Precision determination of the charm quark mass

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HPQCD collaboration

CHARM2013,
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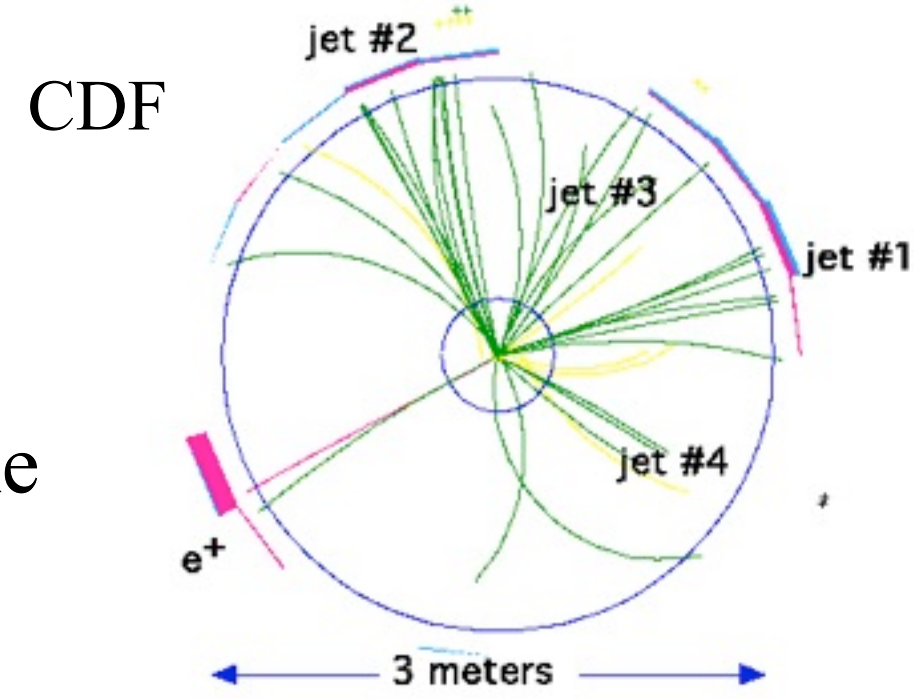
Quark masses are fundamental parameters of the SM but cannot be directly determined from experiment.

Well-defined masses are scheme and scale-dependent.
Convention to use $\overline{MS}$



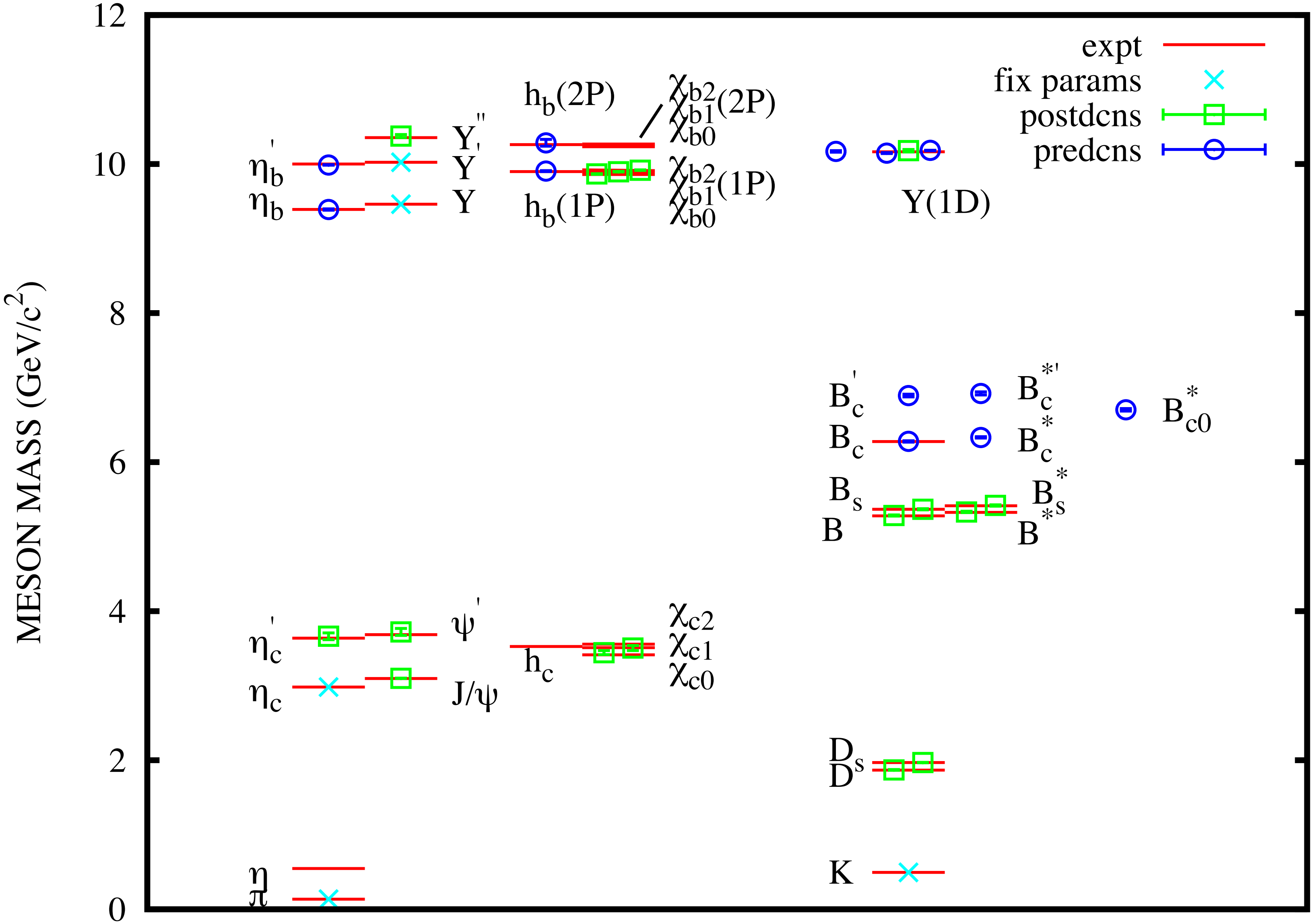
Compare results from multiple approaches for strong test of QCD.

Masses are input to theoretical expressions for SM cross-sections e.g. $H \to c\bar{c}$ (but Higgs WG inflate errors -why?)

	Higgs X-Section WG	PDG	lattice	Karlsruhe (e^+e^-)	world non-lattice
$\delta\ \alpha_s$	0.002	0.0007	0.0007		0.0012
$\delta\ m_c$ (GeV)	0.03	0.025	0.006	0.013	
$\delta\ m_b$ (GeV)	0.06	0.03	0.023	0.016	

P. Mackenzie, Snowmass 2013

Lattice QCD works directly with the QCD Lagrangian. Can tune bare mass parameters very accurately using experimentally very well-determined hadron masses.



R. Dowdall et al, HPQCD, 1207.5149

Conversion of lattice quark masses to $\overline{MS}$ scheme

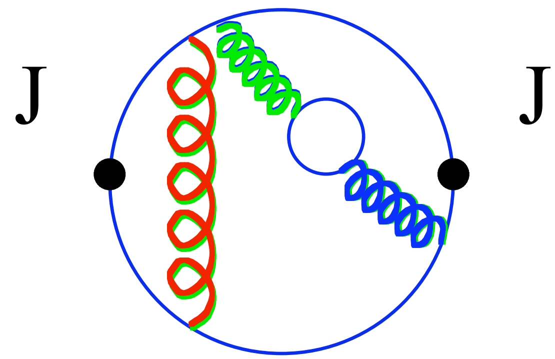
- Direct methods: Determine $m_{q,latt}$ in lattice QCD.

$$m_{\overline{MS}}(\mu) = Z(\mu a) m_{latt}$$

Calculate Z in lattice QCD pert. th. or use 'nonpert' lattice matching.
Error dominated by that of Z and continuum extrapolation.
Note: Z cancels in mass ratios.

- Indirect methods: (after tuning m_{latt}) match a quantity calculated in lattice QCD to continuum pert. th. in terms of $\overline{MS}$ quark mass

e.g. Current-current correlators for heavy quarks known through α_s^3 .



Chetyrkin et al, Maier et al

Issues with handling ‘heavy’ quarks on the lattice:

$$L_q = \overline{\psi}(\not{D} + m)\psi \rightarrow \overline{\psi}(\gamma \cdot \Delta + ma)\psi$$

Δ is a finite difference on the lattice - leads to discretisation errors. What sets the scale for these?

For light hadrons the scale is Λ_{QCD} = few hundred MeV

For heavy hadrons the scale can be m_Q

$$E(a) = E(a=0) \times (1 + A(m_Q a)^2 + B(m_Q a)^3 + \ldots)$$

$m_c a \approx 0.4, m_b a \approx 2$ for $a \approx 0.1\text{fm}$

⇨ need good discretisation of Dirac equation and multiple values of a for accurate continuum extrapolation.

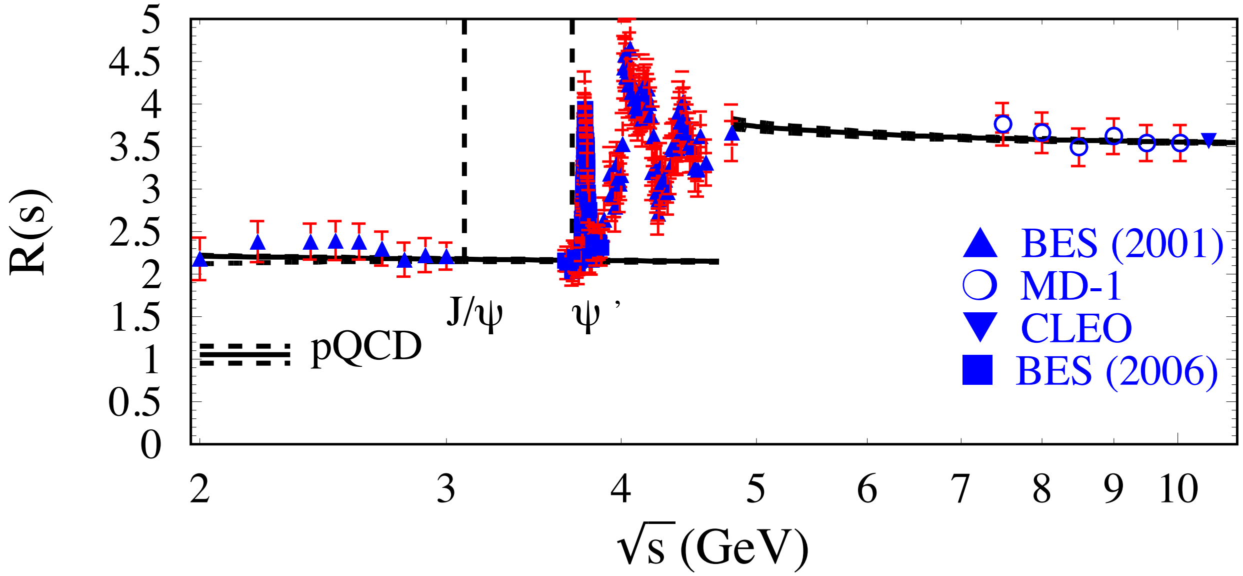
Highly Improved Staggered Quarks (HISQ) formalism has errors improved to $\alpha_s(am)^2, (am)^4$ Follana et al, HPQCD, hep-lat/0610092

Current-current correlator method for m_c

Continuum: extract charm piece of:

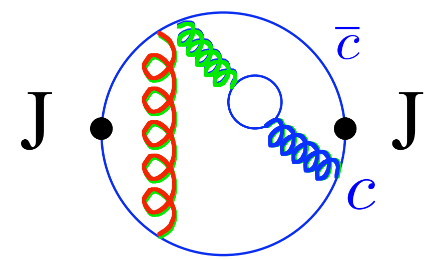
e.g. Kuhn et al, hep-ph/0702103

$R_{e^+e^-}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/(3s)}$ from experiment, then



$$\mathcal{M}_k \equiv \int \frac{ds}{s^{k+1}} R_{e^+e^-}(s)$$

$$= \frac{12\pi^2}{n!} \left(\frac{d}{dq^2}\right)^k \Pi_c(q^2)\Big|_{q^2=0}^2$$



vector coupling to photon

$$\Pi_c(q^2) = \frac{3}{16\pi^2} e_c^2 \sum_{k\geq 0} C_k^V \left(\frac{q^2}{4(m_c(\mu))^2}\right)^k$$

C_k a power series in $\alpha_s(\mu)$, known through α_s^3 for first few values of k

Use k=1: $m_c(m_c) = 1.279(13)\text{GeV}$

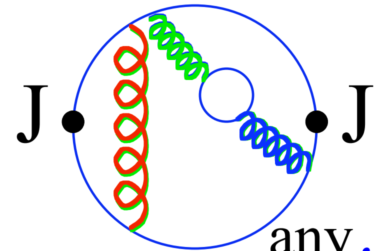
errors: expt + α_s

Chetyrkin et al, 0907.2110

Current-current correlator method for lattice m_c

HPQCD + Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD,1004.4285

- Substitute time-moment of lattice charmonium correlator for experiment. In principle can use any current J now.



any J^{PC} current now

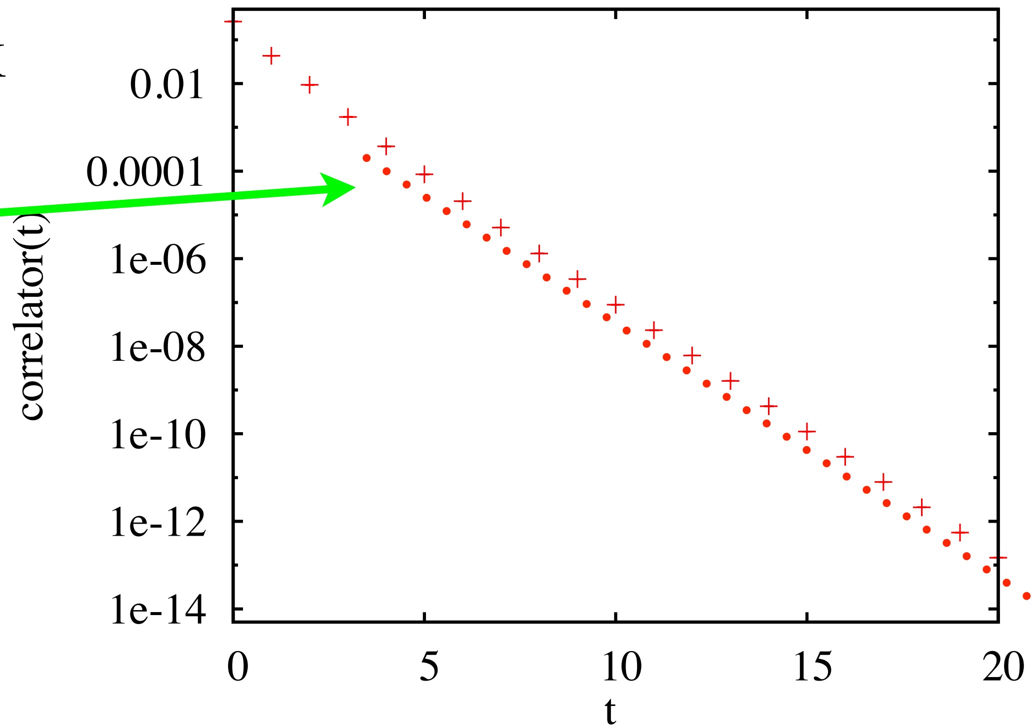
- For HISQ quarks pseudoscalar η_c correlator is most accurate. J is absolutely normalised.

step 1: calculate η_c correlators by combining lattice charm quark propagators

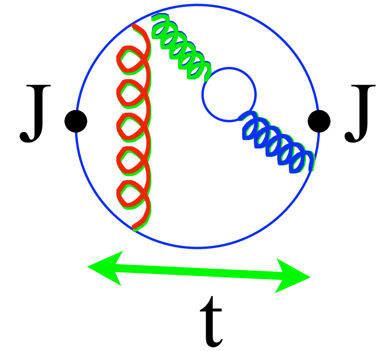
step 2: large time - fit to exponential, gives η_c mass

step 3: tune lattice quark mass so η_c mass correct.

step 4: calculate time moments to compare to QCD pert. theory. Emphasises short-time contribns.



Correlator time-moments:



$$G(t) = a^6 \sum_{\vec{x}} (am_c)^2 < 0|j_5(\vec{x},t) j_5(0,0)|0 >$$

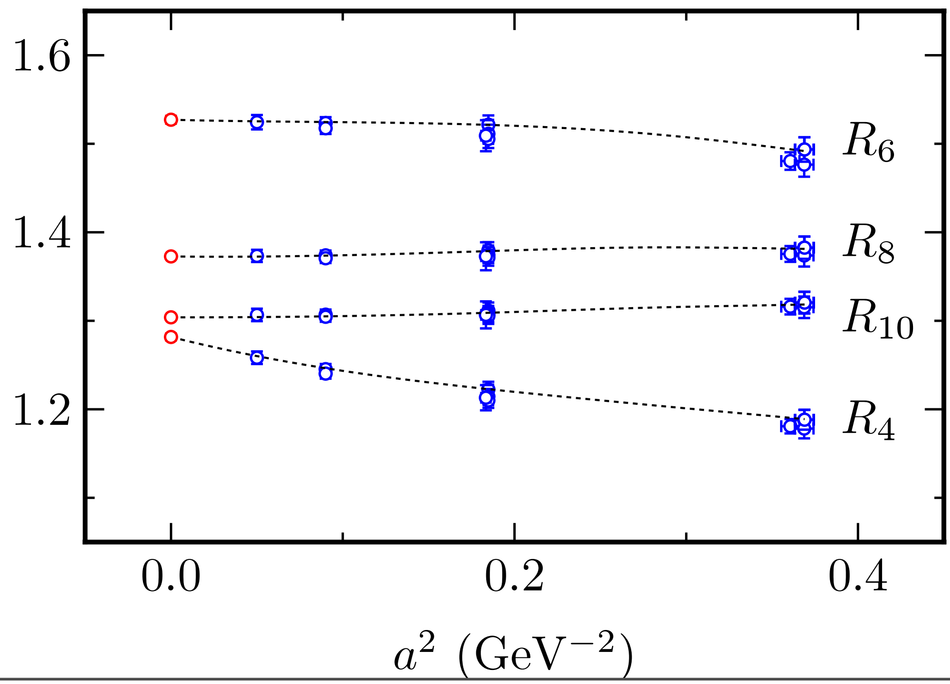
$$G_n = \sum_t (t/a)^n G(t)$$

$$R_{n,latt} = G_4/G_4^{(0)} \quad n = 4$$

ratio to results with no gluon field improves disc. errors

$$= \frac{am_{\eta_c}}{2am_c} (G_n/G_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10 \ldots$$

(match k = 2, 3, 4 ...)



extrapolate to a=0 and compare to contnm pert. th.

$$R_{n,cont} = \frac{m_{\eta_c}}{2m_c(\mu)} \frac{C_k^P}{C_k^{P,0}}$$

$n = 2k + 2$

$$\frac{C_k^P}{C_K^{P,0}} = 1 + \sum c_i \alpha_s^i(\mu)$$

Fit first 4 moments simultaneously, gives $\frac{m_{\eta_c}}{2m_c(\mu)}$ AND $\alpha_s(\mu)$

Result:

$$m_c(m_c) = 1.273(6)\text{GeV}$$

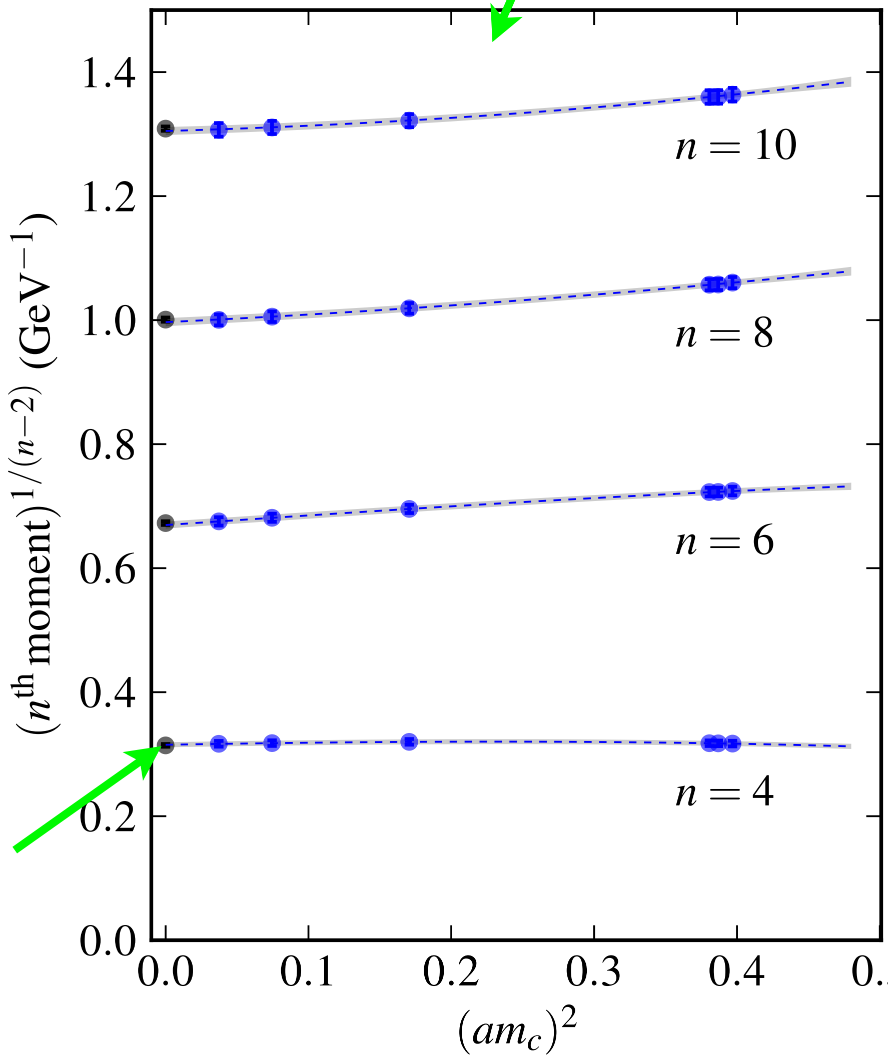
error dominated by unknown higher orders in pert. th.

C. McNeile et al, HPQCD,1004.4285

Further check:
compare vector moments (after normalising current) to those extracted from $R_{e^+e^-}$

Agreement is a 1% test of (lattice) QCD

lattice and expt errors similar size



G. Donald et al, HPQCD, 1208.2855

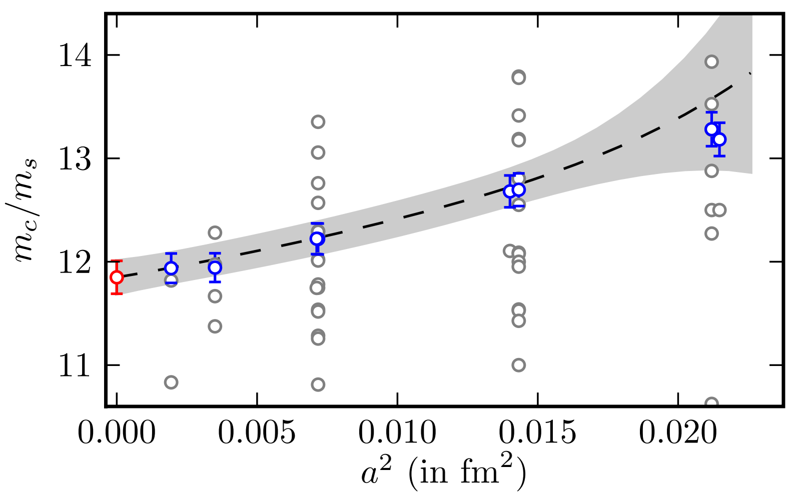
m_c/m_s

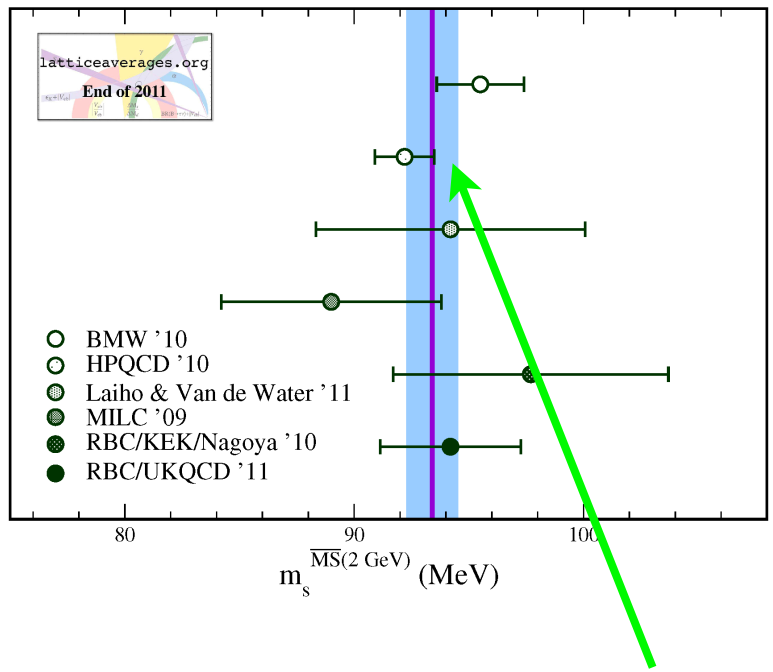
Mass ratio can be obtained directly from lattice QCD if same quark formalism is used for both quarks. Ratio is at same scale and for same n_f.

$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

Not possible with any other method ...

HISQ





$$\frac{m_c}{m_s} = 11.85(16) \quad n_f = 3$$

C. Davies et al, HPQCD, 0910.3102

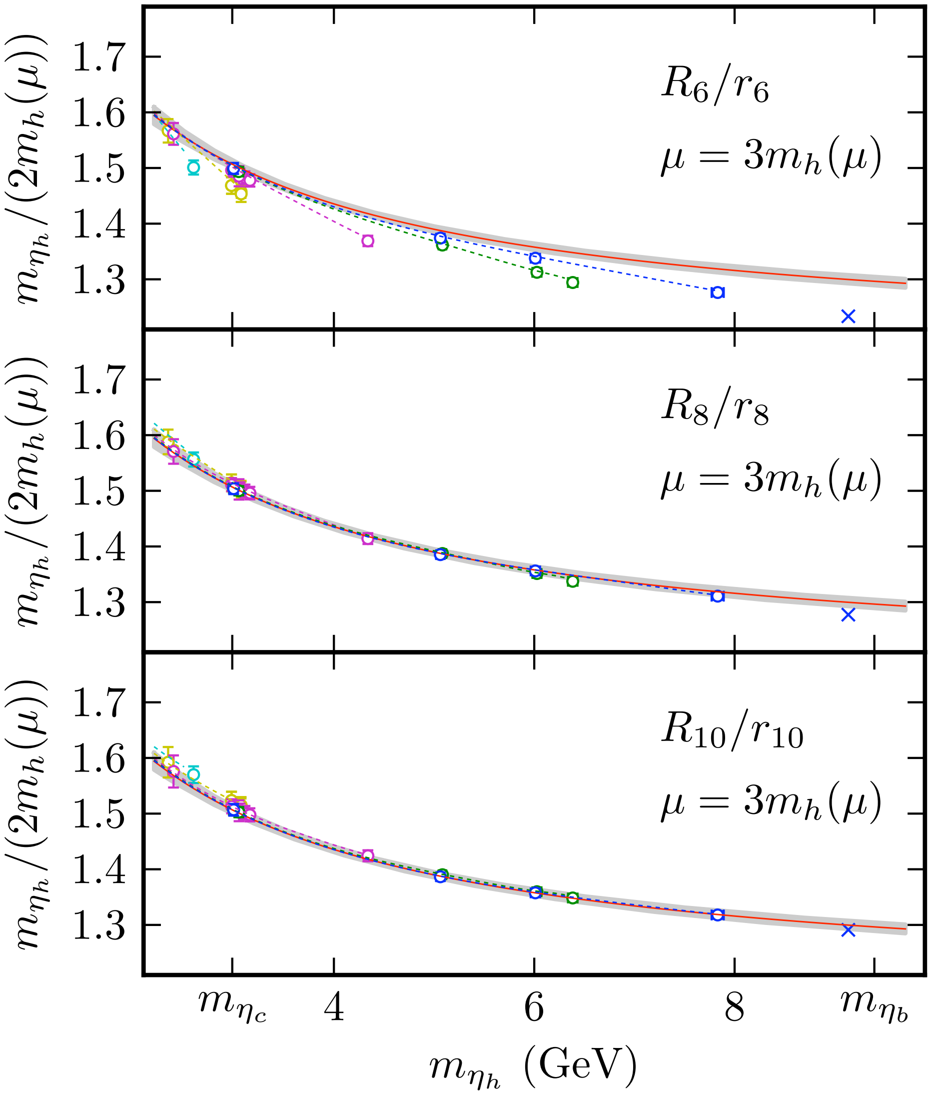
92.2(1.3) MeV

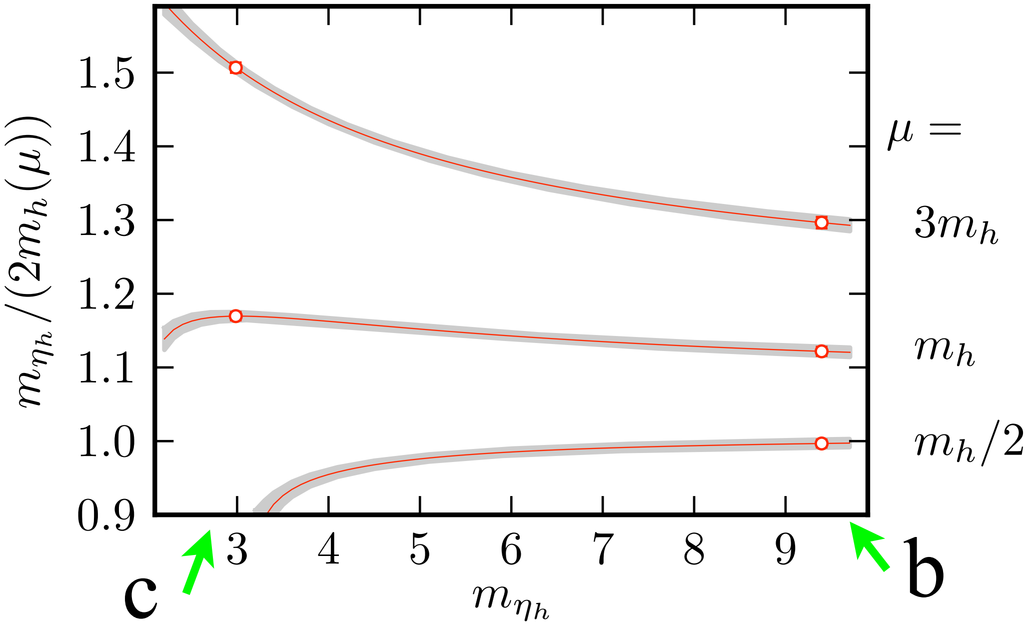
allows 1% accuracy in m_s

Current-current correlator method -HISQ

HPQCD, 1004.4285

- Repeat calcln for $m_q \geq m_c$ inc. ultrafine lattices





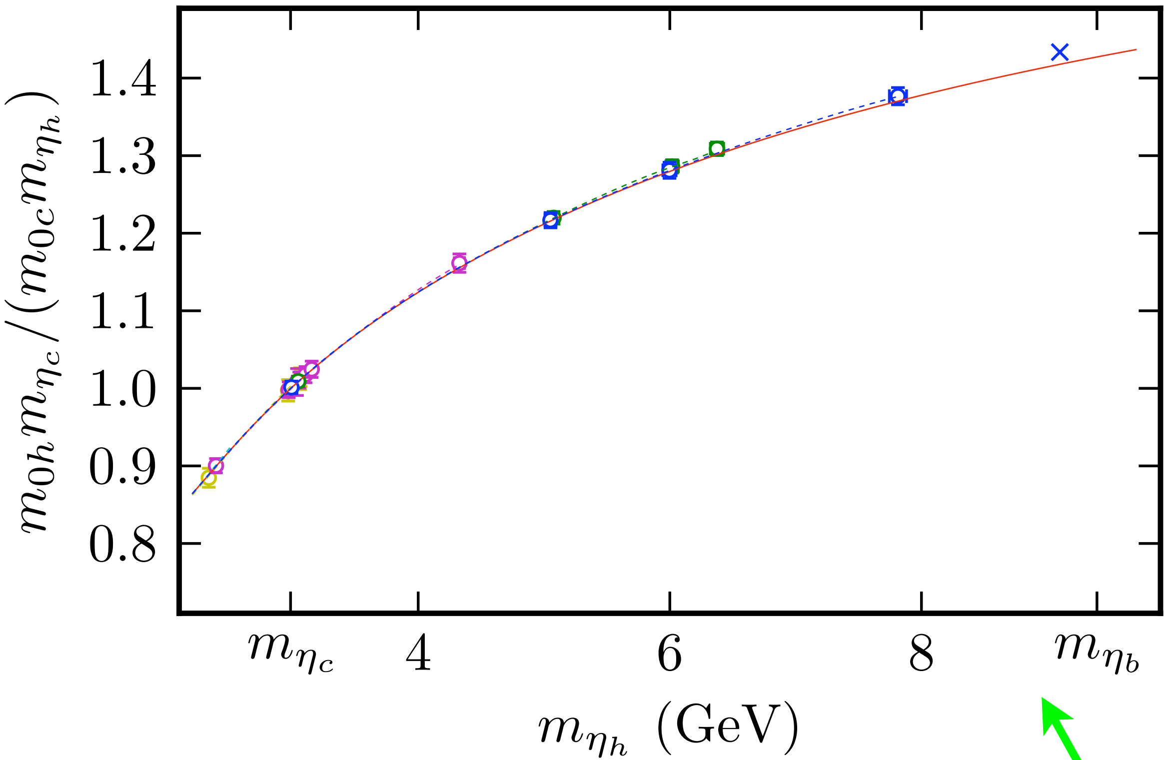
Agrees well with contnm results using $R_{e^+e^-}$

Can determine m_h/m_{η_h} for heavy quarks - extrapolate (slightly) to b.

$$\overline{m}_b^{n_f=5}(\overline{m}_b) = 4.164(23)\text{GeV}$$

key error is now extrapoln in a

m_b/m_c from lattice QCD



$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0}$$

$$= \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

completely nonperturbative determination of ratio gives:

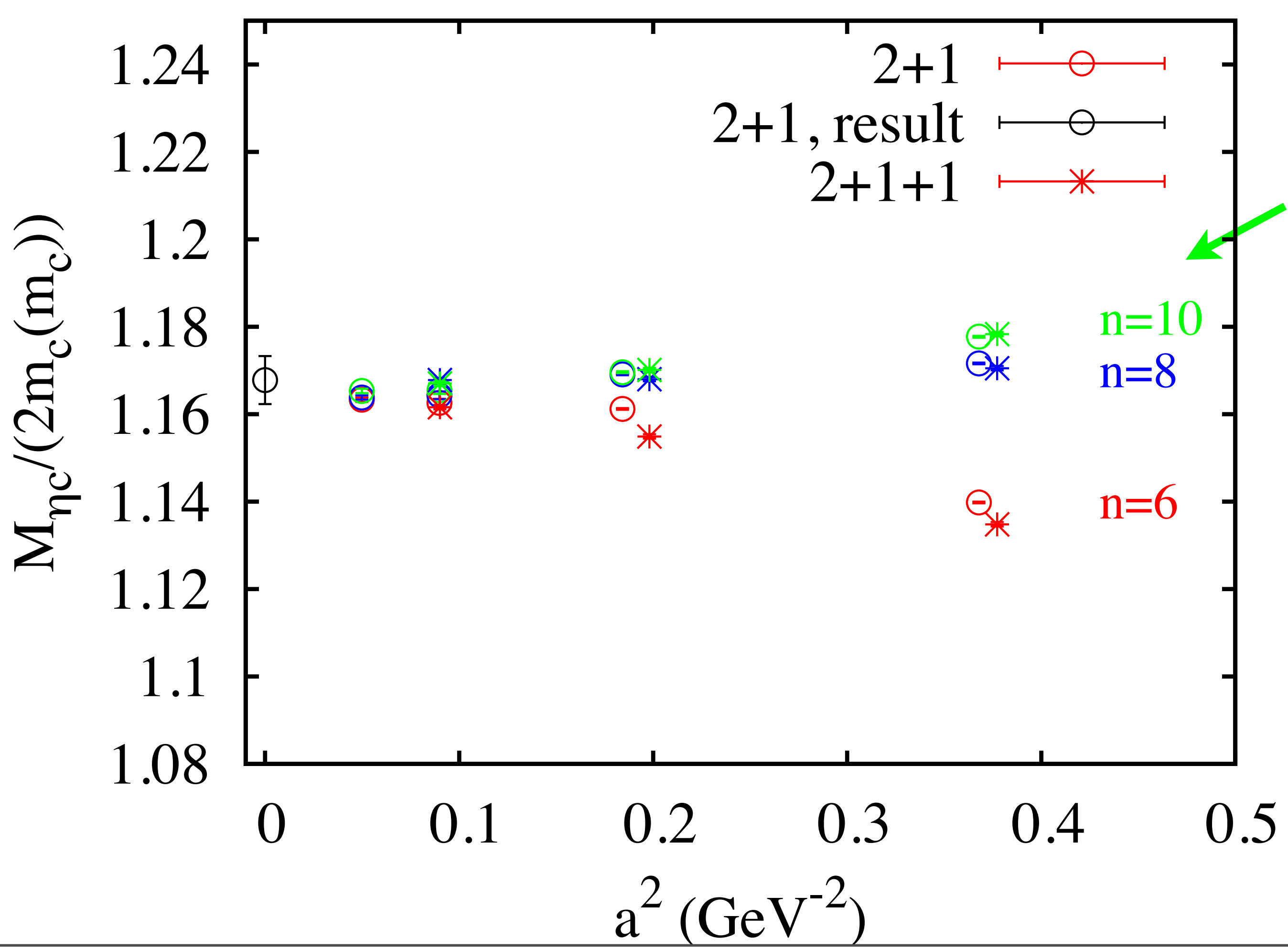
$$\frac{m_b}{m_c} = 4.49(4)$$

Agrees with that from current-current correlator method - test of pert. th.

Ongoing work

Existing lattice QCD results include u, d, s sea quarks with u/d quark masses heavier than their real values.

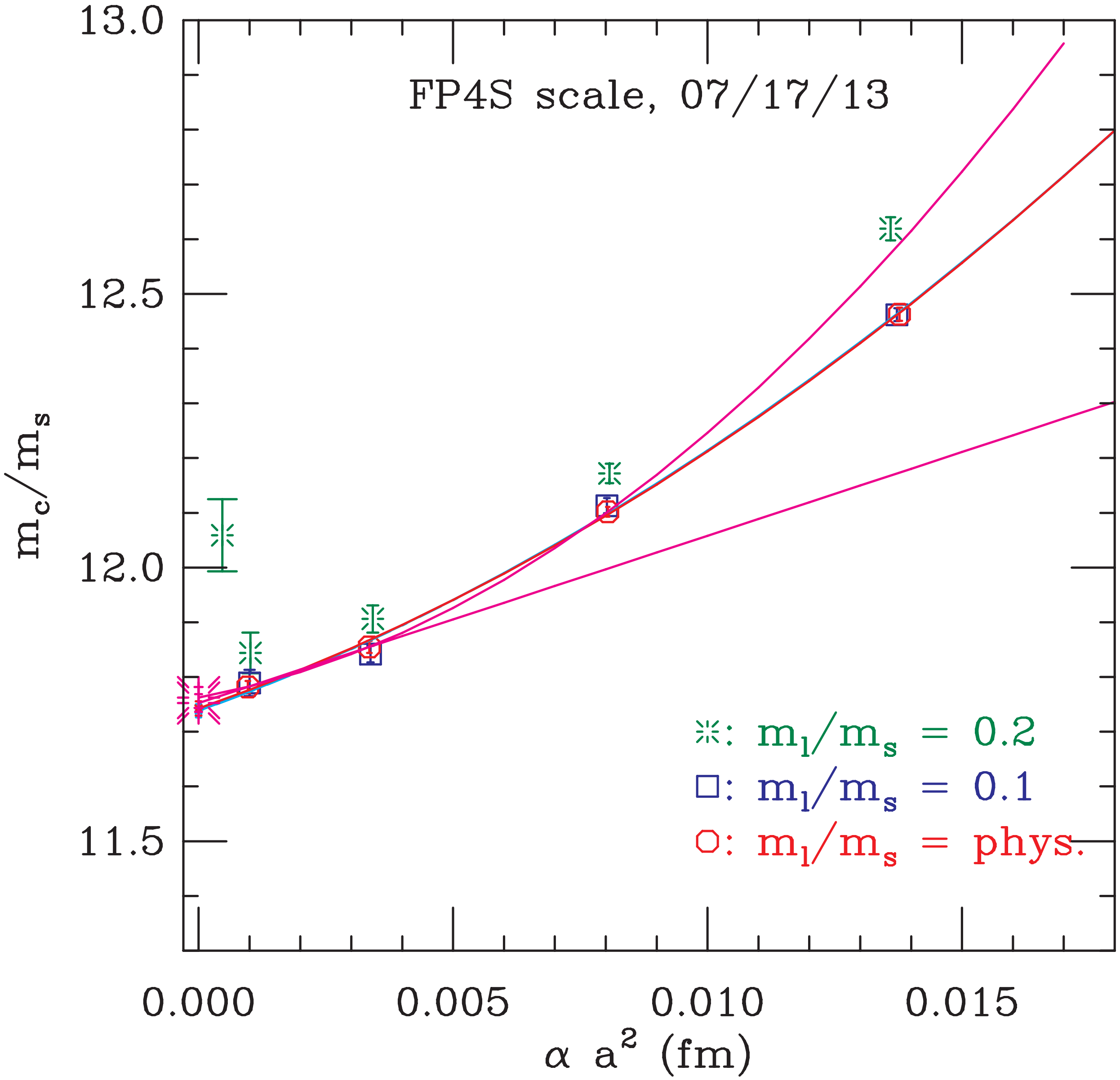
NOW have gluon configurations including 2+1+1 flavours of sea quarks and u/d quark masses at their physical values.



HPQCD preliminary results (HISQ quarks) show very little effect of c in sea (as expected)

ETMC also working on m_c with 2+1+1 quarks in sea.

Improved accuracy on ratio m_c/m_s on $n_f = 2+1+1$ configs with physical u/d quarks:

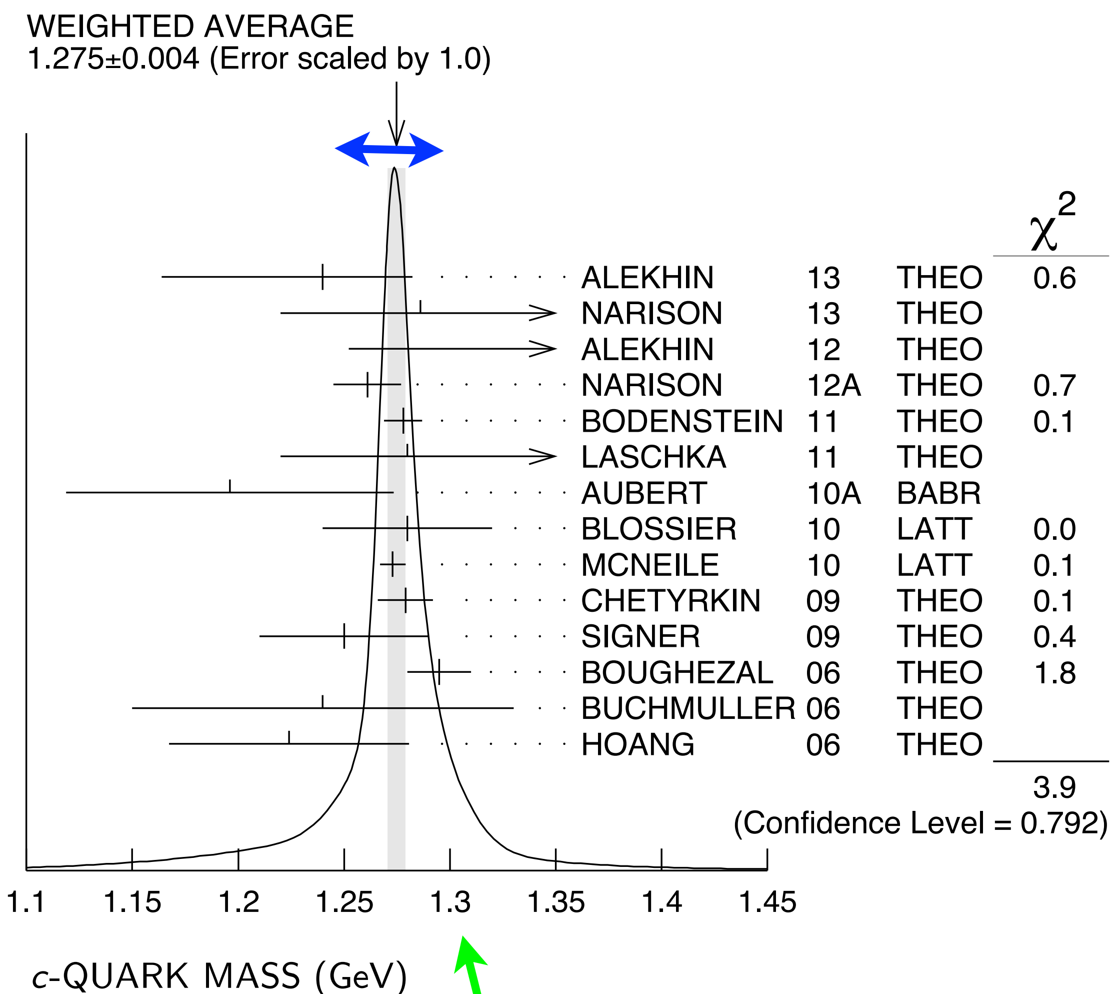


MILC/Fermilab result@LAT13

$$\frac{m_c}{m_s} = 11.75(6)$$

will allow improved m_s from improved m_c determination

PDG compilation of results



$m_c(m_c)$

Their evaluation:
1.275(25) GeV

good agreement between most precise lattice and non-lattice results

NB new result from joint H1+ZEUS charm prodn cross-section:
m_c=1.26(6) GeV
arXiv;1211.1182

Conclusions

$m_c(m_c)$ is determined to 1% and
$m_b(m_b)$ to 0.5% from continuum and lattice methods.

- Will be hard to improve m_c further directly.
- m_b can be improved from lattice QCD with finer lattices reducing/removing extrapolation to b.
- Then determine m_b/m_c ratio nonperturbatively to improve m_c
- Improved m_c will give improved m_s from 0.5% accurate m_c/m_s

New lattice QCD determinations in progress using a variety of formalisms and now with u, d, s and c quarks in sea and physical u/d quarks. Watch this space ...

NOTE: errors are ~a factor of 3 better than Higgs WG assume

Error budget for HISQ current-current method

1004.4285

	$m_c(3)$	$m_b(10)$	m_b/m_c	$\alpha_{\overline{\rm MS}}(M_Z)$
a^2 extrapolation	0.2%	0.6%	0.5%	0.2%
Perturbation theory	0.5	0.1	0.5	0.4
Statistical errors	0.1	0.3	0.3	0.2
m_h extrapolation	0.1	0.1	0.2	0.0
Errors in r_1	0.2	0.1	0.1	0.1
Errors in r_1/a	0.1	0.3	0.2	0.1
Errors in m_{η_c}, m_{η_b}	0.2	0.1	0.2	0.0
α_0 prior	0.1	0.1	0.1	0.1
Gluon condensate	0.0	0.0	0.0	0.2
Total	0.6%	0.7%	0.8%	0.6%