The Υ Spectrum & Semileptonic Decays with NRQCD b Quarks

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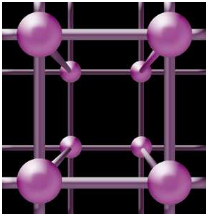
QCD on the Lattice

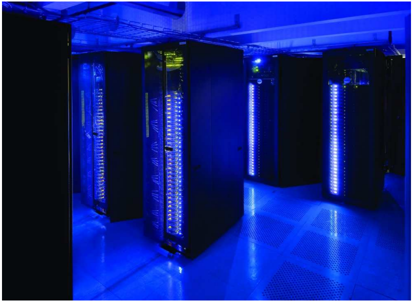
- Space-time lattice with lattice spacing a. Quarks live on the lattice sites. Gluon exist on the links between.
- Put valence quarks on a set of gluon field configurations. We use ~ 1000 of these background snapshots per set.
- Different lattice spacings; finer spacings closer to real world.

- Free parameters: quarks masses, coupling constant; tune these, use the results elsewhere.



Big Machines

- Lattice QCD calculations are computationally expensive.
- Supercomputers are utilized to carry out these calculations.
- Modern computing power allows for effects from sea quarks and for finer lattices.



The Darwin Cluster

bs on a Lattice

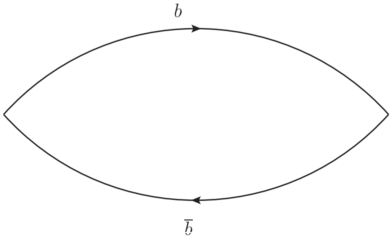
- Nonrelativistic QCD (NRQCD) is useful for heavy quarks on the lattice, so any b quarks bound inside a meson can be simulated with NRQCD.
- It's feasible to consider b quarks as nonrelativistic: $v^2 \approx 0.1$ for Υ.
- NRQCD uses an expansion of powers of v^2.
- No doubling problem!
- It is matched to full QCD and can subsequently be used wherever there is a b quark.

NRQCD Hamiltonian

The NRQCD Hamiltonian I use here ($\mathcal{O}(v^4)$) is:

$$
\begin{aligned}
aH &= aH_0 + a\delta H; \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}, \\
a\delta H &= -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8\left(am_b\right)^3} + c_2 \frac{i}{8\left(am_b\right)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) \\
&\quad - c_3 \frac{1}{8\left(am_b\right)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad - c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{\left(\Delta^{(2)}\right)^2}{16n\left(am_b\right)^2}.
\end{aligned}
$$

Evolution equation



$$G(\vec{x}, t+1) = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n U_t^\dagger(x) \times \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) G(\vec{x}, t)$$

Coefficients

Coefficients $c_i = 1 + c_i^{(1)}\alpha_s + \mathcal{O}(\alpha_s^2)$ fixed to match NRQCD and full QCD.

Set	c_1	c_4	c_5	c_6
very coarse	1.36	(1.22)	1.21	1.36
coarse	1.31	(1.20)	1.16	1.31
fine	1.21	(1.16)	1.12	1.21

Gluon Field Configurations

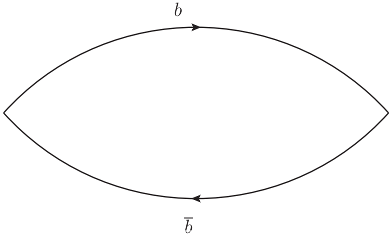
The work here uses improved gluon field configurations with $2+1+1$ flavours of quarks in the sea. The latest calculations are on ensembles with *physical* light quark masses.

Set	β	am_l	am_s	am_c	$L/a \times T/a$
1	5.80	0.013	0.065	0.838	16×48
2	5.80	0.0064	0.064	0.828	24×48
3	5.80	0.00235	0.0647	0.831	32×48
4	6.00	0.0102	0.0509	0.635	24×64
5	6.00	0.00507	0.0507	0.628	32×64
6	6.00	0.00184	0.0507	0.628	48×64
7	6.30	0.0074	0.037	0.440	32×96
8	6.30	0.0012	0.0363	0.432	64×96

Υ and η_b [1110.6887]

We want to apply NRQCD to b quarks, so

- Both η_b and Υ are $b\bar{b}$ states
- η_b is a pseudoscalar meson, Υ a vector; just insert operators
- Experimentally well understood: particularly Υ
- I'll consider here the ground states at various momenta



Fitting

We run several time sources per configuration. For this: 4.

We use a Bayesian fitting approach to fit two-point functions to,

$$C(t) = \sum_{n}^{\text{nexp}} a^2 e^{-E_n t}$$

We have loose priors set for the energies, the energy differences and the amplitudes. All the momenta were fit simultaneously.

Kinetic Mass

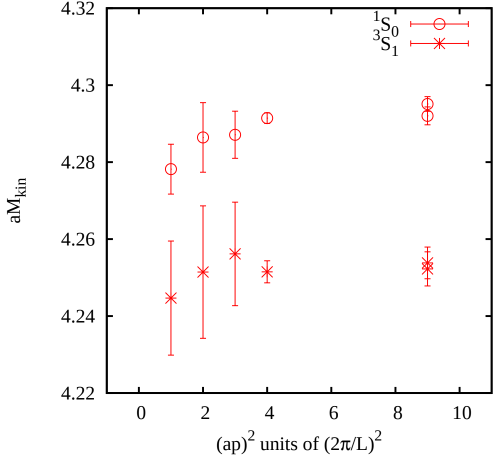
- mass term explicitly removed
    - ground state energy $\neq$ ground state mass
    - energy differences *do* correspond to mass differences

We can use *kinetic mass*:

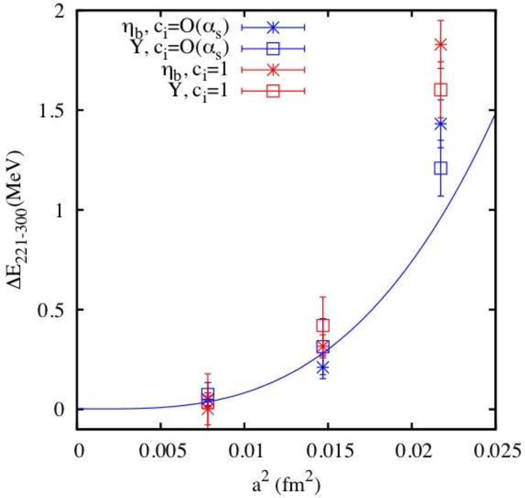
$$M_{\text{kin}} = \frac{a^2 P^2 - (a\Delta E)^2}{2a\Delta E}$$

In principle, this should give results that are stable over a range of momenta. Let's see...

Kinetic Mass Results



Lattice Artifacts



Matrix Elements

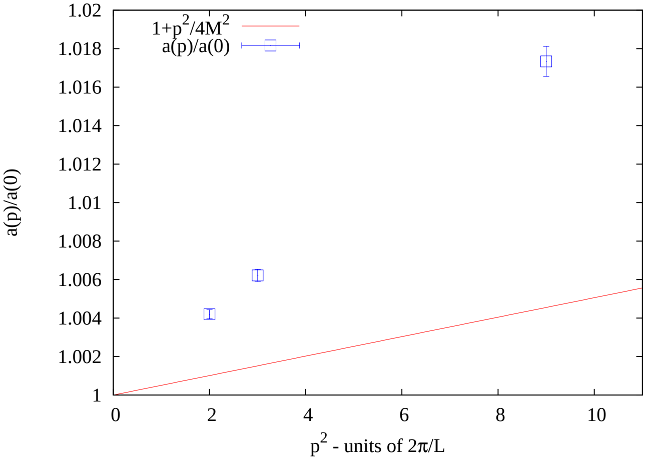
Correlators are of form $\sum_n |\langle 0|\Gamma|n\rangle|^2 e^{-E_n t}$. So amplitudes of fit correspond to matrix element.

We want to ensure the correct behaviour of the amplitudes, so this is an area for correction of the currents. We take our improved currents to be,

$$\mathbf{J}_i = \sigma \left(\frac{\Delta^2}{M^2} \right)^i$$

We need to determine the correct coefficients to match them to full QCD as usual.

Amplitude Corrections



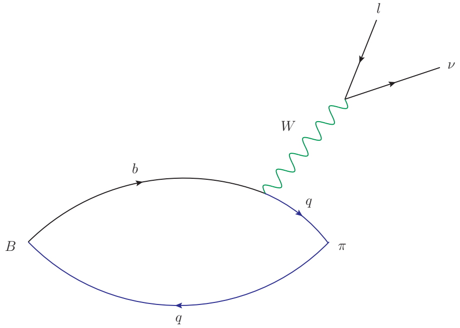
Stuff I didn't do. . . but someone else did

- Excited states for Υ and η_b
- Lattice space determination
  - $\Upsilon(2S)$-$\Upsilon(1S)$
  - From η_s
- Determination of P and D waves.
- Prediction of $\eta_b(2S)$ states.
  - Evidence for this at both Belle and CLEO.

Semileptonic Decays

- Semileptonic decays are flavour changing processes where a W boson is emitted.
- Possible to study this on the lattice.
- When dealing with lattice QCD, we're only seeing the strong force stuff, so when the W Boson leaves we know nothing more about it.
- But this is still useful, and we can get plenty of information about what's going on.

B Decays



$$\frac{\langle 0|P|\pi\rangle\langle\pi|V|B\rangle\langle B|A_0|0\rangle}{2E_\pi 2E_B} e^{-E_\pi t} e^{-E_B(T_B - t)}$$

- The b quark can be studied with NRQCD. Light quarks HISQ (a relativistic formulation)
- Decay constants & form factors from semileptonic B decays
- Get matrix element from lattice QCD → CKM matrix elements from lattice QCD and experiment.

Semileptonic Form Factors

Matrix element relevant to this decay can be parametrised by form factors $f_+(q^2)$ and $f_0(q^2)$:

$$\langle B|V^0|\pi\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}\right) + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

But here I am only considering the case where the B and π 3-momenta are 0, so the W Boson would have maximum momentum, $q_{\max}$.

Currents

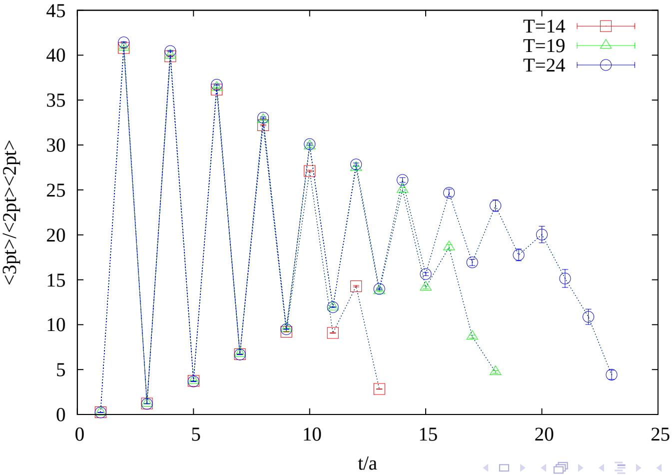
We use the following:

$$
\begin{aligned}
J_0^{(0)}(x) &= \bar{q}(x)\Gamma_0 Q(x) \\
J_0^{(1)}(x) &= \bar{q}(x)\Gamma_0 \gamma\cdot\nabla Q(x) \\
J_0^{(2)}(x) &= \bar{q}(x)\gamma\cdot\overleftarrow{\nabla}\gamma_0\Gamma_0 Q(x)
\end{aligned}
$$

Matched via:

$$
\begin{aligned}
\langle V_0\rangle &= (1+\alpha_s\rho_0^{(0)})\langle J_0^{(0)}\rangle + \\
&\quad (1+\alpha_s\rho_0^{(1)})\langle J_0^{(1),sub}\rangle + \alpha_s\rho_0^{(2)}\langle J_0^{(2)}\rangle
\end{aligned}
$$

3pt, 2pt Correlator Ratios



Fitting

- Fitting very similar to 2-point fits, but fit 2-points for B and π with 3-point simultaneous
  - The 2-point correlators were generated seperately by Rachel Dowdall
- Difference here is addition of quark smearing (on the b quarks)
  - This basically gives ground state quicker
  - All smearings are fit simultaneously, too

Fit function

$$C_{3\mathrm{pt}}(t) = \sum_{i,j}^{\mathrm{nexp}} a_i b_{j,\mathrm{sm}} V_{00} e^{-E_\pi^{(i)} t} e^{-E_B^{(j)} t}$$

f_0 Form Factor

At $q^2_{\max}$, only left with f_0.

$$\langle B|V^0|\pi\rangle = f_0(q^2_{\max})(m_B^2 + m_\pi^2)$$

Can get this directly from the fit through:

$$f_0(q^2_{\max}) = 4\sqrt{2}V_{00}\frac{\sqrt{m_B m_\pi}}{m_B + m_\pi}$$

Soft Pion Theorem

The soft pion theorem relates the decay constants of the B and π to $f_0(q^2_{\max})$

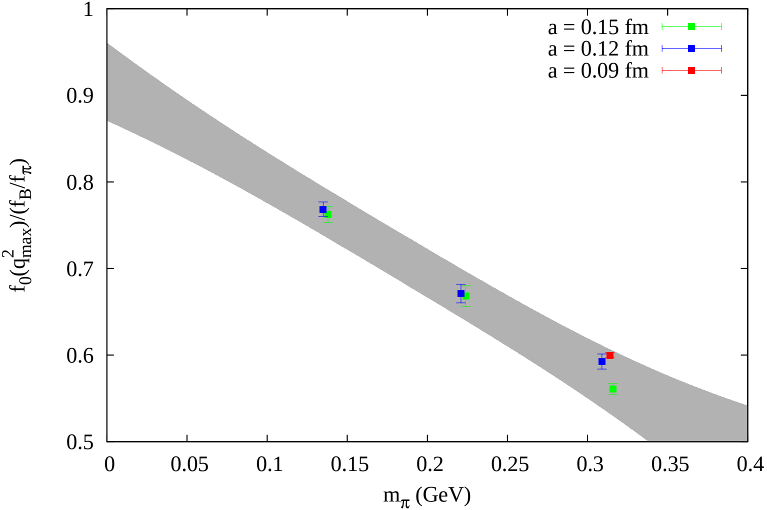
$$f_\pi = 2m_l\sqrt{\frac{2}{E_\pi^3}}a_0$$

$$f_B\sqrt{m_B} = 2b_0$$

In the limit, $m_\pi \to 0$, the soft pion theorem says

$$f_0(q^2_{\max}) = \frac{f_B}{f_\pi}$$

Soft Pion Theorem



Relativistic Heavy Quarks

It is perfectly possible to study heavy quarks relativistically, but it's different:

- Errors are unreasonable at large quark mass, still
- The c quark can now be treated relativistically
- Can get correlators for a range of quark masses, $m_c < m_q < m_b$
- Do this in a range of ensembles, just like usual
- Extrapolate

A relativistic treatment allows direct comparison between methods.

CKM Matrix Elements

At non-zero recoil, i.e. $q^2 < q^2_{\max}$, we can get both $f_0(q^2)$ and $f_+(q^2)$. $(f_0(0) = f_+(0) = 1)$

This allows access to the following:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} \left[\left(M_B^2 + M_\pi - q^2 \right)^2 - 4 M_B^2 M_\pi^2 \right]^{3/2} \left| f_+(q^2) \right|^2 .$$

So in combination with experimental results and known factors, can extract $|V_{ub}|$

Other Semileptonic Decays

We can consider other decays (actually, I am doing):

- $B_s \to K\ell\nu$, different spectator
- $B_s \to \eta_s\ell\nu$, different spectator and active
    - The η_s doesn't exist in the real world
    - On the lattice, we can make it exist and study it

NRQCD Improvement

There are two ways this can be done:

- Do further corrections to coefficients c_i
  - Already have c_4 coefficient to $\mathcal{O}(\alpha_s)$ now
  - Darwin term, c_2, has been improved for use, too
- Extra terms in Hamiltonian
  - Work here is at $\mathcal{O}(v^4)$. Can go to $\mathcal{O}(v^6)$

Summary

- Lattice QCD essential for nonperturbative calculations of the strong force
- Lattice NRQCD is useful for the precision calculations of systems involving a b quark
- Gives good results for bottomonium states
- Same b quarks and ensembles used for other calculations
- Can use it to extract $f_0(q^2_{\mathrm{max}})$
- Soft pion theorem holds
- CKM matrix elements can be determined
- NRQCD can be extended for use on these things plus more