NRQCD and the Upsilon Spectrum

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Nonrelativistic QCD

NRQCD useful for heavy quarks on the lattice – the Upsilon spectrum ($b\bar{b}$ states, or bottomonium) can be studied effectively.

Understanding b quarks crucial to our understanding of CKM matrix elements.

NRQCD uses an expansion in powers of v^2 - the Υ has $v^2 \sim 0.1$

The mesons of interest in this talk are the pseudoscalar, η_b and the vector, Υ

NRQCD

We use NRQCD Hamiltonian of the form:

$$aH_0 = \frac{-\Delta^{(2)}}{2aM_b}$$

$$a\delta H = -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8\left(aM_b\right)^3} + c_2 \frac{ig}{8\left(aM_b\right)^2}\left(\nabla \cdot \tilde{E} - \tilde{E} \cdot \nabla\right)$$

$$-c_3 \frac{g}{8(aM_b)^2} \sigma \cdot (\tilde{\nabla} \times \tilde{E} - \tilde{E} \times \tilde{\nabla})$$

$$-c_4 \frac{g}{2aM_b} \sigma \cdot \tilde{B} + c_5 \frac{a^2 \Delta^{(4)}}{24aM_b} - c_6 \frac{a(\Delta^{(2)})^2}{16n(aM_b)^2}$$

Wilson Coefficients

The coefficients c_i in δH have their values fixed by matching NRQCD to full QCD

This can be done perturbatively, giving c_i the expansion:

$$c_i = 1 + c_i^{(1)} \alpha_s + \mathcal{O}(\alpha_s^2)$$

This has been carried out with tree level coefficients previously, and now with $\mathcal{O}(\alpha_s)$ coefficients.

Set	c_1	c_5	c_6
very coarse	1.36	1.21	1.36
coarse	1.31	1.16	1.31
fine	1.21	1.12	1.21

Time Evolution

Time evolution of the propagator:

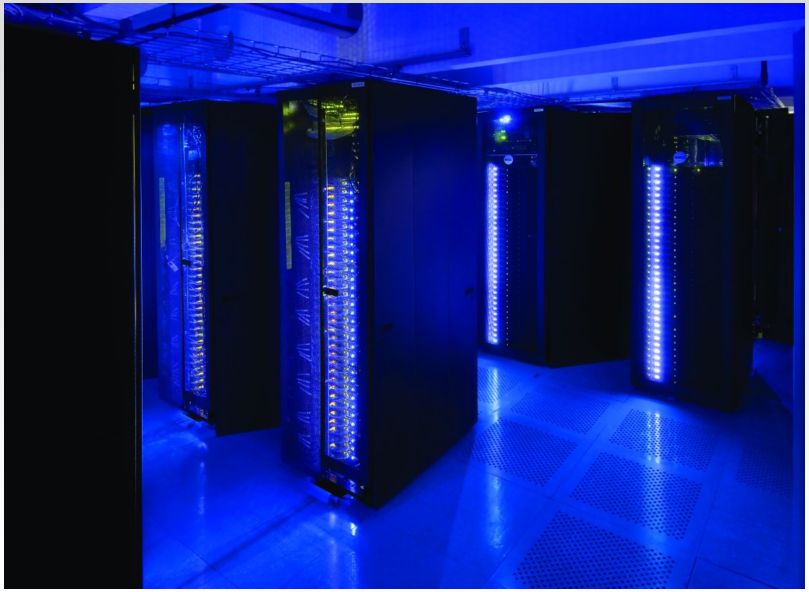
$$G(\vec{x},t+1)=\left(1-\frac{a\delta H}{2}\right)\left(1-\frac{aH_0}{2n}\right)^n U_t^\dagger(x)$$

$$\times\left(1-\frac{aH_0}{2n}\right)^n\left(1-\frac{a\delta H}{2}\right)G(\vec{x},t)$$

Initial value problem!

Starting condition: $G(\mathbf{x},0)=\phi(\mathbf{x})\mathbf{1}$

Darwin



Work performed on Cambridge High Performance Computing Cluster, *Darwin*.

Each set of lattices has in the region of 1000 configurations.

Run several correlators per configuration and averaged.

Gluon Configurations

MILC collaboration created gluon configurations

2+1+1 flavours quarks in the sea: up and down, strange and charm

Up and down quarks given same mass on the lattice – heavier than they are in the real world.

Strange and charm quarks tuned close to their real world values.

Gluon Configurations

Some details about the ensembles that we used:

Set	β	am$_l$	am$_s$	am$_c$	L/a $\times T/a$
1	5.80	0.013	0.065	0.838	16 $\times$48
2	5.80	0.0064	0.064	0.828	24 $\times$48
3	6.00	0.0102	0.0509	0.635	24 $\times$64
4	6.00	0.00507	0.0507	0.628	32 $\times$64
5	6.30	0.0074	0.037	0.440	32 $\times$96

Other Lattice Details

Tadpole improvement: QCD in the real world and QCD on the lattice are different. On the lattice we have divided all the gluon fields by the mean trace of the gluon field in Landau gauge, u_{0L}

Fixing the spacing: Not everything is a prediction. We use the difference in the ground state and first excited state energy of the upsilon to fix the lattice spacing.

NRQCD Parameters

Parameters used in the NRQCD action:

Set	am_b	u_{0L}	n_{cfg}	n_t	T_p
1	3.42	0.8195	1021	16	40
2	3.39	0.82015	1000	16	40
3	2.66	0.834	1053	16	40
4	2.62	0.8349	1000	16	40
5	1.91	0.8525	874	16	48

Fitting

Meson 2-point functions:

$$C(t) = \sum_{\vec{x}} \langle \bar{\psi}(t,\vec{x}) \Gamma \psi(t,\vec{x}) (\bar{\psi}(0)\Gamma\psi(0))^\dagger \rangle$$

Bayesian fit to the function:

$$\boxed{C(t) = \sum_{n=1}^{n_{\mathrm{exp}}} A_n \exp(-E_n t)}$$

The fits can be performed with different numbers of exponentials – the errors equilibrate with only a few exponentials in the fit.

Kinetic Mass

We can look at how things change with meson momentum. Zero momentum energy does not correspond to a mass – there is an energy offset.

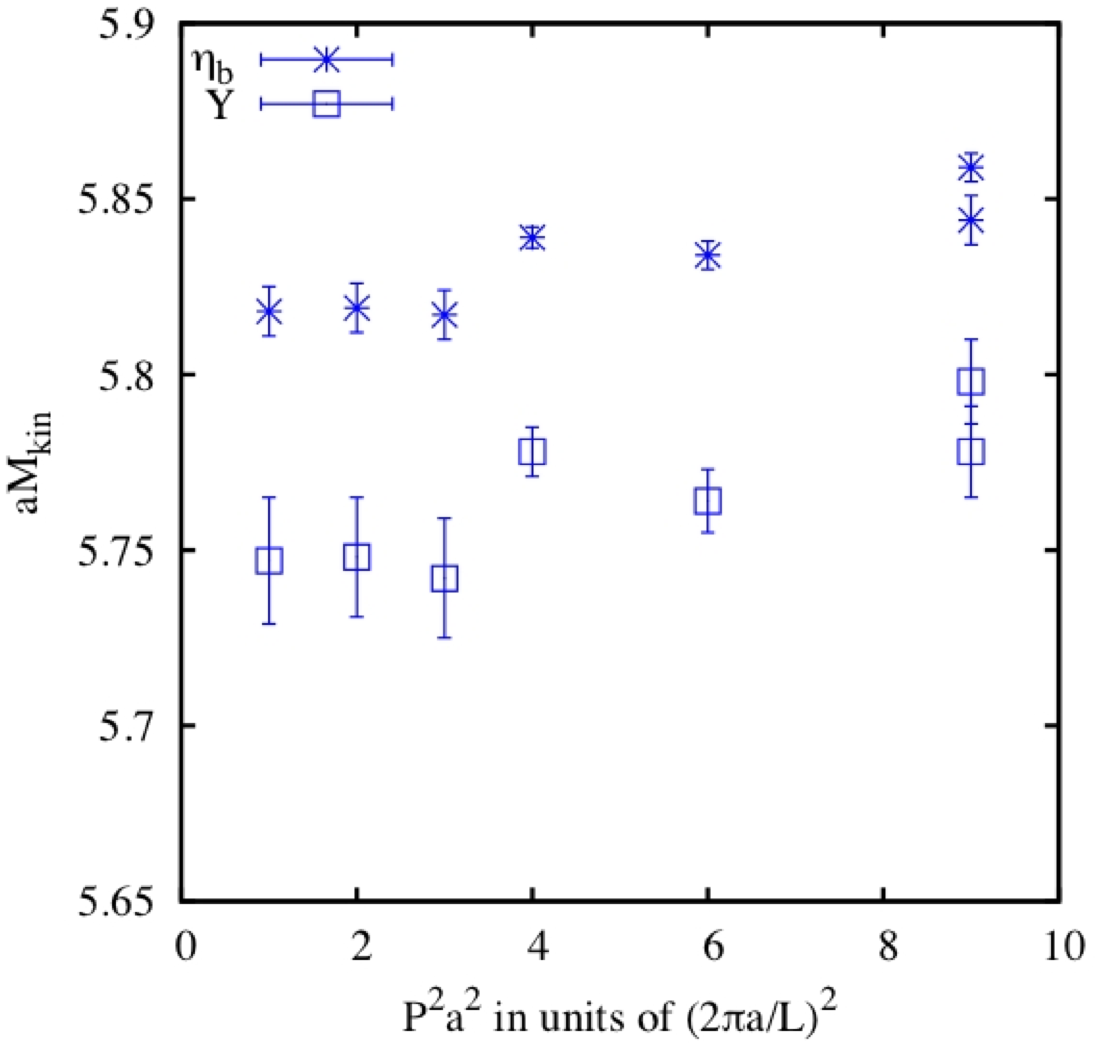
We can, however, consider the kinetic mass given by:

$$\boxed{M_{\text{kin}} = \frac{p^2a^2 - (\Delta E a)^2}{2\Delta E a}}$$

where

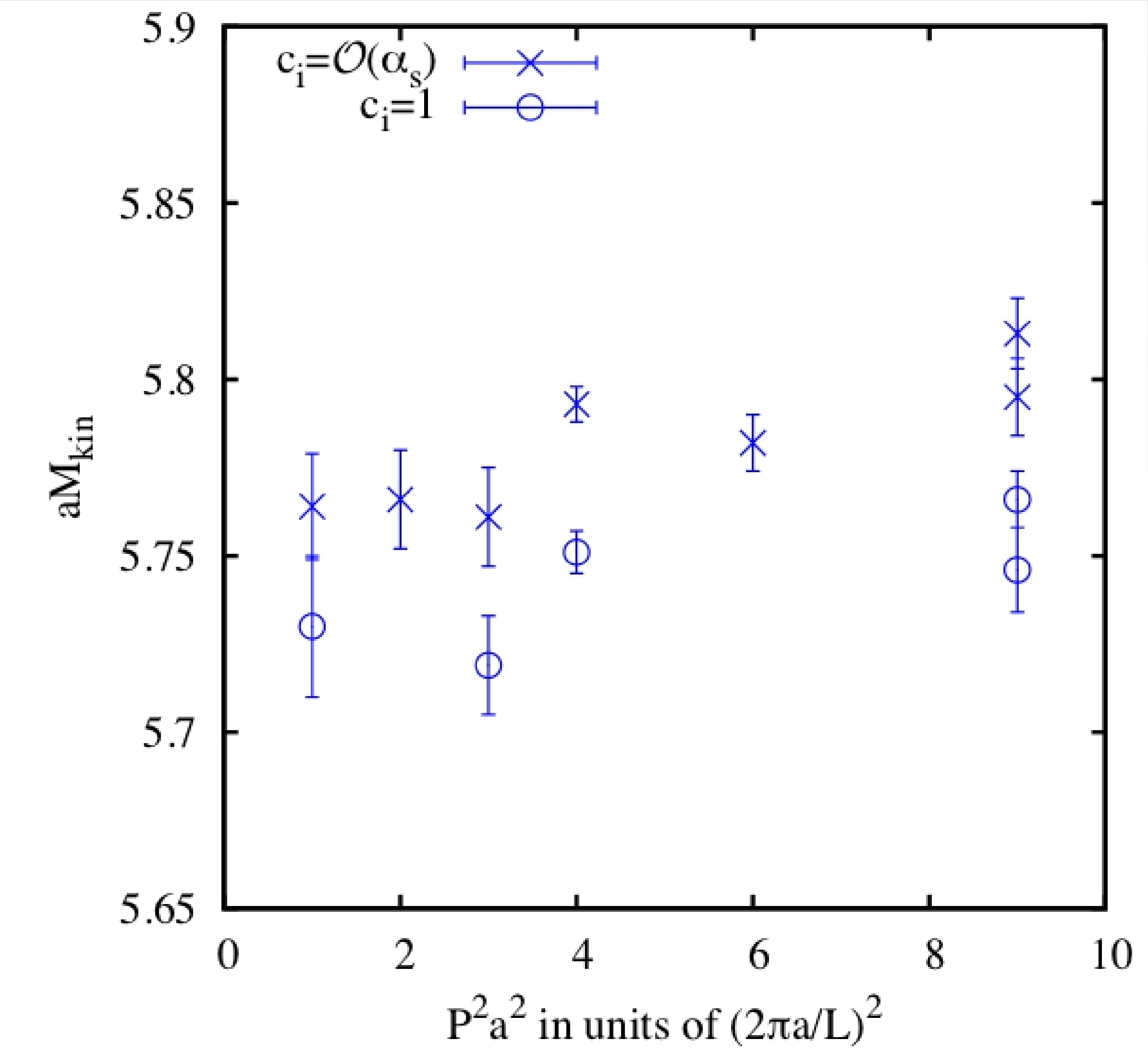
$$\Delta E = E(p) - E(0)$$

Some Results



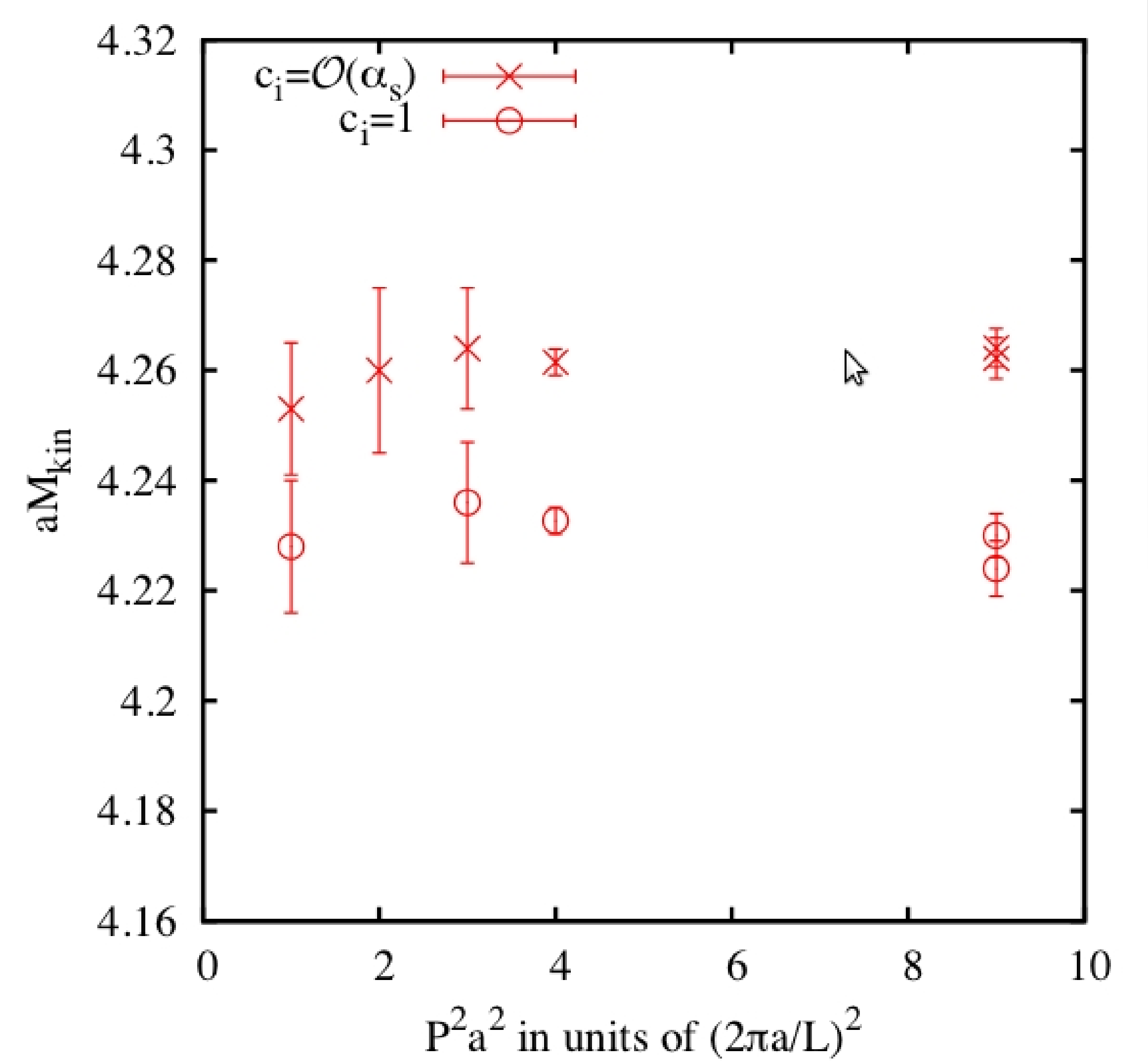
The η_b and Υ kinetic masses on the coarse ensemble in lattice units

Some Results



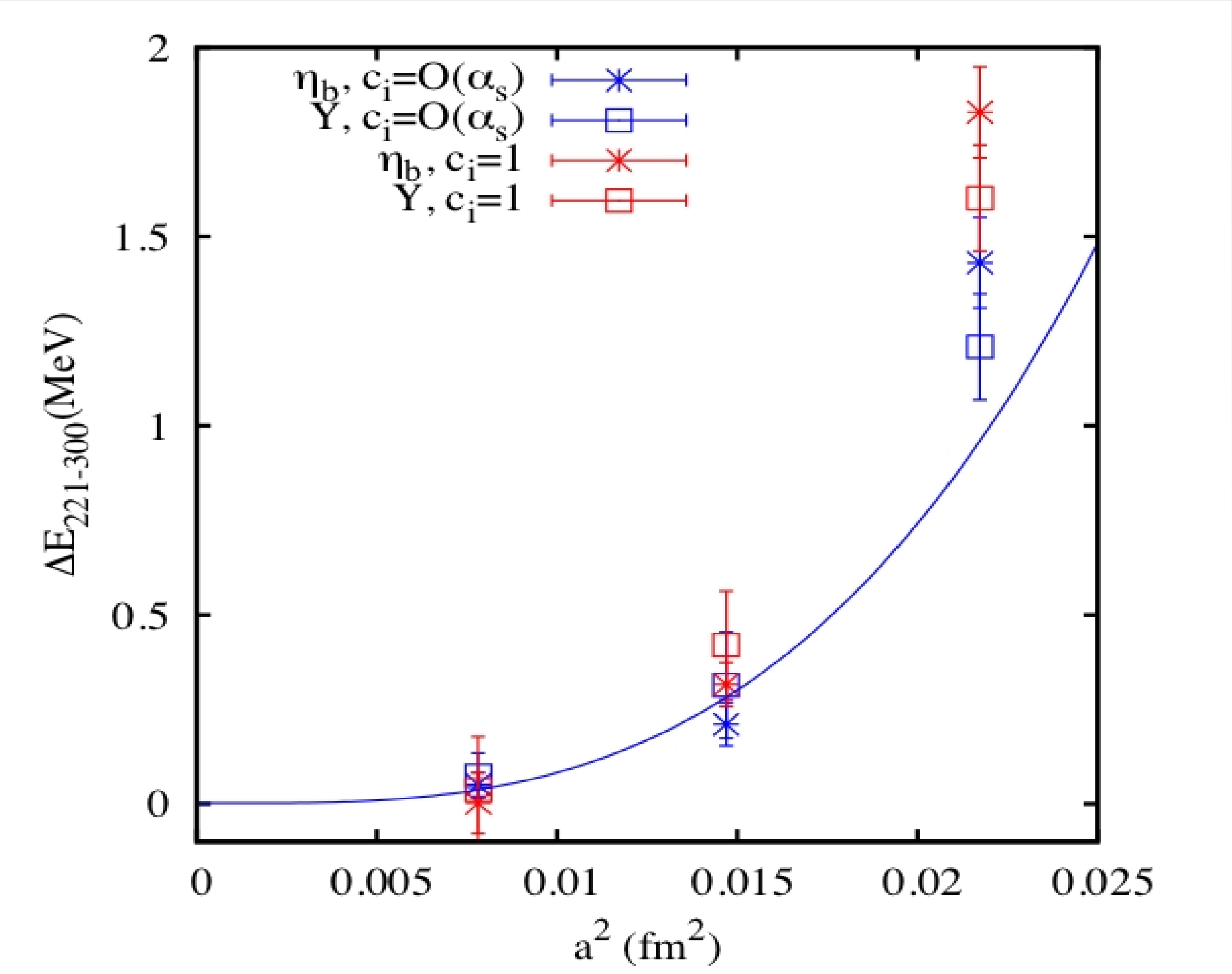
Spin averaged results with tree level and α_s improved coefficients on coarse lattice

Some Results



Spin averaged results with tree level and α_s improved coefficients on fine lattice

Some Results



The splitting between energies for P=(2,2,1) and P=(3,0,0) plotted against lattice spacing

And there's more...

This is just the tip of the iceberg – as well as the s-wave states discussed here, much work has been done on the p- and d-wave states.

The smearing of the quarks is also important. The NRQCD code is run with various smearings at the source and the sink. A simultaneous matrix fit is carried out. This improves the results.

The Future

More of the same: determine where the biggest errors are coming from and take them into consideration.

Currently looking at corrections from leptonic width.

Heavy-light correlators: using the NRQCD action for the heavy quarks and a different action for light quarks in a meson – light quarks are more costly on the lattice.

Thank you!