

Tutorial Questions for Modern and Nonlinear Optics

1) Explain the terms spatial coherence and temporal coherence as applied to laser systems.

2) By comparing the exponential gain in the laser medium to the exponential loss of the laser cavity, show that

$$k_{\text{threshold}} = \frac{n}{c t_{\text{cavity}}}$$

where $k_{\text{threshold}}$ is the gain required to reach threshold and t_{cavity} is the decay time for the light intensity within the cavity

3) By considering a light beam passing through a cubic metre of gain medium show that the small signal gain is given by

$$k = (N_2 - N_1) \sigma$$

where N_1 and N_2 are the population densities of levels 1 and 2 and σ is the cross section for stimulated emission.

4) With the aid of an energy momentum diagram, briefly explain the difference between a direct and an indirect band gap semiconductor.

5) Explain the roles of heterojunction and stripe geometries in reducing the threshold of laser diodes.

6) Sketch a stability diagram which shows the allowed mirror curvatures and separations to obtain a stable laser cavity

An argon ion laser (514,5nm) has a cavity comprising of a output mirror, 10m radius of curvature concave mirror and a plane mirror separated by 1m.

For a Gaussian beam we have

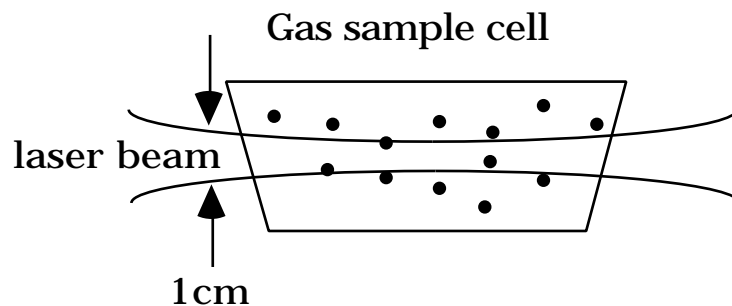
$$z = z_0^2 (1 + z^2/z_r^2)$$

$$R = z \left(1 + z_r^2 / z^2 \right)$$

$$z_r = \quad 0^2 /$$

Calculate the size and position of the beam waist formed within the laser cavity. Calculate the size of the beam at the output mirror.

7) Calculate the inhomogeneous and homogeneous linewidths as measured within the system below



wavelength $10.6\mu\text{m}$
excited state lifetime 1ms
5000 collision per second between molecules
average speed of molecules 100m/s

8) Use simple ray optic diagrams to show how optical tweezers can exert forces on small transparent objects.

9) With reference to the polarisation states of the light fields and the optic axis of the crystal, draw two figures to illustrate the difference between type-I and type-II phase matching.

10) What is meant by the term index ellipsoid

Use diagrams to show how the index ellipsoid can be used to calculate the phase matching angle for second harmonic generation in type I and type II geometry.

11) The second order nonlinear relation between E and D requires a tensor of 27 components, why?

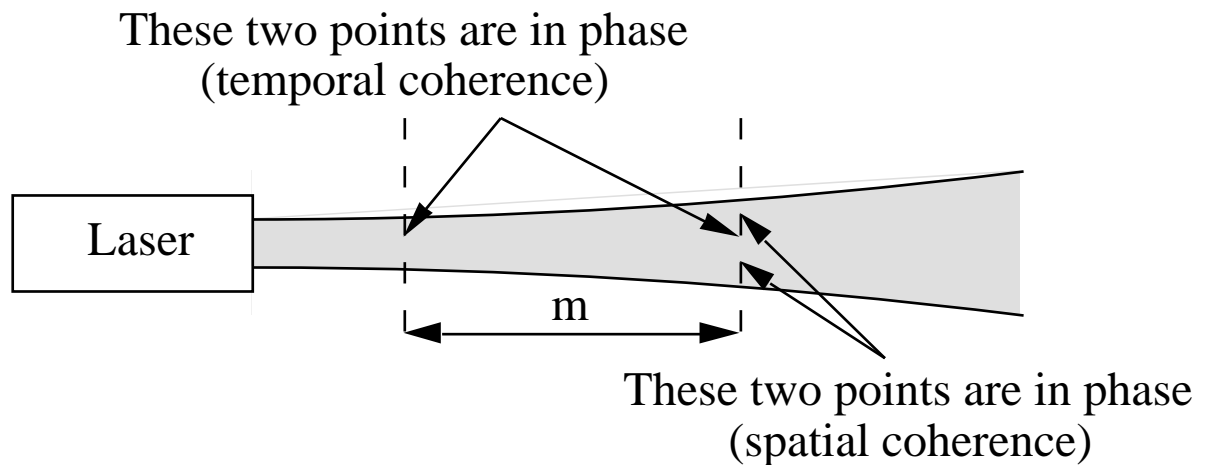
What assumptions can be made to reduce this number to 18?

List and explain the constraints for a doubly resonant optical parametric oscillator.

12) What is an optical parametric oscillator. How does the process in optical parametric generation differ from that of a conventional laser.

To form an OPO the nonlinear material is placed within an optical cavity. What are the merits and drawbacks of singly and doubly resonant oscillators regarding pump power threshold and tuning the OPO output

Outline a design of a pulsed OPO based on a diode pumped solid state laser



Gain (measure per metre) within the laser cavity gives an exponential rise in the intracavity intensity $I = I_0 \exp(kz)$

Cavity losses (measured per unit time) within the laser cavity gives an exponential loss in the intensity $I = I_0 \exp(t/t_c)$.

At threshold, the loss rate is exactly compensated for by the gain

$$k_{thres}z = t/t_c$$

relating z to t via the speed of light we get

$$k_{thres} = \frac{n}{c t_c}$$

Slightly differently from the notes!

$$\text{Rate of stimulated emission} = N_2 \sigma D \frac{c}{n}$$

$$\text{Rate of stimulated absorption} = N_1 \sigma D \frac{c}{n}$$

The difference in these two rates gives the rate of change in the number of photons

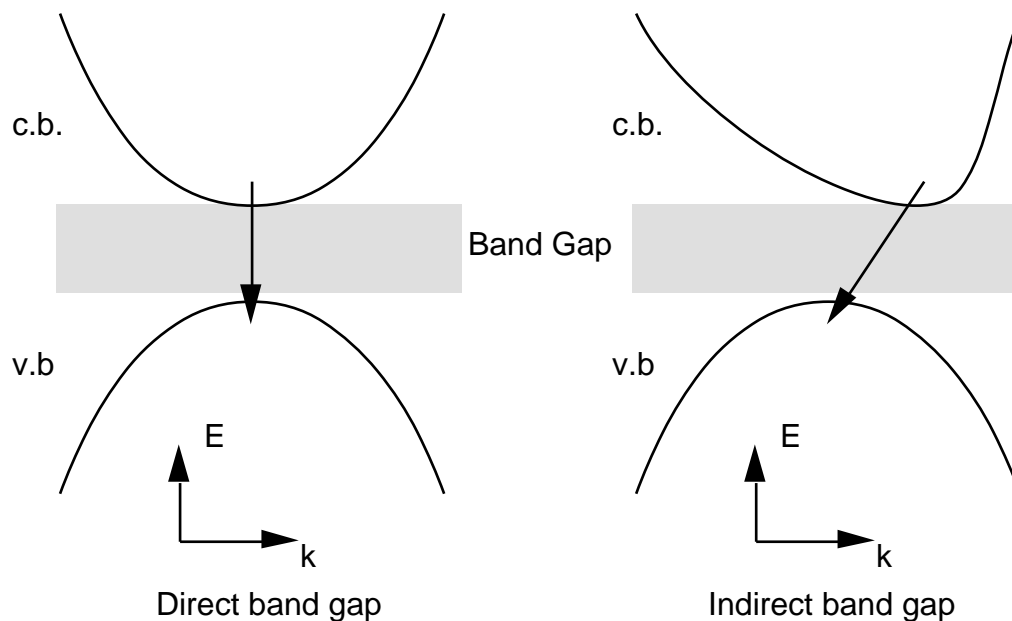
$$\frac{dD}{dt} = D \cdot \frac{c}{n} \sigma (N_2 - N_1)$$

Note that $D \frac{c}{n}$ is the photon flux (number per square metre per unit time)

k defined as $I = I_0 \exp(kz)$, hence

$$k = \frac{1}{I} \frac{dI}{dz} = \frac{1}{D} \frac{dD}{dz} = \frac{1}{D} \frac{dD}{dt} \cdot \frac{dt}{dz} = \frac{1}{D} \frac{dD}{dt} \cdot \frac{n}{c}$$

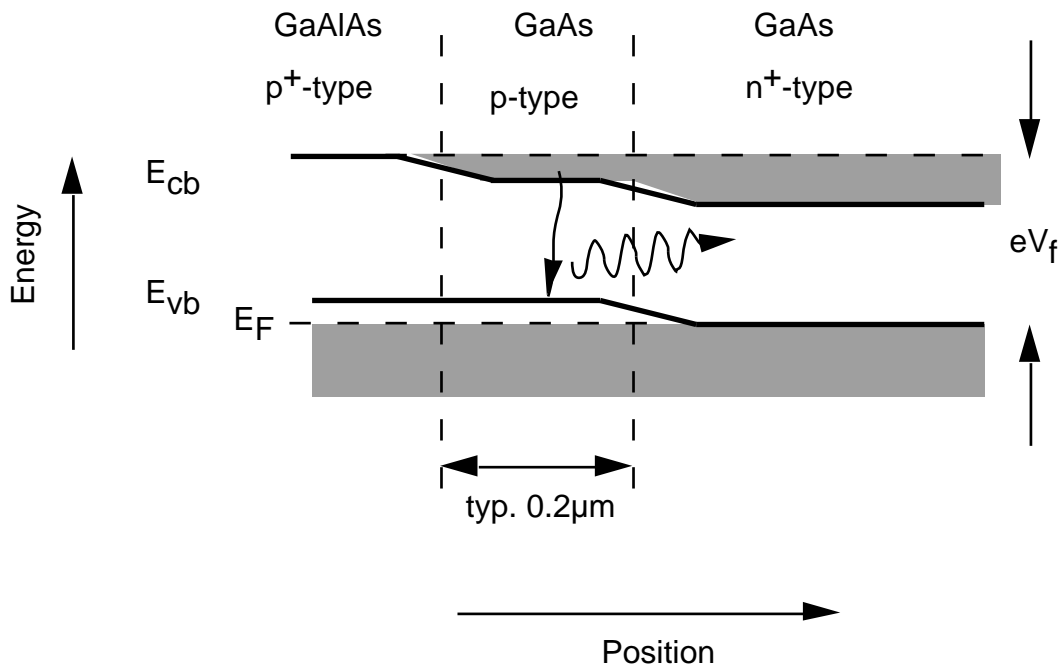
this gives $k = \sigma (N_2 - N_1)$



In a direct band gap the electron can transfer across the band gap without changing momentum. In many real solid, the bottom of the conduction band does not have the same momentum as the top of the valence band. These are indirect band gap semiconductors.

A heterojunction is a junction between materials of different types.

In a heterojunction laser an extra thin layer of material creates a region of higher refractive index as the active region which also acts as a waveguide for the photons



Points to note:

- GaAs acts as waveguide for emitted photons
- GaAlAs has larger band-gap than GaAs, therefore design reduces photon absorption in p⁺-type material
- threshold reduces to 10A mm⁻²

Heterojunction designs allow the threshold to be reduced to a low current density. To reduce the current to a low value it is necessary to reduce the area of the laser diode. If the length is reduced the round trip gain may fall below threshold, therefore need to reduce the width.

The current flow into the active region can be restricted to a single stripe along the length of the laser which may only be a few microns wide.

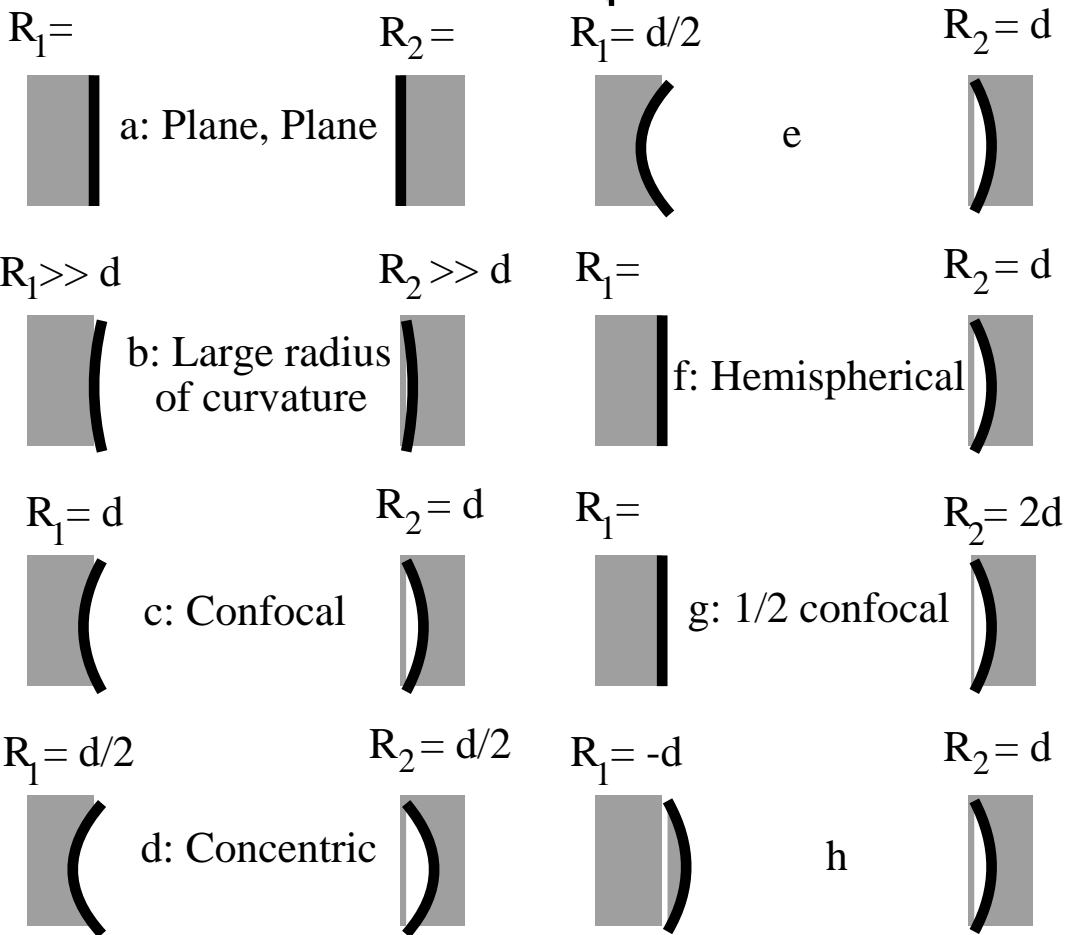
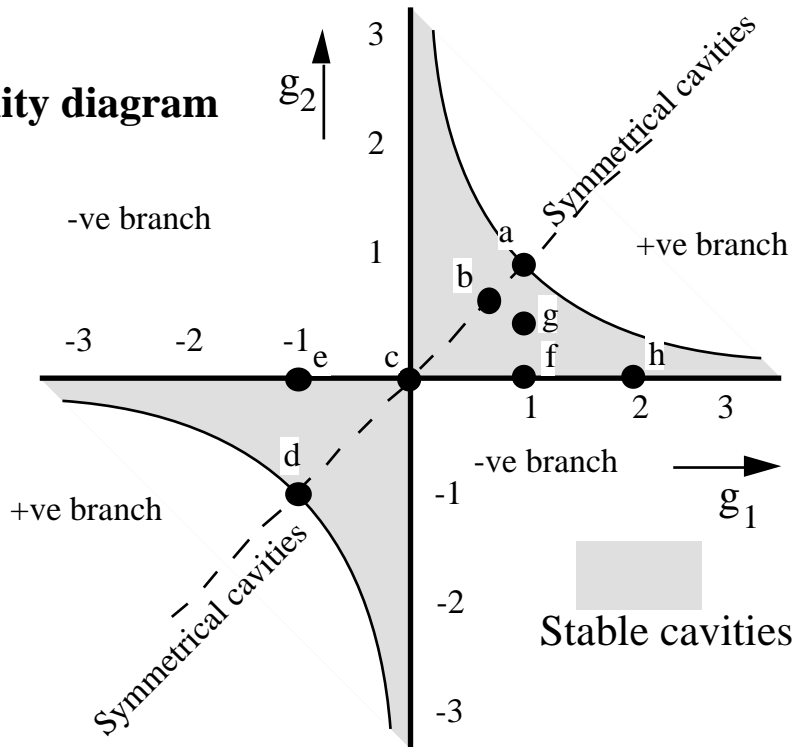
The stripe can be formed using a patterned electrode or by selective processing of the semiconductor material.

A stripe geometry gives threshold currents as low as 50mA with power outputs of 10mW.

Other advantages of stripe geometry are

- power emitted from small area therefore collimation and focusing are simplified
- power output can be more stable since lateral stability of active region is improved.

Cavity stability diagram



In a stable laser, the radius of curvature of the oscillating mode matches that of the mirrors. Therefore the beam waist of the mode lies at the surface of the plane mirror.

We are given $R = z (1 + z_r^2/z^2)$

Data in the question tells us that at $z=1$, $R=10$. Substituting these values into the previous question we can calculate that $z_r = 3$ m.

From $z_r = \omega_0^2 / \dots$ we calculate that $\omega_0 = 0.7$ mm

From $z = \omega^2 (1 + z^2/z_r^2)$ we calculate that at $z=1$, $\omega = 0.74$ mm

Homogeneous contributions

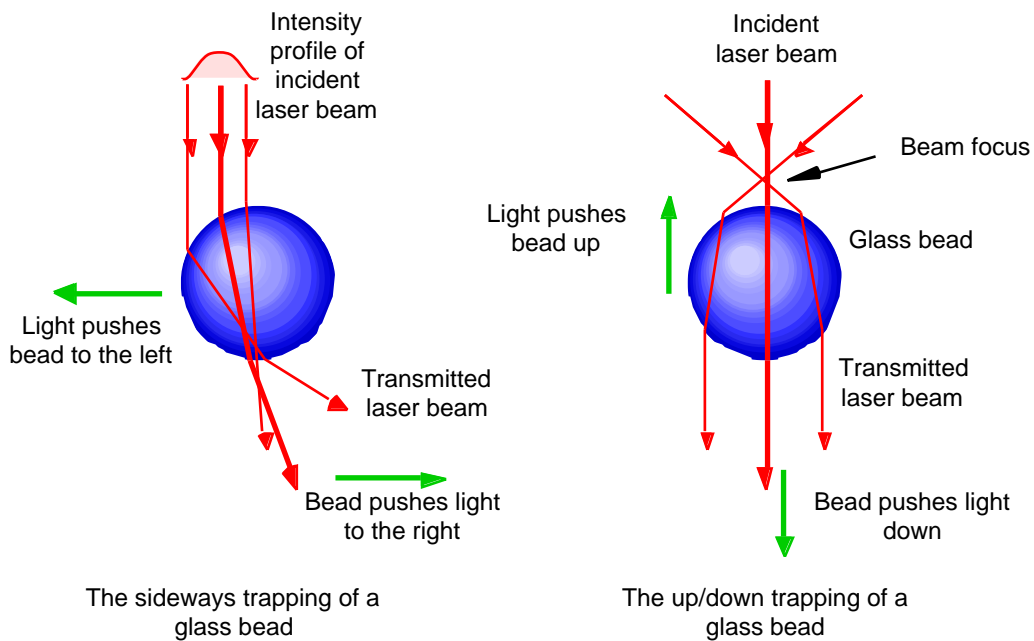
These can be calculated in terms of the uncertainty principle $\Delta E \Delta t \sim \hbar$ gives $\Delta \nu \sim 1/\Delta t$.

Therefore the finite upperstate lifetime of 1ms gives a linewidth of 1 kHz and the time between collisions gives 5 kHz.

The finite size of the beam also gives a finite interaction time of 1cm/100m/s i.e. 0.1ms. This gives a linewidth of 10 kHz. Therefore the transit time broadening is the dominant homogeneous component.

The main source of inhomogeneous broadening is the Doppler broadening,

$$\nu = \frac{v}{c} \nu_0, \text{ gives } \Delta \nu = \pm 10 \text{ MHz}.$$



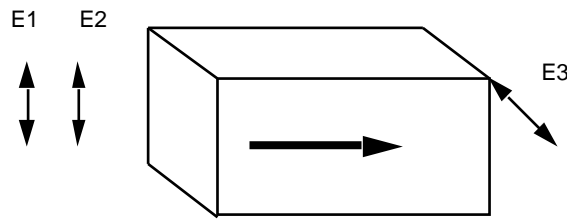
Type-I phase-matching ($k_3 = k_1 + k_2$)

- $E(1)$ and $E(2)$ have parallel polarisations
- $E(3)$ is orthogonally polarised with respect to $E(1)$ and $E(2)$

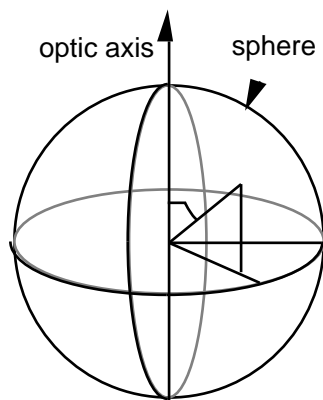
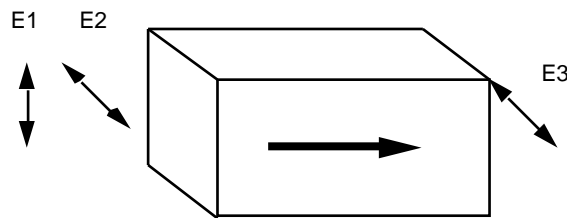
Type-II phase-matching ($k_3 = k_1 + k_2$)

- $E(1)$ and $E(2)$ have orthogonal polarisations
- $E(3)$ has a polarisation parallel to $E(1)$ or $E(2)$

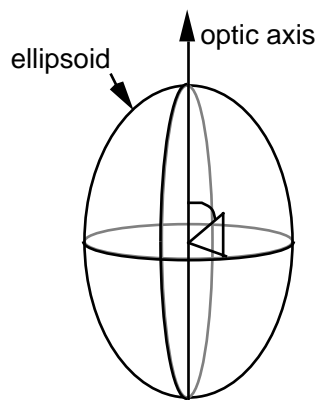
Type-I



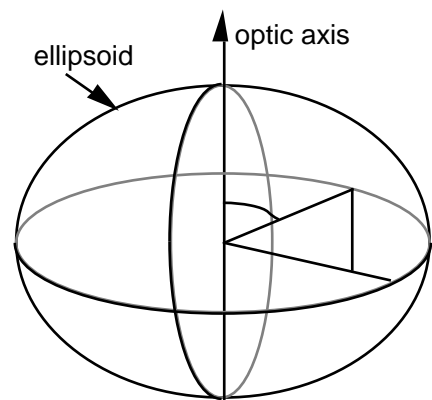
Type-II



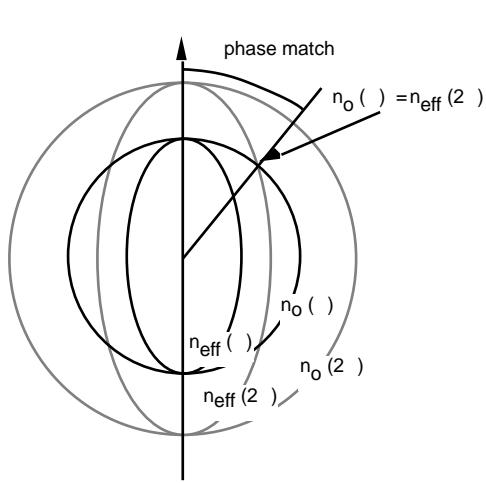
n , as a function of propagation direction for the o-ray



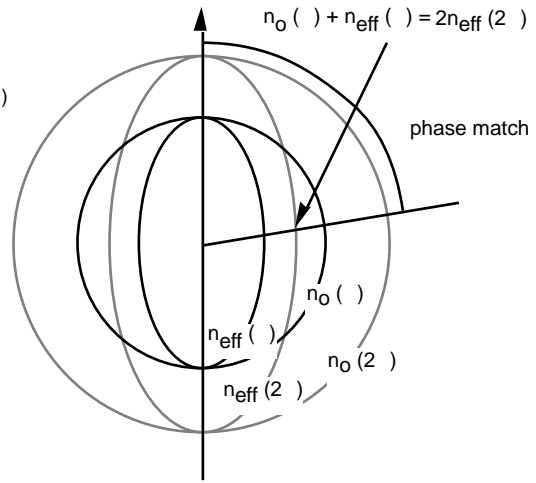
n , as a function of propagation direction for the e-ray, $n_e < n_o$ (-ve birefringence)



n , as a function of propagation direction for the e-ray, $n_e > n_o$ (+ve birefringence)



Type I



Type II



The polarisation in the x-direction can be related to the incident fields in the x, y and z-directions by

$$\begin{aligned}
 P_x(3) = & d_{xxx}(3)E_x(1)E_x(2) + d_{xyy}(3)E_y(1)E_y(2) \\
 & + d_{xzz}(3)E_z(1)E_z(2) + d_{xzy}(3)E_z(1)E_y(2) \\
 & + d_{xyz}(3)E_y(1)E_z(2) + d_{xzx}(3)E_z(1)E_x(2) \\
 & + d_{xxz}(3)E_x(1)E_z(2) + d_{xxy}(3)E_x(1)E_y(2) \\
 & + d_{xyx}(3)E_y(1)E_x(2)
 \end{aligned}$$

9 components for each component of P, ie. 27 in total

In a lossless medium, the order of the multiplication of the fields is not significant, therefore

$$d_{ijl} = d_{ilj}$$

Consequently the matrix expression for P_i can be simplified (slightly) to

$$\begin{array}{rcccccccc}
 P_x & & & & & & & & E_x(1)E_x(2) \\
 & & & & & & & & E_y(1)E_y(2) \\
 P_y = & d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & & E_z(1)E_z(2) \\
 & d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} & E_y(1)E_z(2) + E_z(1)E_y(2) \\
 P_z & d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} & E_x(1)E_z(2) + E_z(1)E_x(2) \\
 & & & & & & & E_x(1)E_y(2) + E_y(1)E_x(2)
 \end{array}$$

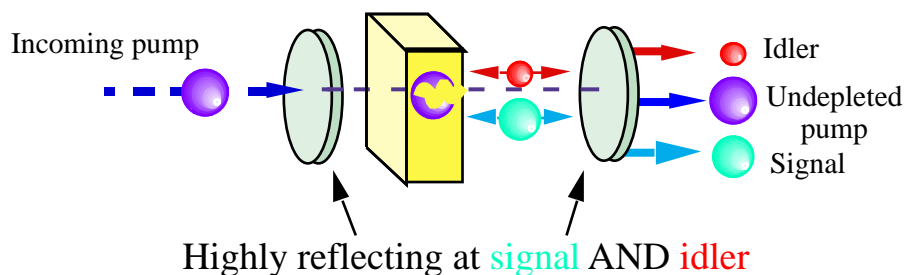
Optical parametric down conversion is an example of a second order process. In optical parametric down-conversion, an input pump wave at frequency ω_3 (ω_{pump}) is converted into two outputs at frequencies ω_1 and ω_2 , these are termed the signal and idler frequencies ($\omega_{\text{sig}} > \omega_{\text{idl}}$). As with other nonlinear interactions, these frequencies must obey the energy conservation relation, i.e.

$$\omega_{\text{pump}} = \omega_{\text{sig}} + \omega_{\text{idl}}$$

For a given ω_{pump} the ω_1 and ω_2 are determined by the phase-matching within the crystal, i.e.

$$k_{\text{pump}} - k_{\text{sig}} - k_{\text{idl}} = k = 0$$

To further increase the conversion efficiency the OPO cavity can be made resonant at both signal and idler frequencies, a doubly-resonant OPO (DRO)



However, to operate a DRO, four conditions must be satisfied simultaneously

- energy-conservation (see above)
- phase-matching (see above)
- cavity resonance for the signal field $m(\omega_{\text{sig}}/2) = L_{\text{cavity}}$
- cavity resonance for the idler field $n(\omega_{\text{idl}}/2) = L_{\text{cavity}}$

a) Optical Parametric Oscillators (OPO)

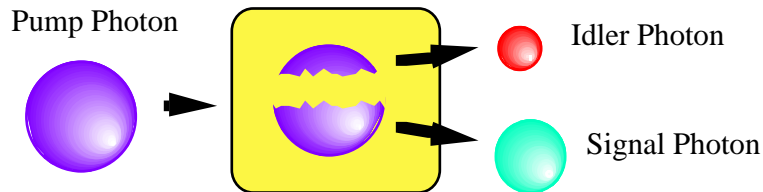
Another example of a second order nonlinear process is optical parametric down conversion. In optical parametric down-conversion, an input pump wave at frequency ω_3 (ω_{pump}) is converted into two outputs at frequencies ω_1 and ω_2 , these are termed the signal and idler frequencies ($\omega_{\text{sig}} > \omega_{\text{idl}}$). As with other nonlinear interactions, these frequencies must obey the energy conservation relation, i.e.

$$k_{\text{pump}} = k_{\text{sig}} + k_{\text{idl}}$$

For a given k_{pump} the k_{sig} and k_{idl} are determined by the phase-matching within the crystal, ie.

$$k_{\text{pump}} - k_{\text{sig}} - k_{\text{idl}} = k = 0$$

Within a photon picture, the OPO can be thought of as a photon splitter

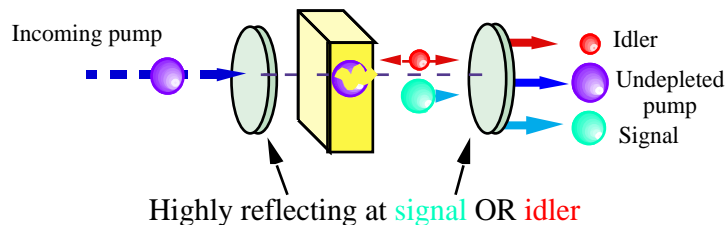


To form an optical parametric oscillator, resonance is provided by feedback at cavity mirrors for either, or both, the signal and idler frequencies. This significantly increases the efficiency of the conversion from pump frequency to signal and idler frequencies.

Note that unlike a laser, stimulated emission plays no role in an OPO

b)

A device with feedback at only one of the signal or idler frequencies is referred to as a singly-resonant OPO (SRO).

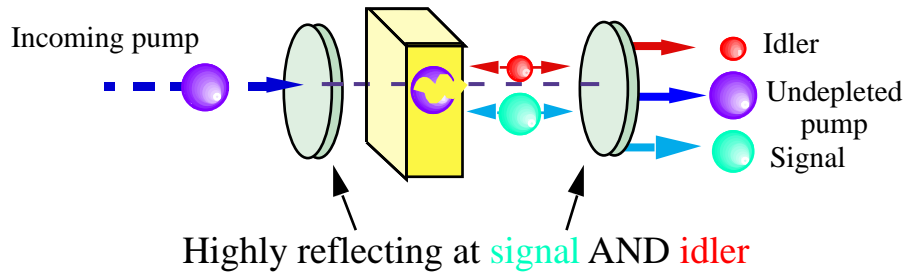


SRO's have comparatively high thresholds which are often difficult to reach with cw pump sources.

On the positive side: For an SRO, the frequency of the resonant wave corresponds to the cavity mode closest to optimum phase-matching. As has been demonstrated by a number of reported SRO configurations, no additional frequency selection is required to ensure single mode output. In a true SRO, the required stability for cavity length to maintain single mode output is identical to that of a conventional laser, namely,

$$L_{\text{stab}} \pm \frac{\lambda_{\text{res}}}{4}$$

To further increase the conversion efficiency the OPO cavity can be made resonant at both signal and idler frequencies, a doubly-resonant OPO (DRO)

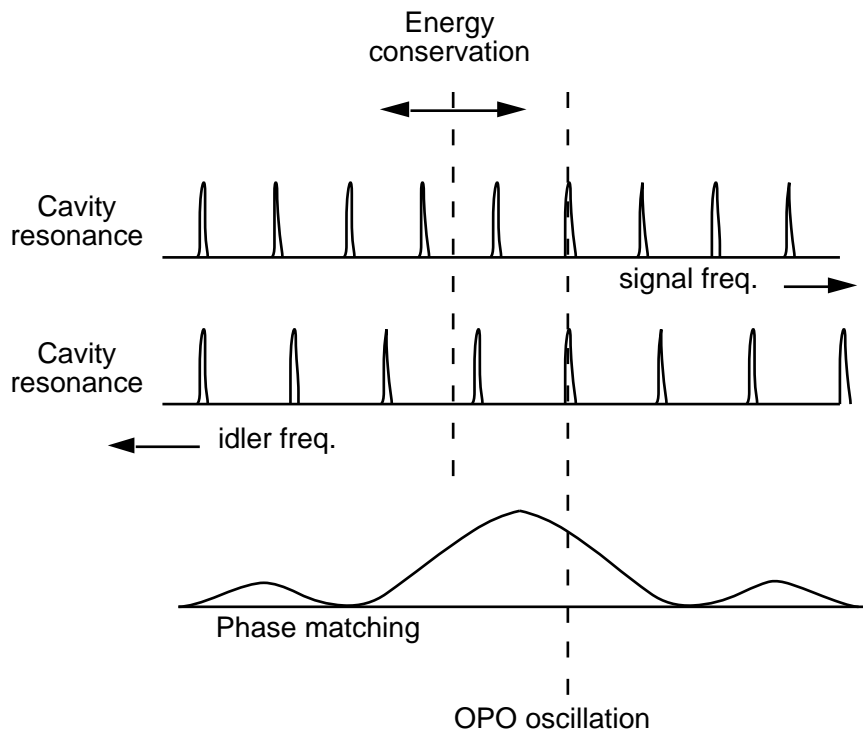


However, to operate a DRO, four conditions must be satisfied simultaneously

- energy-conservation
- phase-matching
- cavity resonance for the signal field
- cavity resonance for the idler field

In general, a DRO is over-constrained, and this introduces complications in the tuning of these devices. To hold a single output frequency a cavity length stability of better than 1nm is required.

The figure below shows the interrelation between the various constraints. (Note that the axes for the signal and idler frequencies are reversed.)



c) An example of an SRO OPO incorporating a frequency tripled diode-pumped laser as the pump source is shown below

