Control Of Optical Aberrations With Coded Apertures

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Outline

• Introduction and motivation
• Design of binary-amplitude masks
  – Contour masks (analytical)
  – Multi-mask (numerical)
• Examples
  – Fundamental aberrations
  – Ocular aberrations
  – Aberrations in conformal optics
• Conclusions
Control of Aberrations

• High-cost optics
• Hybrid/computational imaging
  – Wavefront coding
  – Coded apertures
    • With lens!
    • Agile/adaptive
      – Cf. adaptive optics
• Foveal
• Low cost

SPIE Coded Aperture 2011
Aberrations: decomposition of OTF

- Defocus:
  - At spatial frequency \( \nu \), OTF components form a circle
  \[
  OTF(\nu) = \int P(u + \nu)P^*(u - \nu)\,du
  \]

- Cubic phase modulation:
  - OTF components curl into generalised Cornu spiral
  - Defocus unwinds spiral
    - no zeros in MTF

M Demenikov and A R Harvey Opt. Exp. 18, 17, pp 18035 (2010)
M Demenikov and A R Harvey Opt. Exp. 18, 8, pp 8207 (2010)
Decomposition of the OTF

$$\text{OTF}(\nu) = \iint P^*(u+v)P(u-v)du, \quad \text{with } P(u) = \exp(\text{i}l(u)), \quad \forall u \|u\| \leq 1$$

$$= \iint_{\Omega(\nu)} \exp(\text{i}\Delta(u,v))du, \quad \text{with } \Delta(u,v) = l(u+v) - l(u-v)$$

Optical path length variation

Difference in optical path length between two points on the pupil 2\nu apart.

OTF with defocus

OTF with defocus and mask

$W_{20} = 1/2$
Mitigating aberrations with amplitude masks

- **Binary-amplitude masks**
  - attenuate 2\textsuperscript{nd} order image-plane interference
  - enable sufficient information to be recorded for good image restoration.

- **Advantages**
  - Wide variety of fixed or agile amplitude modulation techniques
  - Wideband implementation: UV to far-infrared
  - Lower complexity and cost

**Example:**
Coded mask for defocus
Design of the binary mask

• The ideal mask permits only constructive 2nd-order interference (of phasors within OTF integral):

\[
\left(\Delta(u, v) + \pi/2\right) \mod 2\pi \leq \pi
\]

• Thus, for an aberration characterized by an optical path length variation \(l(u)\), the mask \(M(u)\) blocks areas of the pupil along contour lines:

\[
M(u) = 1, \quad \forall u \left| l(u) - \phi_0 - t \cdot u + \frac{\Delta \phi}{2} \right| \mod 2\pi \leq \Delta \phi
\]

• Parameters to optimise
  – Tip-tilt \(t\) and reference phase \(\phi_0\)
OTF of a contour mask

- Contour mask can be designed with no nulls in OTF
  - in the limit for large aberrations\(^1\)
    \[
    \text{OTF}_c(v) \approx \frac{2\pi}{\Delta \phi} \frac{\text{MTF}}{\pi^2} \int dl(v) \sin^2 \left( \frac{\Delta \phi}{2} \right) \leq \frac{2}{\pi^2} \text{MTF}_{dl}(v)
    \]
  - For \(\Delta \phi = \pi\), \(\text{MTF} \approx 0.2 \text{MTF}_{dl}\)

- MTF is independent of the magnitude of aberrations

Optimisation

\[ M(u) = 1, \quad \forall u \mid \left( l(u) - \varphi_0 - t \cdot u + \frac{\Delta \varphi}{2} \right) \mod 2\pi \leq \Delta \varphi \]

- Low dimensional optimization
  - Tip-tilt, \( t \)
  - Phase reference, \( \varphi_0 \)
- minimizes expected imaging error \( \varepsilon \).

Cost function for astigmatism as a function of tip-tilt \((t_x, t_y)\) and minimized for phase-reference.
Examples of contour masks

- Pupil phase (hue)
- Mask (black)
- MTF (no mask)
- MTF (mask)

RMS OPD of 2λ

- Astigmatism
  - T = 52.17%
- Coma
  - T = 56.02%
- Eye
  - T = 52.56%

RMS OPD of 2λ

RMS OPD of 3.5λ
Example: doublet and broadband illumination

- F/5 cemented doublet, 5° off-axis at $\lambda_0=550\text{nm}$
  - Astigmatism and field curvature with a peak-to-valley OPD of 10$\lambda_0$
  - Broadband MTF (495nm$\leq\lambda\leq$605nm) strongly suppressed
  - Higher MTF for extended bandwidth
- Contrast lower than for monochromatic operation
Benefit from multiple MTFs?

- Häusler (1972)
  - Scan image plane through volume
- Reduced MTF
  - No zeros
  - Approximately constant
  - Inverse filter to recover high-quality image
**Mask design: numerical optimisation**

- Optimization of pixelated mask
  - High dimensional
  - Maximize MTF
- Optimisation of a set of masks with complementary MTFs\(^1\)
  - Some mask MTFs may contain nulls

\[
\{M_i:i=1,...,N\} = \arg\min_{\sigma+\sum_{i} T_{i} \text{MTF}(M_{i})} \left( \frac{1}{\sigma+\sum_{i} T_{i} \text{MTF}(M_{i})} \right)^{2}
\]

\(^1\) \text{J. W. Stayman, N. Subotic, and W. Buller, Proc. SPIE 7468, 74680D (2009).}
Image restoration algorithm for a sequence of N coded images

- The sequence of coded images are subsequently processed via a multi-frame restoration algorithm to produce a sharp image.

- Forward model,

  $$\tilde{g} = \begin{pmatrix} \tilde{g}_1 \\ \tilde{g}_2 \\ \vdots \\ \tilde{g}_N \end{pmatrix} = H\tilde{f} + \tilde{n} = \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_N \end{pmatrix} \tilde{f} + \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_N \end{pmatrix}$$

- Tikhonov-type regularization scheme leads us to a least-squares restoration algorithm:

  $$\tilde{f}' = (H^*H + \sigma I)^{-1} H^*\tilde{g}$$
Application: Conformal optics

- Imaging through ellipsoidal dome in MWIR ($\lambda_0=4$ microns)
  - Imaging system with $\pm45^\circ$ FoV
  - Aberrations: PV OPD of $4\lambda_0$ at $\pm45^\circ$
- Two 16×16 binary-amplitude masks for each FoV
  - Reconfigurable shutter array at the aperture stop that optimises mask with FoV
    - Foveal imaging
  - Optimisation up to Nyquist frequency uses a differential evolution algorithm with 20000 iterations
Optimised coded masks

Average mask transmission of 60%
PSFs across FoV

Coded aperture system
(after spatially-variant restoration)

Conventional system

90°

90°

90°

90°
MTF at \((0^\circ, 45^\circ)\)

Solid black line: Diffraction limited MTF
Solid lines (red and blue): mask 1 and mask 2 MTFs
Dashed blue line: aberrated MTF (no masks)
Restored images for (0°, 45°) deg

With masks

Final restored image

RMSE=15%

SNR of one recorded conventional image~37dB
SNR of one coded image~30dB (signal scaled down by transmission of coded masks)

Without masks

Final restored image

RMSE=35%
Restored images for \((0^\circ, 45^\circ)\)

**With masks**

RMSE=13%

**Without masks**

RMSE=27%
Conclusions

• Where aberrations are unavoidable binary coded masks can be optimised to yield improved overall MTF.
  – MTF typically 20% of diffraction limited MTF
  – Absence of nulls and modest reduction in MTF allows recovery of high quality images.
  – Agile - adaptable programmable masks are a low cost alternative to deformable mirrors

• Good performance with broadband illumination is possible.