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Big bangs

Closing in on gamma-ray bursts

No limits for slow light
A star is born
First among equals

Angular uncertainty passes test

The uncertainty principle states that we cannot know the state of a system with arbitrary precision. Now the uncertainty relation between angular momentum and position has been measured in an experiment for the first time using specially shaped beams of light

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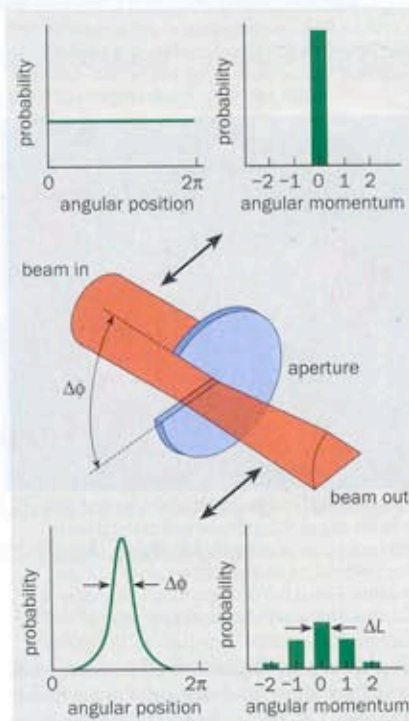
If you asked members of the public to name a concept or idea from modern physics, the most popular answer would probably be $E=mc^2$. However, the runner-up could well be Heisenberg's uncertainty principle. This principle states that the position and linear momentum of a particle cannot be known simultaneously with arbitrary precision. More precisely, it sets a lower bound for the product of the uncertainties of these quantities: $\Delta x \Delta p \geq \hbar/2$, where Δx is the uncertainty in the position, Δp is the uncertainty in the momentum and \hbar is Planck's constant divided by 2π .

One consequence of the uncertainty principle is that quantum objects such as electrons do not follow classical trajectories, which is in clear conflict with our everyday experience. A similar, but less well known, uncertainty relation also exists between energy and time. Now, researchers at the universities of Glasgow and Strathclyde in the UK have studied another manifestation of the uncertainty principle. In a beautiful experiment involving light beams, the team has verified the uncertainty relation between angular position and angular momentum for the first time (S Franke-Arnold *et al.* 2004 *New J. Phys.* **6** 103).

Uncertain world

The angular position of a particle, ϕ , is simply the angle at which it is located in 2D "polar" co-ordinates. The angular momentum, L , is the rate of change of this angle with time multiplied by the mass of the particle and the square of its polar radius. A light beam consists of many photons, each of which has an angular position and an angular momentum, so the beam is best described by a probability distribution of angles. The uncertainty in the angular position, $\Delta\phi$, is the typical spread of the angles among the photons.

In its general form, the uncertainty principle applies to every pair of "conjugate" variables (in quantum mechanics two variables, A and B , are said to be conjugate if $AB - BA = i\hbar$). It might therefore seem that the uncertainty relation between angular position and angular momentum is exactly the same as that between linear position and momentum. But things are not quite this simple because the angular position is



Shedding light on uncertainty – in the Glasgow experiment an input light beam with a flat angular-position distribution (top) passes through an aperture that cuts out a certain range of angular positions. As a result of the uncertainty principle, the output beam has a narrow angular-position distribution and a broad angular-momentum distribution (bottom).

restricted to values between $-\pi$ and $+\pi$, whereas linear position can take any value. This has two interesting consequences. First, the uncertainty in the angular position is always finite. Second, the angular momentum is quantized and can only have certain values known as eigenvalues, whereas linear momentum, like linear position, can take any value. When the angular momentum assumes only one of these eigenvalues, the system is said to be in an angular-momentum eigenstate.

Since the uncertainty in the angular momentum, ΔL , is zero for any angular-momentum eigenstate, the product of $\Delta\phi$ and ΔL is also zero. It therefore seems as if these two conjugate variables can beat the uncertainty relation. However, when a system is in an angular-momentum eigenstate, we know absolutely nothing about its angular position. Conversely, if the angular

position is confined to a narrow range of values, then the angular momentum must be distributed over a broad range of values (i.e. the system must be in a superposition of many different angular-momentum eigenstates). Mathematically, this is expressed as $\Delta\phi\Delta L \geq (\hbar/2)[1 - 2\pi P(\pi)]$, where $P(\pi)$ is the probability that the angular position has a value of π .

It is natural to ask, which states satisfy the equality in this relation? Sonja Franke-Arnold and Stephen Barnett at Strathclyde have now calculated that the only distribution of angular position that satisfies this "minimum uncertainty" state is a Gaussian that is truncated at $\pm\pi$. When the uncertainty in ϕ is large, the probability of obtaining one eigenvalue of L is large and the probability for all other eigenvalues of L is small. In this case, the product of the two uncertainties is small. Conversely, for small uncertainties in angular position, the distribution of ϕ is narrow and $P(\pi)$ is small. Here, the angular-momentum distribution is broad and closely resembles a normal Gaussian. This situation is reminiscent of the normal position-momentum uncertainty relation, and the value of the uncertainty product $\Delta\phi\Delta L$ is indeed close to $\hbar/2$.

Experimental certainty

Guided by the Strathclyde group's theoretical work, Eric Yao and colleagues at Glasgow prepared a beam in which the angular position followed a Gaussian distribution, and then measured the resulting distribution of angular momentum. To do this, they inserted an absorber in the path of a broad light beam that let different amounts of light through depending on the light's angular position. The absorber has a wedge shape that looks like a cake slice, and it imprints the desired angular distribution onto the intensity profile of the beam. The easiest way to make such a spatial light absorber is to use an array of liquid-crystal pixels that can be programmed with a computer. Diffraction from the absorber changes the angular-momentum distribution.

The angular momentum can then be measured using an appropriate holographic grating followed by a lens and a pinhole (the light intensity passing through the pinhole indicates the probability of finding a specific eigenvalue of the angular momentum). For a series of absorbers with dif-

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ferent uncertainties in angular position, the observed angular-momentum distributions agree well with theory and are consistent with the uncertainty relation derived by the Strathclyde group.

This work beautifully illustrates that uncertainty relations for quantities other than

position and momentum are not just theoretical constructs but can also be explored in the lab. The Glasgow experiment illustrates this uncertainty relation for the wave nature of light, which can be fully described by classical electrodynamics. But a lot of the relevant quantum physics is

already captured in the present set-up because the wave description of light and the de Broglie picture are similar. The Glasgow group is now repeating the experiment for single photons, in order to investigate the angular-uncertainty relation for a genuine quantum system.

Microscopes focus on single electrons

Magnetic resonance force microscopy has been used to detect the spin of a single electron

From **John A. Marvin** in the Department of Chemistry and Chemical Biology at Cornell University, Ithaca, New York, US

For chemists, the starting point in understanding the function of a molecule is its structure. Molecular biologists are obsessed with structure as well. Their interest usually lies in determining the structure and function of proteins – the beautiful, machine-like molecules of life. However, nature's molecules tend to be so large that determining their detailed molecular structure can often take years. For example, after putting decades of effort into understanding membrane proteins – the gate-keepers of cells – we know the full 3D structure of only a handful of the thousands that exist, despite the fact that this is a prerequisite for defeating a huge number of diseases.

Historically, both chemists and molecular biologists have turned to physics when new tools for determining molecular structure are needed. In the 1980s the technique of magnetic resonance imaging was perfected, and imaging techniques such as scanning tunnelling and atomic force microscopy were invented. In the early 1990s medical physicist John Stiles, now at the University of Washington in Seattle, married these two technologies and invented magnetic resonance force microscopy. This technique has generated considerable excitement recently because it represents a new route to imaging the full 3D structure of a single frozen molecule.

Now Dan Rugar, John Marvin, Raffi Budakian and Ben Chui at IBM's Almaden Research Center in San Jose, California have used a magnetic resonance force microscope to image the spin of a single electron. This is an important milestone on the way to imaging proteins in a single molecule, and represents some exciting and hard-won new physics (*Nature* **430** 329–332).

Rethinking spin

In magnetic resonance force microscopy (MRFM) a tiny cantilever with a magnetic tip is brought close to the surface of a sample, where it senses the force of a nearby electron or nuclear spin. Compared with electrostatic and van der Waals forces in the system, how-



Spin sensitive – IBM researchers Raffi Budakian, John Marvin and Dan Rugar (left to right) have increased the sensitivity of magnetic resonance force microscopy to detect the spin of a single electron. This is the smallest magnetic moment that has ever been detected mechanically.

ever, these tiny "spin forces" are too weak to deflect the cantilever by a measurable amount. The trick is to modulate the magnetization of the tip so that the tiny spin force is time dependent, and therefore more easily distinguishable from background forces.

To do this, the sample is placed in a large magnetic field and subjected to an additional, perpendicular radio-frequency (RF) field. Spins in the sample are tipped away from the static field by tuning the frequency of the RF field to match the difference in energy between the spin "up" and spin "down" states. For electrons, this corresponds to a frequency of 28 GHz, while for protons it is much lower at 42 MHz. Since the static magnetic field is enhanced near the tip of the cantilever, the spins can be modulated up and down at will by sweeping either the magnitude of the static magnetic field or the frequency of the RF field.

By the middle of the 1990s a number of groups had detected both electron spin resonance and nuclear magnetic resonance using this approach. But the sensitivity of the technique was still many orders of magnitude away from that required to image a single electron. A major improvement in sensitivity was achieved when Rugar teamed up with Tom Kenny's group at Stanford University to make cantilevers capable of detecting forces as small as 10^{-16} N. This is

many thousands of times smaller than the forces that can be observed in atomic force microscopy using commercially available cantilevers. To reach one-electron sensitivity, the next step was to meticulously re-engineer the cantilever and to rethink the way it is used to detect spins in a sample.

In order to detect a signal, the spins in the sample need to be modulated for at least 1 s. The IBM team therefore chose glassy silica for the sample because it has an unpaired electron that stays "locked" in step with the applied RF field for a long time. In an effort to make the force per spin as large as possible, Rugar and co-workers sharpened the cantilever's magnetic tip to produce a larger magnetic-field gradient.

However, when the tip became too small, the magnetization of the sample could not be modulated for as long as was needed due to random spin flips associated with fluctuations in the magnetization of the tip. To eliminate this effect, the team fabricated the tip from a high-coercivity magnetic alloy and worked with specially designed beam cantilevers that have suppressed vibrational modes. The researchers also began working at cryogenic temperatures to minimize the thermal motion of the cantilever, which is the main source of noise in MRFM experiments.

The particularly clever part, however, is that the IBM team figured out how to flip spins in the sample while leaving the RF field almost unchanged. Modulating the spins in an MRFM experiment usually requires varying the frequency or amplitude of the applied RF field, which can interfere with the motion of the cantilever. Instead, Rugar and co-workers showed that the moving cantilever itself could be used to sweep the static field seen by the spins. By turning the RF on and off for just a fraction of a cantilever cycle every once in a while, they demonstrated that the cantilever frequency could be made time dependent.

Towards single-molecule imaging

After years of continually improving their apparatus, Rugar and his team have finally detected and imaged (in one dimension) a single electron spin in vitreous silica. This is the smallest magnetic moment that has