Bottomonium and B Physics with Lattice NRQCD b Quarks

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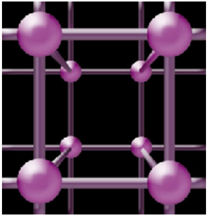
QCD on the Lattice

- Space-time lattice with lattice spacing a. Quarks live on the lattice sites. Gluon exist on the links between.
- Put valence quarks on a set of gluon field configurations. We use ~ 1000 of these background snapshots per set.
- Different lattice spacings; finer spacings closer to real world.

- Free parameters: quarks masses, coupling constant; tune these, use the results elsewhere.



Big Machines

- Lattice QCD calculations are computationally expensive.
- Supercomputers are utilized to carry out these calculations.
- Computing power allows calculations including light (u/d) quarks down to physical mass

The Darwin Cluster

Problem:

Errors grow with mass of heavy quark (in lattice units)

2 Possible Solutions:

1) Various quark masses between m_c and as close to m_b as possible $\rightarrow$ extrapolate to m_b.

2) Use formalism that *somehow* allows physical b quark mass to be used.

bs on a Lattice

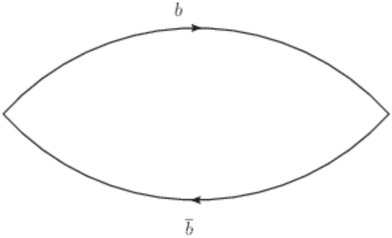
- Nonrelativistic QCD (NRQCD) is useful for heavy quarks on the lattice, so any b quarks bound inside a meson can be simulated with NRQCD.
- It's feasible to consider b quarks as nonrelativistic: $v^2 \approx 0.1$ for Υ.
- NRQCD uses an expansion of powers of v^2 to desired order.
- Match to full QCD and can subsequently be used wherever there is a b quark.

NRQCD Hamiltonian

The NRQCD Hamiltonian I use here ($\mathcal{O}(v^4)$) is:

$$
\begin{aligned}
aH &= aH_0 + a\delta H; \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}, \\
a\delta H &= -c_1 \frac{\left(\Delta^{(2)}\right)^2}{8\left(am_b\right)^3} + c_2 \frac{i}{8\left(am_b\right)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) \\
&\quad - c_3 \frac{1}{8\left(am_b\right)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad - c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{\left(\Delta^{(2)}\right)^2}{16n\left(am_b\right)^2}.
\end{aligned}
$$

Evolution equation



$$G(\vec{x}, t+1) = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n U_t^\dagger(x) \times \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2}\right) G(\vec{x}, t)$$

Gluon Field Configurations

The work here uses improved gluon field configurations with $2+1+1$ flavours of quarks in the sea. The latest calculations are on ensembles with *physical* light quark masses.

Set	β	am_l	am_s	am_c	$L/a \times T/a$
1	5.80	0.013	0.065	0.838	16×48
2	5.80	0.0064	0.064	0.828	24×48
3	5.80	0.00235	0.0647	0.831	32×48
4	6.00	0.0102	0.0509	0.635	24×64
5	6.00	0.00507	0.0507	0.628	32×64
6	6.00	0.00184	0.0507	0.628	48×64
7	6.30	0.0074	0.037	0.440	32×96
8	6.30	0.0012	0.0363	0.432	64×96

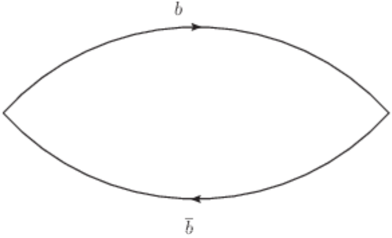
b Quark Parameters

Set	am_b	u_{0L}	c_1	c_4	c_5	c_6
1	3.297	0.8195	1.36	1.22	1.21	1.36
2	3.263	0.82015	1.36	1.22	1.21	1.36
3	3.25	0.819467	1.36	1.22	1.21	1.36
4	2.66	0.834	1.31	1.20	1.16	1.31
5	2.62	0.8349	1.31	1.20	1.16	1.31
6	2.62	0.834083	1.31	1.20	1.16	1.31
7	1.91	0.8525	1.21	1.16	1.12	1.21
8	1.89	0.851805	1.21	1.16	1.12	1.21

Bottomonium Physics [1110.6887],[1408.5768]

We want to apply NRQCD to b quarks

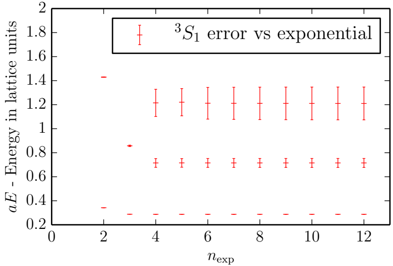
- Both η_b and Υ are bottomonium mesons: $b\bar{b}$ states
- Experimentally well understood: particularly Υ

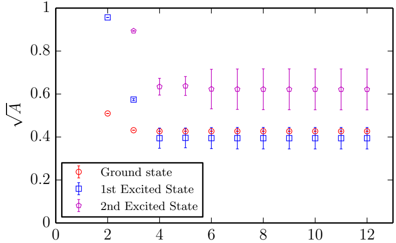


Fitting

We use a Bayesian fitting approach to simultaneously fit η_b and Υ two-point correlators to,

$$C(t) = \sum_{n=0}^{n_{\text{exp}}-1} A e^{-E_n t}$$
$$= \sum_{n=0}^{n_{\text{exp}}-1} c(\phi_{\text{sc}}, n) c^*(\phi_{\text{sk}}, n) e^{-E_n t}$$





Kinetic Mass

- mass term explicitly removed
  - ground state energy $\neq$ ground state mass
  - energy differences *do* correspond to mass differences

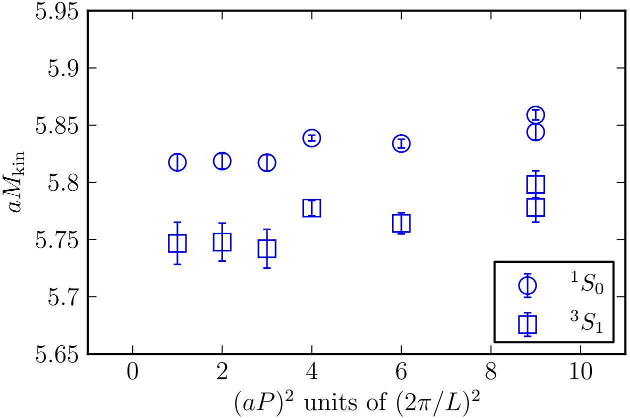
We can use *kinetic mass*.

$$aE(P) = \sqrt{a^2P^2 + a^2M_{\rm kin}}$$

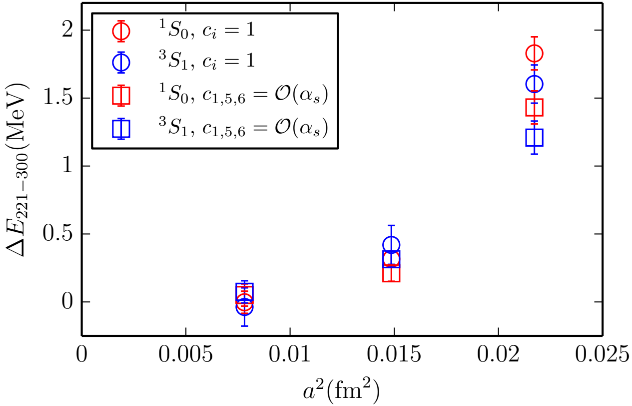
$$\Downarrow$$

$$M_{\rm kin} = \frac{a^2P^2 - (a\Delta E)^2}{2a\Delta E}$$

Kinetic Mass Results



Lattice Artifacts



Υ Decay Constant & Leptonic Width

Leptonic Width

$$\Gamma(\Upsilon^{(n)} \to e^+e^-) = 16\pi\alpha_{\text{em}}^2 e_b^2 \frac{|\langle 0|J_{V,\text{NRQCD}}|\Upsilon^{(0)}\rangle|^2}{M_{\Upsilon^{(n)}}^2} Z_V^2$$

Decay constant:

$$\langle 0|J_V|\Upsilon\rangle = f_{\Upsilon^{(n)}} M_{\Upsilon^{(n)}}$$

$$\Gamma(\Upsilon^{(n)} \to e^+e^-) = \frac{4\pi}{3}\alpha_{\text{em}}^2 e_b^2 \frac{f_{\Upsilon^{(n)}}^2}{M_{\Upsilon^{(n)}}}$$

Vector Currents

$$\mathbf{J_i} = \sigma \left(\frac{\Delta^2}{m_b^2} \right)^i$$

$$\mathbf{J_0} = \sum_{x;i=1}^{3} \chi_x^\dagger \sigma \Psi_x$$

$$\mathbf{J_1} = \sum_{x;i=1}^{3} \chi_x^\dagger \frac{\sigma}{(am_b)^2} \times \left(\Psi_{x+\hat{\imath}} + \Psi_{x-\hat{\imath}} - 2\Psi_x \right),$$

$$\langle 0 | \mathbf{J}^{\mathrm{QCD}} | \bar{Q}Q \rangle = \sum_i k_i \langle 0 | \mathbf{J}_i | \bar{Q}Q \rangle, \qquad k_i = \sum_n \alpha_s^n k_i^{(n)}$$

Temporal Moments & Z_V Matching Factor

$$\begin{aligned} J_V &= Z_V \, J_{V,\mathrm{NRQCD}} \\ &\equiv Z_V \left(J^{(0)}_{V,\mathrm{NRQCD}} + k_1 J^{(1)}_{V,\mathrm{NRQCD}} \right), \end{aligned}$$

$$G_n^{V,\mathrm{NRQCD}} = 2 \sum_t (t/a)^n \, C_{V,\mathrm{NRQCD}}(t) e^{(\overline{E_0} - \overline{M}_{\mathrm{kin}})t}.$$

for $n = 4, 6, 8, \ldots$

Matching to continuum QCD:

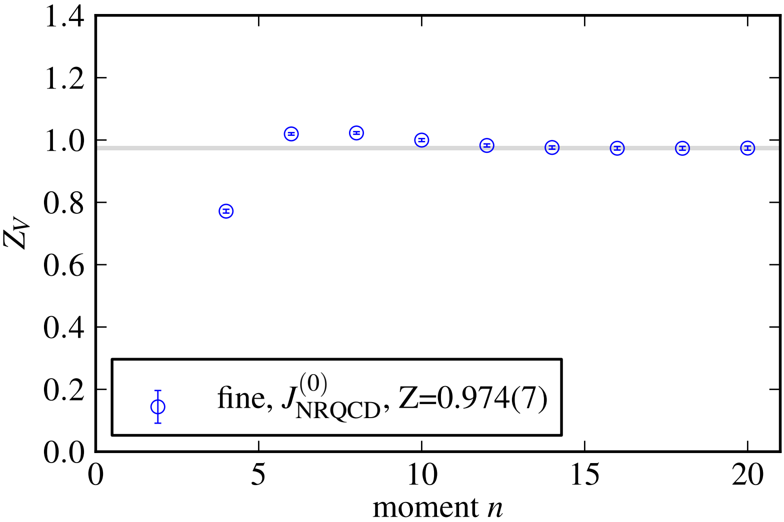
$$G_n^V = Z_V^2 G_n^{V,\mathrm{NRQCD}}$$

Temporal Moments & Z_V Matching Factor

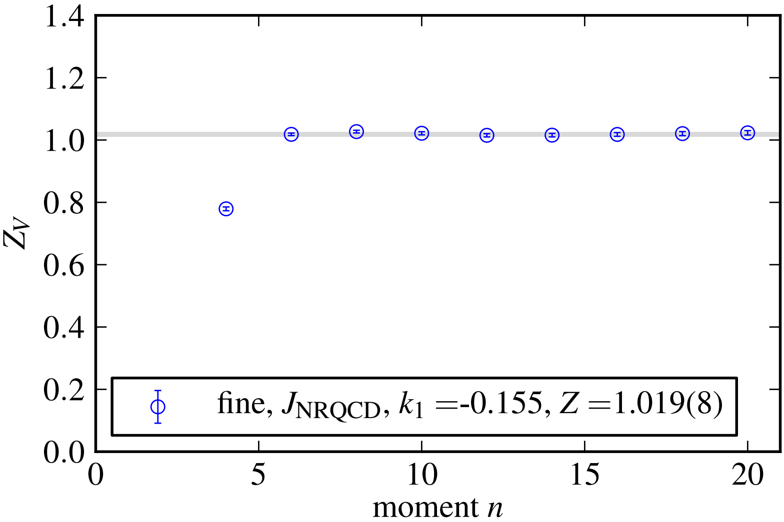
$$G_n^V = \frac{g_n^V(\alpha_s, \mu/m_b)}{[a\overline{m}_b(\mu)]^{n-2}}.$$

$$Z_V = \left(\frac{G_n^V}{G_n^{V,(0)} r_n^V}\right)^{\frac{(n'-2)}{2(n-n')}} \left(\frac{G_{n'}^{V,(0)} r_{n'}^V}{G_{n'}^V}\right)^{\frac{(n-2)}{2(n-n')}},$$

Z_V

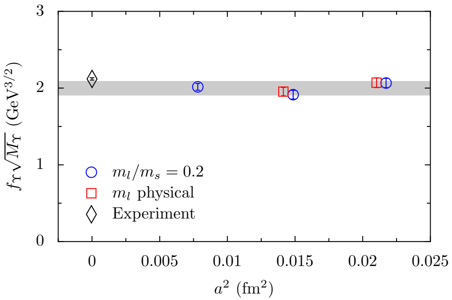

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z_V
0
5
10
15
20
moment n
fine, $J^{(0)}_{\mathrm{NRQCD}}$, Z=0.974(7)

Z_V


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z_V
0
5
10
15
20
moment n
fine, J_{NRQCD}, k_1 =-0.155, Z =1.019(8)

Υ Leptonic Width

$$Z_V c(J_{V,\mathrm{NRQCD}}, 0) = \frac{\langle 0|J|\Upsilon\rangle}{\sqrt{2M_\Upsilon}}, \qquad \langle 0|J_V|\Upsilon\rangle = f_\Upsilon M_\Upsilon$$



Decay Constant

$$f_\Upsilon = 0.649(31)\ \mathrm{GeV}$$

Leptonic Width

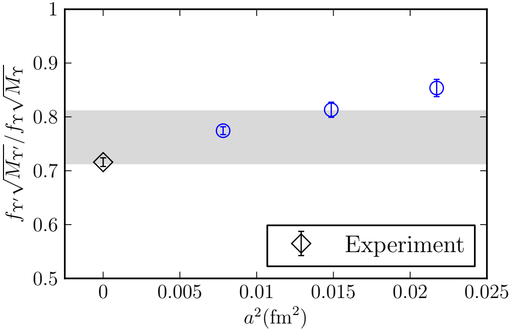
$$\Gamma(\Upsilon \to e^+e^-) = 1.19(11)\ \mathrm{keV}$$

Υ' Leptonic Width

Harder to get excited states: so use 3×3 fit. Convenient to take ratio, cancel Z_V:

$$A = \frac{\langle 0|J_V|\Upsilon'\rangle}{\langle 0|J_V|\Upsilon\rangle} = \frac{f_{\Upsilon'}}{f_{\Upsilon}}\sqrt{\frac{M_{\Upsilon'}}{M_{\Upsilon}}}$$

Υ' Leptonic Width



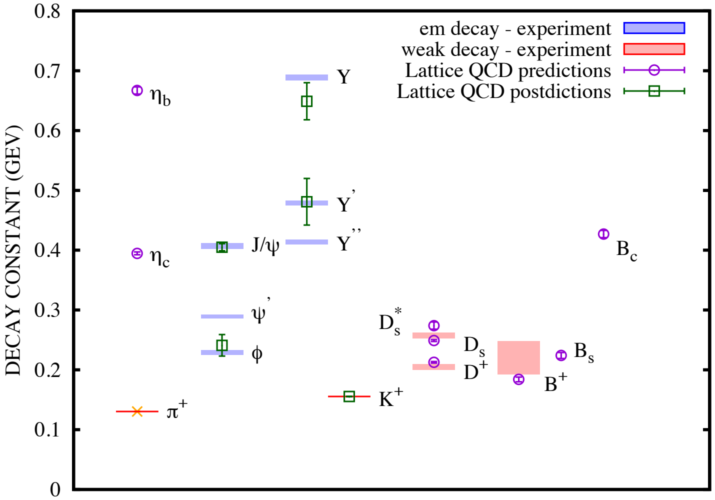
Decay Constant

$$f_{\Upsilon'} = 0.481(39)\ \mathrm{GeV}$$

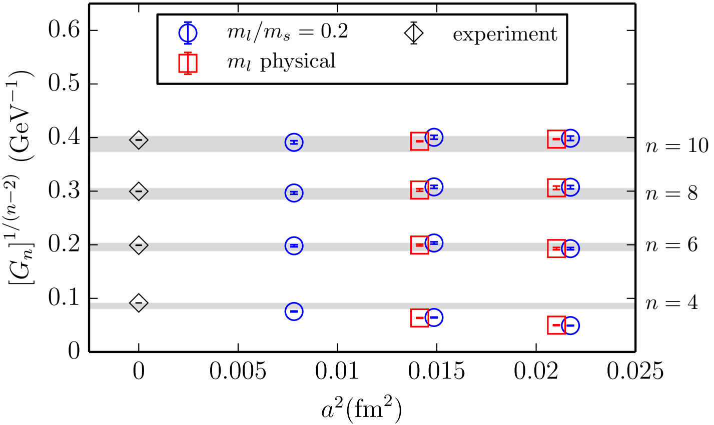
Leptonic Width

$$\Gamma(\Upsilon' \to e^+e^-) = 0.69(9)\ \mathrm{keV}$$

LQCD Decay Constants



$R_{e^+e^-}$



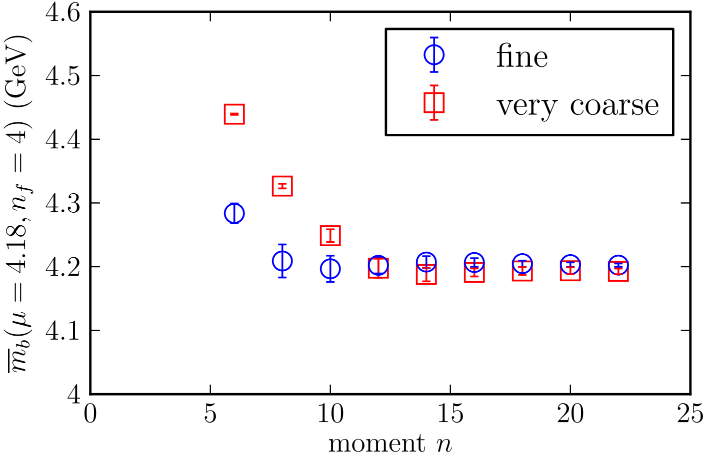
$m_l/m_s = 0.2$
m_l physical
experiment
$[G_n]^{1/(n-2)}$ (GeV^{-1})
0.6
0.5
0.4
0.3
0.2
0.1
0
$n = 10$
$n = 8$
$n = 6$
$n = 4$
0
0.005
0.01
0.015
0.02
0.025
$a^2(\text{fm}^2)$

Determination of m_b

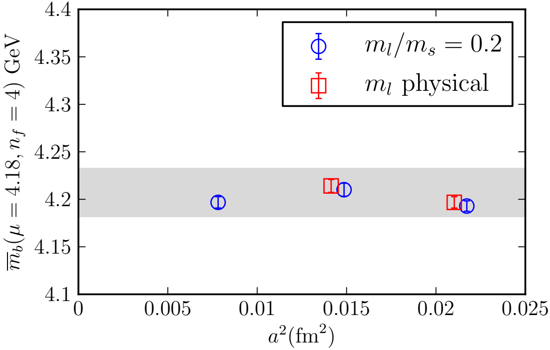
These temporal moments can be used to calculate the mass of the b quark.

$$\overline{m}_b(\mu) = \frac{\overline{M}_{\Upsilon,\eta_b}}{2}\left[\frac{R_{n-2}r_n}{R_n r_{n-2}}\right]^{1/2}\frac{2m_b}{\overline{M}_{\mathrm{kin}}}$$

Determination of m_b

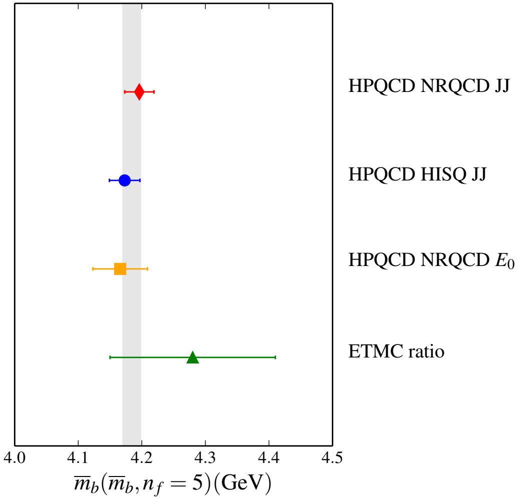


Determination of m_b

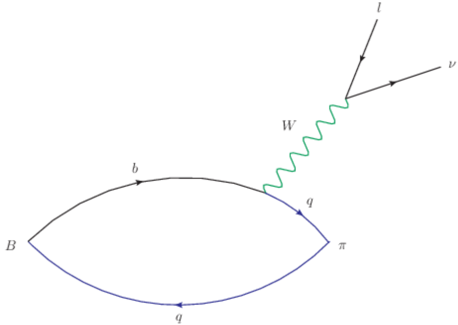


$$\bar{m}_b(\mu = \bar{m}_b \text{ GeV}, n_f = 5) = 4.196(23) \text{ GeV}$$

m_b Summary


HPQCD NRQCD JJ
HPQCD HISQ JJ
HPQCD NRQCD E_0
ETMC ratio
4.0
4.1
4.2
4.3
4.4
4.5
$\overline{m}_b(\overline{m}_b, n_f = 5)(\mathrm{GeV})$

B Decays



$$\frac{\langle 0|P|\pi\rangle\langle\pi|V|B\rangle\langle B|A_0|0\rangle}{2E_\pi 2E_B} e^{-E_\pi t} e^{-E_B(T_B-t)}$$

- The b quark can be studied with NRQCD. Light quarks HISQ (a relativistic formulation)
- Decay constants & form factors from semileptonic B decays
- Get matrix element from lattice QCD → CKM matrix elements from lattice QCD and experiment.

Semileptonic Form Factors

Matrix element relevant to this decay can be parametrised by form factors $f_+(q^2)$ and $f_0(q^2)$:

$$\langle B|V^0|\pi\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}\right) + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu$$

But here I am only considering the case where the B and π 3-momenta are zero

Currents

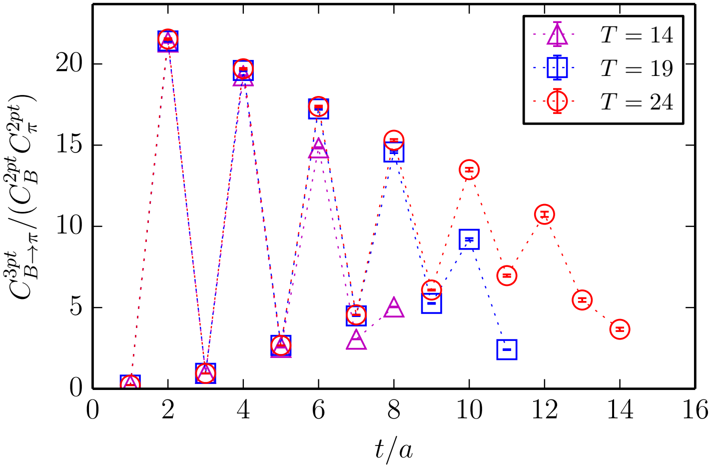
We use the following:

$$\begin{aligned} J_0^{(0)}(x) &= \bar{q}(x)\Gamma_0 Q(x) \\ J_0^{(1)}(x) &= \bar{q}(x)\Gamma_0 \gamma\cdot\nabla Q(x) \\ J_0^{(2)}(x) &= \bar{q}(x)\gamma\cdot\overleftarrow{\nabla}\gamma_0\Gamma_0 Q(x) \end{aligned}$$

Matched via:

$$\begin{aligned} \langle V_0 \rangle = &(1+\alpha_s\rho_0^{(0)})\langle J_0^{(0)}\rangle + \\ &(1+\alpha_s\rho_0^{(1)})\langle J_0^{(1),sub}\rangle + \alpha_s\rho_0^{(2)}\langle J_0^{(2)}\rangle \end{aligned}$$

3pt, 2pt Correlator Ratios



Fitting

- Fitting very similar to 2-point fits, but fit 2-points for B and π with 3-point simultaneous
- Fit correlators with various smearings, and T simultaneously

Fit function

$$C(t) = \sum_{k=0}^{n_{\exp}-1} c_k^2 \left(e^{-E_k t} + e^{-E_k (T-t)}\right) - (-1)^{t/a} \sum_{ko=0}^{n_{\exp}-1} \tilde{c}_{ko}^2 \left(e^{-\tilde{E}_{ko} t} + e^{-\tilde{E}_{ko} (T-t)}\right),$$

f_0 Form Factor

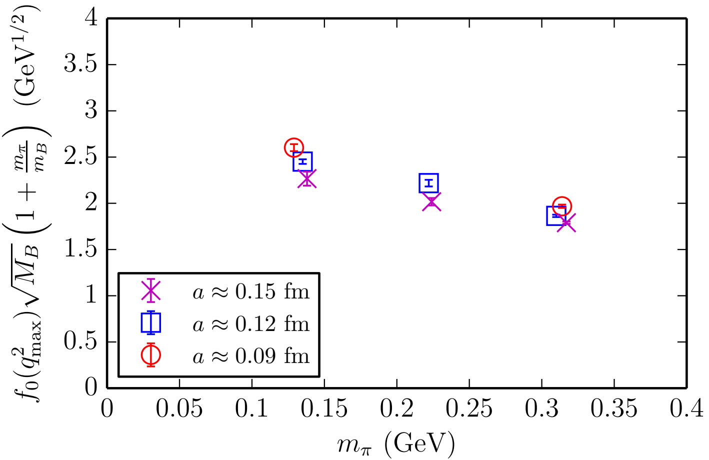
At q^2_{max}, only left with f_0.

$$\langle B|V^0|\pi\rangle = f_0(q^2_{\mathrm{max}})(m_B^2 + m_\pi^2)$$

Can get this directly from the fit through:

$$f_0(q^2_{\mathrm{max}}) = 4\sqrt{2}V_{00}\frac{\sqrt{m_B m_\pi}}{m_B + m_\pi}$$

f_0 Form Factor



Soft Pion Theorem

The soft pion theorem relates the decay constants of the B and π to $f_0(q^2_{\text{max}})$

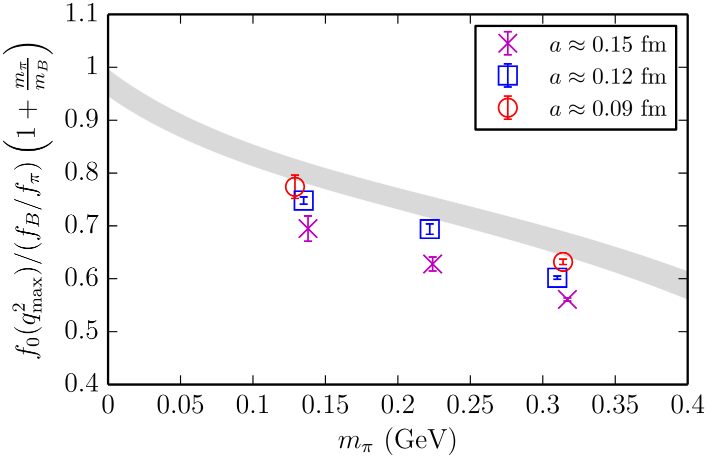
$$f_\pi = 2m_l\sqrt{\frac{2}{E_\pi^3}}a_0$$

$$f_B\sqrt{m_B} = 2b_0$$

In the limit, $m_\pi \to 0$, the soft pion theorem says

$$f_0(q^2_{\text{max}}) = \frac{f_B}{f_\pi}$$

Soft Pion Theorem



CKM Matrix Elements

At non-zero recoil, i.e. $q^2 < q^2_{\max}$, we can get both $f_0(q^2)$ and $f_+(q^2)$. ($f_0(0) = f_+(0) = 1$)

This allows access to the following:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} \left[\left(M_B^2 + M_\pi - q^2 \right)^2 - 4M_B^2 M_\pi^2 \right]^{3/2} \left| f_+(q^2) \right|^2 .$$

So in combination with experimental results and known factors, can extract $|V_{ub}|$

Other Semileptonic Decays

We can consider other decays (actually, I am doing):

- $B_s \to K\ell\nu$, different spectator
- $B_s \to \eta_s\ell\nu$, different spectator and active
  - The η_s doesn't exist in the real world
  - Can be studied on the lattice

NRQCD Improvement

There are different ways in which to improve on the work using NRQCD:

- Do further corrections to coefficients c_i
- Extra terms in Hamiltonian
  - Work here is at $\mathcal{O}(v^4)$. Can go to $\mathcal{O}(v^6)$

Summary

- Lattice QCD essential for nonperturbative calculations of the strong force
- Lattice NRQCD is useful for the precision calculations of systems involving a b quark
- Gives good results for bottomonium states
- Same b quarks and ensembles used for other calculations
- Can use it to extract $f_0(q^2_{\text{max}})$
- Soft pion theorem holds
- V_{ub} matrix element can be determined if pion given momentum