NRQCD Studies of the Υ Spectrum

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Nonrelativistic QCD

Nonrelativistic QCD (NRQCD) is useful for heavy quarks on the lattice, so the Υ spectrum (that is, $b\bar{b}$, or bottomonium, states) can be studied effectively. To this end I will talk about the pseudoscalar (η_b) and vector (Υ) states.

NRQCD uses an expansion of powers of v^2.

It is matched to full QCD and can subsequently be used wherever there is a b quark.

This talk is largely based on HPQCD paper, R. Dowdall et al., Phys. Rev. **D85**, 054509 (2012), 1110.6887.

Motivation

Heavy quarks can now be treated relativistically, but this is quite costly.

It is reasonable to consider heavy quarks (for the moment, I mean both c and b quarks) in a nonrelativistic formulation:

- $v_{\psi}^2 \sim 0.3$
- $v_{\Upsilon}^2 \sim 0.1$

If we understand how to treat b quarks on the lattice, we can use the same action in heavy-light mesons $\rightarrow$ input for CKM matrix elements.

NRQCD Action

The NRQCD Hamiltonian is:

$$
\begin{aligned}
aH &= aH_0 + a\delta H; \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}, \\
a\delta H &= -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8\left(am_b\right)^3} + c_2 \frac{i}{8\left(am_b\right)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) \\
&\quad -c_3 \frac{1}{8\left(am_b\right)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad -c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{\left(\Delta^{(2)}\right)^2}{16n\left(am_b\right)^2}.
\end{aligned}
$$

Time Evolution

Heavy quark propagator described by a time evolution equation:

$$G(\vec{x}, t+1) = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n U_t^\dagger(x) \times \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) G(\vec{x}, t)$$

with the starting condition,

$$G(\mathbf{x}, 0) = \phi(\mathbf{x})\mathbf{1}.$$

This is then just an initial value problem!

Improved $2+1+1$ Gluon Field Configurations

We use gluon fields from the MILC collaboration. Ensembles wih various values of the lattice spacing, a, and they all now include c quarks in the sea.

Set	β	am_l	am_s	am_c	$L/a \times T/a$
1	5.80	0.013	0.065	0.838	16×48
2	5.80	0.0064	0.064	0.0828	24×48
3	6.00	0.0102	0.0509	0.635	24×64
4	6.00	0.00507	0.0507	0.0628	32×64
5	6.30	0.0074	0.037	0.440	32×96

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I will (at some point) refer to sets 1 and 2 as 'very coarse' sets 3 and 4 as 'coarse' and set 5 as 'fine'.

NRQCD parameters

Various parameters used in the NRQCD action:

Set	am_b	u_{0L}	n_{cfg}
1	3.42	0.8195	1021
2	3.39	0.82015	1000
3	2.66	0.834	1053
4	2.62	0.8349	1000
5	1.91	0.8525	874

Coefficients improved through $\mathcal{O}(\alpha_s)$

For each of the coefficients in the NRQCD Hamiltonian, the tree level value is 1. We have used the $\mathcal{O}(\alpha_s)$ improved values for c_1, c_5 and c_6.

Set	c_1	c_5	c_6
very coarse	1.36	1.21	1.36
coarse	1.31	1.16	1.31
fine	1.21	1.12	1.21

Fitting

Meson 2-point functions:

$$C(t) = \langle \bar{\psi}(t,\vec{x})\Gamma\psi(t,\vec{x})(\bar{\psi}(0)\Gamma\psi(0))^{\dagger}\rangle$$

Bayesian fitting is used to extract energies from our correlator data. We use functions of the form:

$$G_{\mathrm{meson}}(n_{sc}, n_{sk}; t) = \sum_{k=1}^{n_{exp}} a(n_{sc}, k)a^{*}(n_{sk}, k)e^{-E_k t}$$

We can fit for various numbers of exponentials. The errors quickly equilibrate with only a few exponentials in the fit.

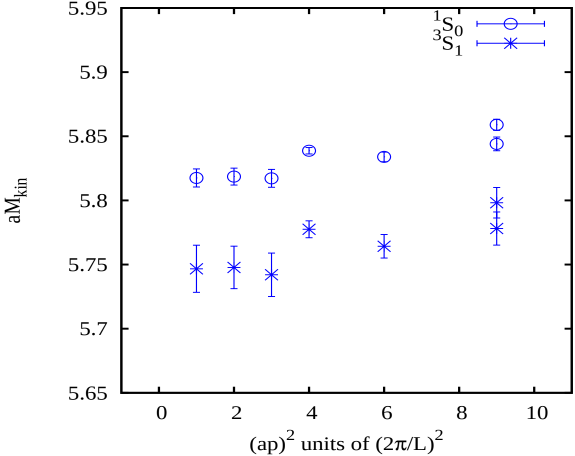
Kinetic Mass

- There is no mass term in the NRQCD action $\rightarrow$ ground state energy $\neq$ meson mass.
- To deal with that we obtain correlators with a finite momentum to calculate the *kinetic mass*,

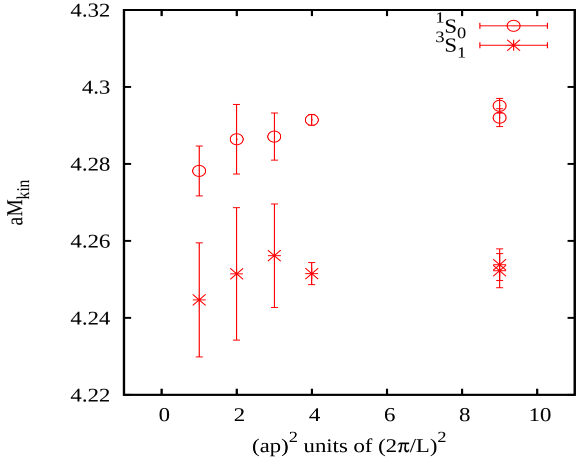
$$aM_{\text{Kin}} = \frac{a^2P^2 - (a\Delta E)^2}{2a\Delta E},$$

where $a\Delta E$ is the energy difference between the finite momentum and zero momentum meson.

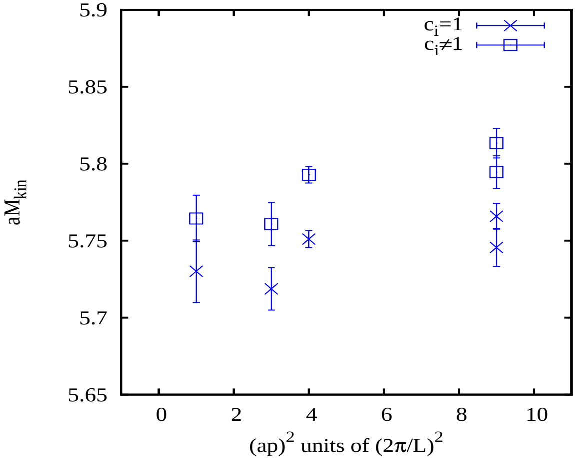
Kinetic Mass



Kinetic Mass



Spin Average

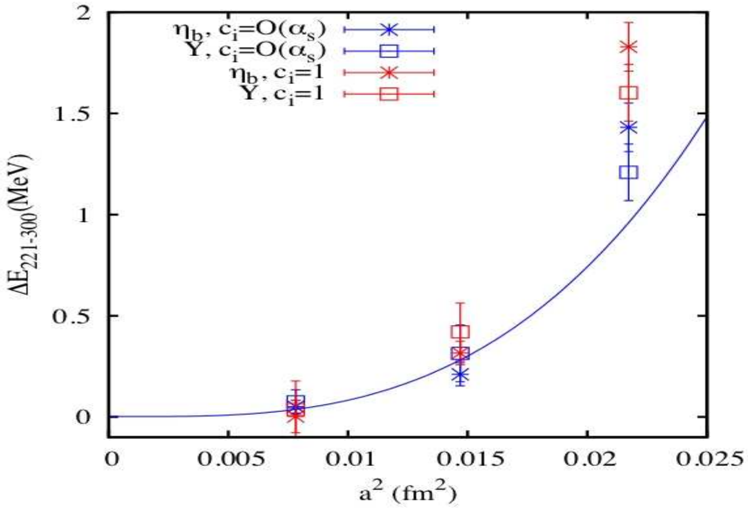


$P^2 = 9$ Splitting

There is a discretisation error in the form of rotational invariance and we can look at this effect.

- There are two different ways that we can get $P^2 = 9$: $(3, 0, 0)$ and $(2, 2, 1)$
- On-axis and off-axis momenta give slightly different values of the kinetic mass

$P^2 = 9$ Splitting



$\eta_b(2S)$ Prediction

Prediction of the $\eta_b(2S)$ state through the ratio of the hyperfine splitting:

$$R_H = \frac{M(\Upsilon') - M(\eta_b')}{M(\Upsilon) - M(\eta_b)}$$

This deals with effects from the c_4 term and any slight mistunings of m_b. Using the experimental average for the $1S$ hyperfine splitting we get a $2S$ hyperfine splitting of $35 \pm 3 \pm 1$ MeV. This leads to:

$$m_{\eta_b'} = 9988 \pm 3 \text{ MeV}$$

Evidence of η_b' state at Belle

[arXiv:1205.6351v1]

The Belle Collaboration have reported evidence of the $\eta_b(2S)$ state!

	Belle	HPQCD	Meinel[1]
$M(\Upsilon') - M(\eta_b')$	$24.5^{+4.0}_{-4.5}$ MeV	$35 \pm 3 \pm 1$ MeV	23.5 ± 4.7 MeV 28.0 ± 4.7 MeV
$m_{\eta_b'}$	$9999.0 \pm 3.5^{+2.8}_{-1.9}$ MeV	9988 ± 3 MeV	-

[1] Phys Rev D **82** 114502, 1002.3966

Matching for Momenta

- Missing relativistic information and this needs sorted to reduce errors.
- Match lattice temporal moments to continuum moments. Lattice moments calculated using,

$$G_n \equiv \sum_t (t/a)^n \, G(t),$$

where $G(t)$ is the correlator at lattice time t,

$$G(t) \equiv a^6 \sum_x (am_b)^2 \, \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle,$$

- Could use to extract the mass, m_b, or extract the value of the strong coupling, α_s.

Future Work & Stuff I Left Out!

(Or stuff that, actually, others worked on)

- Lattice spacing determined from our calculations. . .
  - $\Upsilon(2S) - \Upsilon(1S)$
  - η_s meson
- In addition to the work on S-wave states, some work has been done on P- and D-wave states.
- Heavy-light mesons

Summary

- Lattice NRQCD is useful for the precision calculations of systems involving a b quark
- The improved action and improved gluon field configurations used here reduces errors on previous calculations
- NRQCD could be used for the extraction of m_b and α_s
- The action can be used for other mesons that include a b quark $\rightarrow$ B physics and CKM matrix elements

Tuning of the Darwin Term