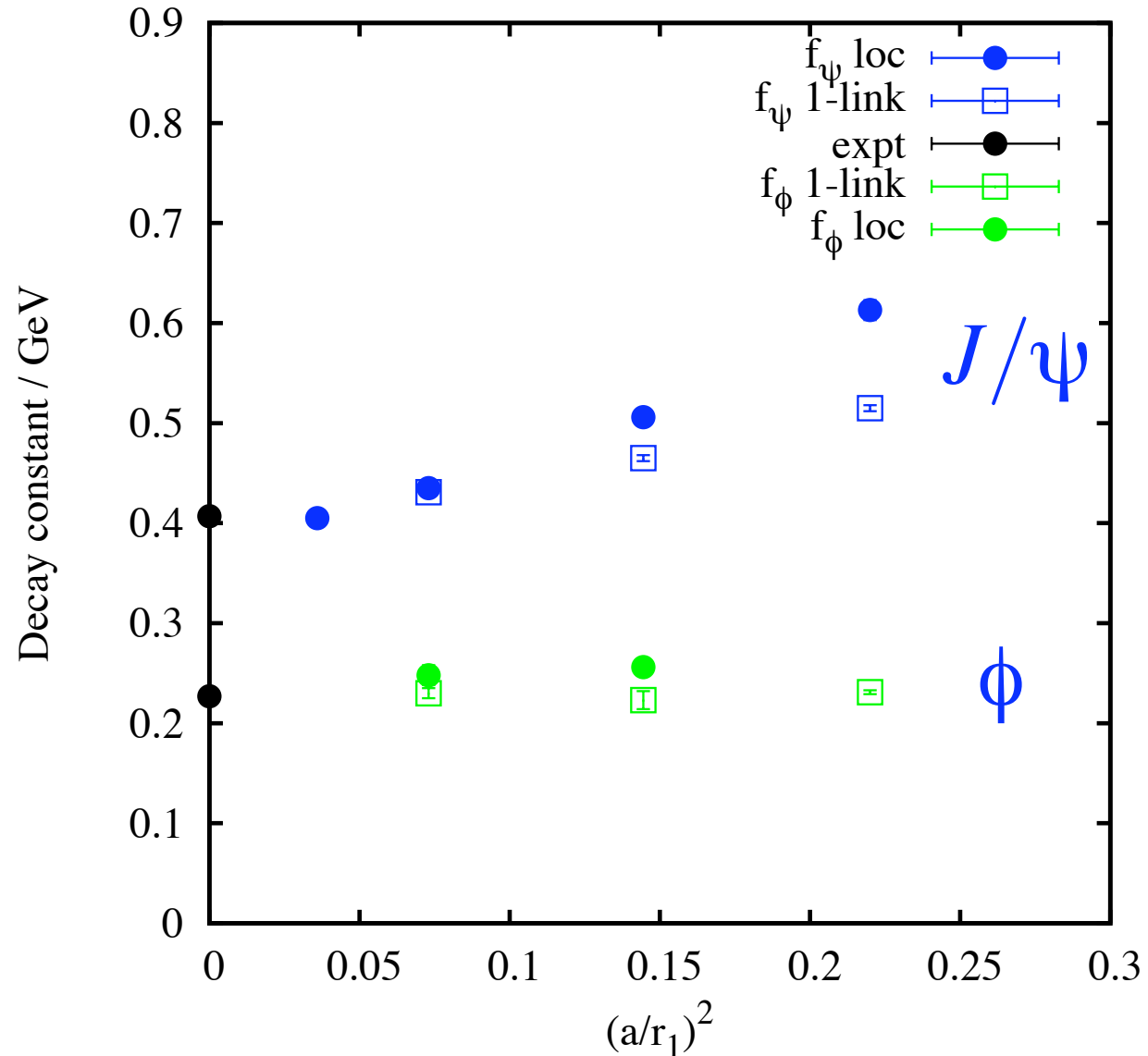
$$\Gamma_{e^+e^-} = \frac{4\pi}{3} \alpha_{QED}^2 e_Q^2 \frac{f_V^2}{m_V}$$

$$f_V m_V = \langle 0 | J | V \rangle$$

Good agreement with  
expt for all tastes as

$$a \rightarrow 0$$

Need to complete with  
conserved vector current



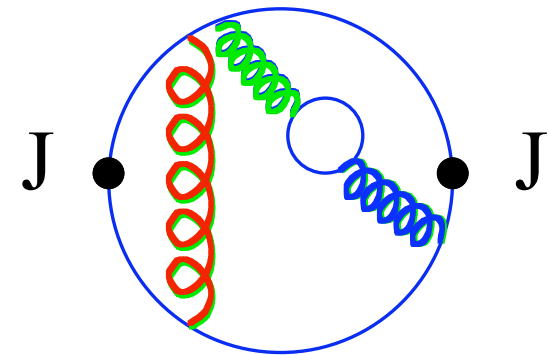
3. Compare charmonium correlators to perturbation theory - allows accurate determin  $m_c, \alpha_s$

J. Kühn talk  
QCD/Lattice

Small t correlators perturbative - take t moments

$$G_n = \sum_t (t/a)^n G(t) \longrightarrow \langle 0 | J^\dagger(0) J(t) | 0 \rangle$$

$$\longrightarrow \frac{\partial^n}{\partial E^n} \Pi(E=0)$$



continuum pert. th.  
(4-loop for low n)

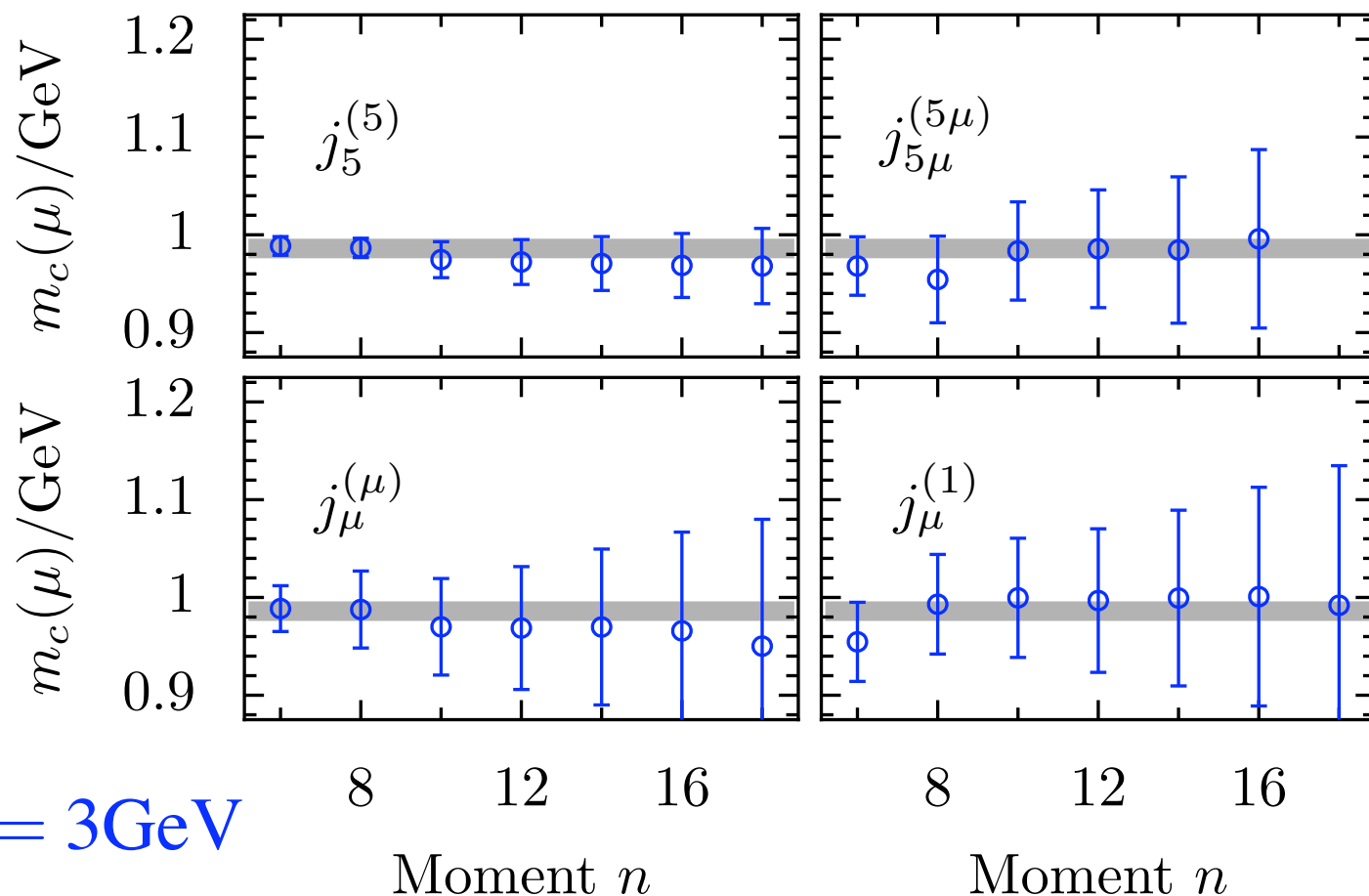
$$G_n = \frac{g_n(\alpha_{\overline{MS}}(\mu), \mu/m_c)}{(am_c(\mu))^{n-4}}$$

lattice calcn.  
extrapolated to  $a=0$

I.Allison et al, 0805.2999[hep-lat]  
HPQCD + Karlsruhe/Brookhaven  
+ new results here

Gives 1% accurate value for  $m_c$

PRELIMINARY



4 different  
currents  
agree

Full error  
budget –  
biggest is  
determined of a

$\mu = 3\text{GeV}$

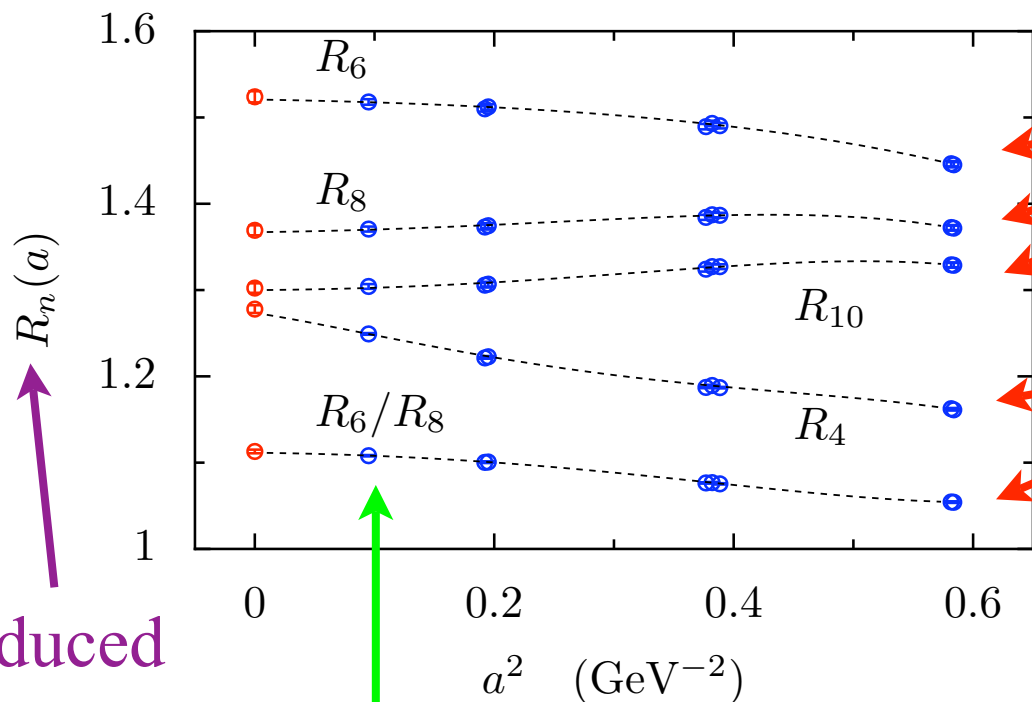
Best lattice result from pseudoscalar

$$m_c(3\text{GeV}) = 0.986(10)\text{GeV} \quad m_c(m_c) = 1.267(9)\text{GeV}$$

Continuum uses vector,  $R(e^+e^-) = 0.986(13)\text{GeV}$  Kühn et al  
hep-ph/0702103

# $\alpha_S$ determination

PRELIMINARY



Reduced moments have less a dependence

New superfine results

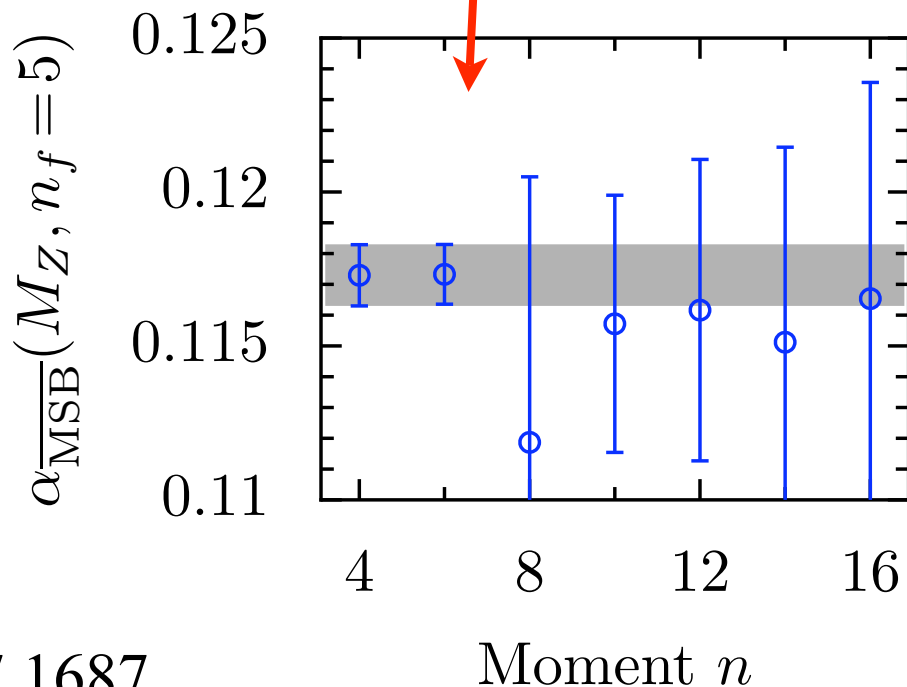
agrees with deternm from Wilson loops (2008)

$$\alpha_{\overline{MS}}(M_Z) = 0.1183(7)$$

Davies et al, 0807.1687

Give  $m_c$

$$\alpha_{\overline{MS}}(M_Z) = 0.1174(12)$$



# Conclusions

- We now have lattice results in charm physics with accuracy (2%) similar to that for light hadrons
- $D_s$  decay constant is the *only* result (from  $\sim 15$  quantities) that disagrees with experiment.
- Further tests this year confirm confidence in the lattice calculation  $\longrightarrow$  must take this seriously.

## Future:

- Need significantly improved experimental error on  $f_{D_s}$  - currently 3x lattice error.
- Further lattice calculations in other formalisms needed.
- Similarly accurate semileptonic form factors for  $D/D_s/K$  need to be calculated.

# Error budgets

E.Follana et al,  
HPQCD  
0706.1726[hep-lat]

	$f_K/f_\pi$	$f_K$	$f_\pi$	$f_{D_s}/f_D$	$f_{D_s}$	$f_D$	$\Delta_s/\Delta_d$
$r_1$ uncertainty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
$a^2$ extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
$m_s$ evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
$m_d$ , QED etc	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

	$m_c(\mu)$		$\alpha_{\overline{\text{MS}}}(M_Z)$	
	$R_6$	$R_8$	$R_4$	$R_6/R_8$
$a^2$ extrapolation	0.3%	0.3%	0.4%	0.2%
perturbation theory	0.4	0.3	0.6	0.6
$\alpha_{\overline{\text{MS}}}$ uncertainty	0.3	0.4	0.0	0.0
$m_c(\mu)$ uncertainty	0.0	0.0	0.1	0.1
gluon condensate	0.3	0.0	0.4	0.7
statistical errors	0.1	0.0	0.2	0.1
$m_{0c}$ errors from $r_1/a$	0.5	0.5	0.3	0.4
$m_{0c}$ errors from $r_1$	0.6	0.6	0.1	0.1
$m_{u/d/s}$ extrapolation	0.2	0.2	0.1	0.2
finite volume	0.1	0.1	0.0	0.3
$\mu \rightarrow M_Z$ evolution	0.0	0.0	0.1	0.1
Total	1.0%	1.0%	1.0%	1.1%

Update of:  
I.Allison et al,  
0805.2999[hep-lat]  
HPQCD + Karlsruhe/  
Brookhaven