Control of optical aberrations with coded apertures

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ABSTRACT

We report the design of binary-amplitude masks that in conjunction with digital restoration enable mitigation of optical aberrations. Essentially, the design process aims to reduce destructive interference of optical transfer function phasors.

Keywords: Fourier optics and signal processing; Image processing; Imaging systems; Computational imaging; Image formation theory; Optical transfer functions.

1. INTRODUCTION

The designer of modern optical imaging systems has a range of options available for optimizing image quality. Using traditional optical design techniques, for a given combination of field of view, optical speed and image quality, further enhancements can be achieved only by the introduction of increased optical complexity[1]. In recent years hybrid imaging, in particular, so-called wavefront coding[2], techniques, in which optical encoding is combined with digital decoding, have been demonstrated to enable diffraction-limited imaging in the presence of high levels of optical aberrations[3]. This has enabled, for example, extended field-of-view imaging with a simple singlet lens[4] and miniaturization of a zoom lens using a single moving element[5]. Wavefront-coded hybrid imaging involves the use of refractive phase masks close to the aperture stop to encode the recorded image with specific aberration-insensitive point-spread functions. The relative insensitivity of these point-spread functions can be understood in terms of the way that spatial-frequency phasors composing the optical-transfer function are phase modulated by the wavefront-coding process so that they are wrapped into spirals which prevents the formation of nulls[6]. In this paper we seek instead to prevent the formation of nulls by selective amplitude shading of the pupil: whereas wavefront coding avoids nulls by phase modulation of the decomposed OTF components, the amplitude coding is introduced in such a way as to reduce attenuate of those spatial-frequency phasor components that cause destructive interference and nulls in the decomposed OTF[7].

Such binary-amplitude masks located at the aperture stop can be optimized to mitigate optical aberrations and allow sufficient information to be recorded by the detector for the recovery of a sharp image using digital image restoration. In comparison to phase-based correction techniques that employ reflective or transmissive wavefront modulators, this technique offers lower complexity and cost, and can be accomplished efficiently from ultra-violet to far-infrared wavelengths with a wide variety of fixed or agile spatial-light amplitude-modulation techniques.

In the next section, we describe two approaches for deriving coding masks to mitigate a given optical aberration. In the first approach, significant image contrast across all spatial frequencies is obtained by employing a low-dimensional mask optimization that involves selective blocking to reduce destructive interference from parts of the aperture[7], we call these *contour masks* and they can be considered to be the amplitude-mask equivalent of the phase masks proposed by Love *et al*[8,9]. An analytical expression for the upper limit of the modulation transfer function (MTF) of an imaging system with large aberrations and a contour mask is also reported. In the second approach, the masks are numerically optimized by exhaustive search over a high-dimensional discretised space of binary values as proposed by Stayman *et al.*[10]. In section 3, we show examples of the performance of coded masks in the presence of common aberrations with monochromatic light. Finally, the performance of a set of agile coded masks is evaluated for imaging through the aberrated conformal surface of a missile dome.

2. DERIVATION OF BINARY AMPLITUDE MASKS

2.1 Contour masks

The contour mask $M(u)$ for an aberration characterized by an optical path length variation $l(u)$, will tend to block contours of equal phase and it is given by

$$M(u) = 1, \forall u \mid \left(l(u) - \varphi_0 - t \cdot u + \frac{\Delta\varphi}{2} \right) \bmod 2\pi \le \Delta\varphi \tag{2.1}$$

where u is the normalized pupil coordinate and $\Delta\varphi$ is the maximum permitted phase-difference between any two points in the pupil. The parameters φ_0 and t are the reference phase and the tip-tilt respectively and are the free parameters to be optimized. The introduction of tip-tilt, t, simply displaces the imaged field-of-view. As demonstrated by Vettenburg *et al.*[1], the optical-transfer function of a given contour mask contains no nulls and can be approximated by

$$OTF_m(v) \approx \frac{2\pi}{\Delta\varphi} \frac{MTF_{dl}(v)}{\pi^2} \sin^2\left(\frac{\Delta\varphi}{2}\right) \le \frac{2}{\pi^2} MTF_{dl}(v) \tag{2.2}$$

,

where the *dl* subscript denotes diffraction-limited. Note that the magnitude of the mask MTF is maximized when $\Delta\varphi = \pi$ and in the limit for large aberrations transmission is 50%. This mask will yield approximately 20% of the contrast of the diffraction-limited MTF. More importantly, the expected MTF is independent of the magnitude of the aberrations: it is limited only by the contrast and spatial resolution of the amplitude modulator.

2.2 Discretised Masks

An alternative approach involves optimization of a discretized mask for maximization of the MTF [4]. With redolence of an approach initially explored by Häusler[11] multiple masks and associated MTFs may be used to form a single image such that the MTF is effectively that that is obtained by combining the masks. Optimisation may then yield sets of masks with complementary MTF characteristics, where nulls in the MTF of one mask are compensated by high MTFs for other masks. Such imaging systems require a reconfigurable binary mask, as may be implemented by a shutter array or digital mirror device, to acquire a sequence of coded images that are subsequently processed via a multi-frame restoration algorithm to produce a sharp image. The recorded data is given by

$$\vec{g} = \begin{pmatrix} \vec{g}_1 \\ \vec{g}_2 \\ \vdots \\ \vec{g}_N \end{pmatrix} = \mathbf{H}\vec{f} + \vec{n} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_N \end{pmatrix} \vec{f} + \begin{pmatrix} \vec{n}_1 \\ \vec{n}_2 \\ \vdots \\ \vec{n}_N \end{pmatrix} \tag{2.3}$$

where $\vec{g}_1 \ldots \vec{g}_N$ are N vectorised images recorded for a source intensity distribution $\vec{f}$ and a set of mask PSFs represented by matrices $\mathbf{H}_1 \ldots \mathbf{H}_N$. Tikhonov-like restoration is obtained by inversion of this equation:

$$\vec{f}' = \left(\mathbf{H}^*\mathbf{H} + \sigma \mathbf{I}\right)^{-1} \mathbf{H}^* \vec{g} \tag{2.4}$$

The masks $\mathbf{M}_i \ldots \mathbf{M}_N$ are optimized against a cost function given by

$$CF\left\{\mathbf{M}_i : i = 1, \ldots, N\right\} = \arg\min \left\| \frac{1}{\sigma + \sum_i^N T_i \, \mathrm{MTF}(M_i)} \right\|^2 \tag{2.5}$$

where the T_i is the mask transmissivity.

3. RESULTS

3.1 Contour masks

The optimization of contours masks is illustrated by the masks displayed in the left column of Fig.1. In the first two rows are shown optimized contour masks for two representative fundamental aberrations: astigmatism and coma. Black areas represent opaque areas of the mask and are overlain on colour-coded aberrations where the phase of the aberration is indicated by hue. The plots in the central column Fig.1 show the MTFs for the aberrated pupils: the low values and nulls in the MTFs are readily apparent. The MTFs in the right column correspond to the masked, aberrated pupils and in agreement with equation (2.2) they are approximately equal to 20% of the diffraction-limited MTF; that is, much higher than the MTFs of the non-coded pupils and also no nulls are apparent. In consequence, the use of coding enables high-quality image recovery.

The last row of Fig. 1 shows a typical phase function for an aberrated eye together with the calculated contour mask and MTFs for the aberrated and coded pupil. This is an illustration of a potential use of this technique for low-cost mitigation of arbitrary aberrations – in this case for high resolution imaging of the retina.

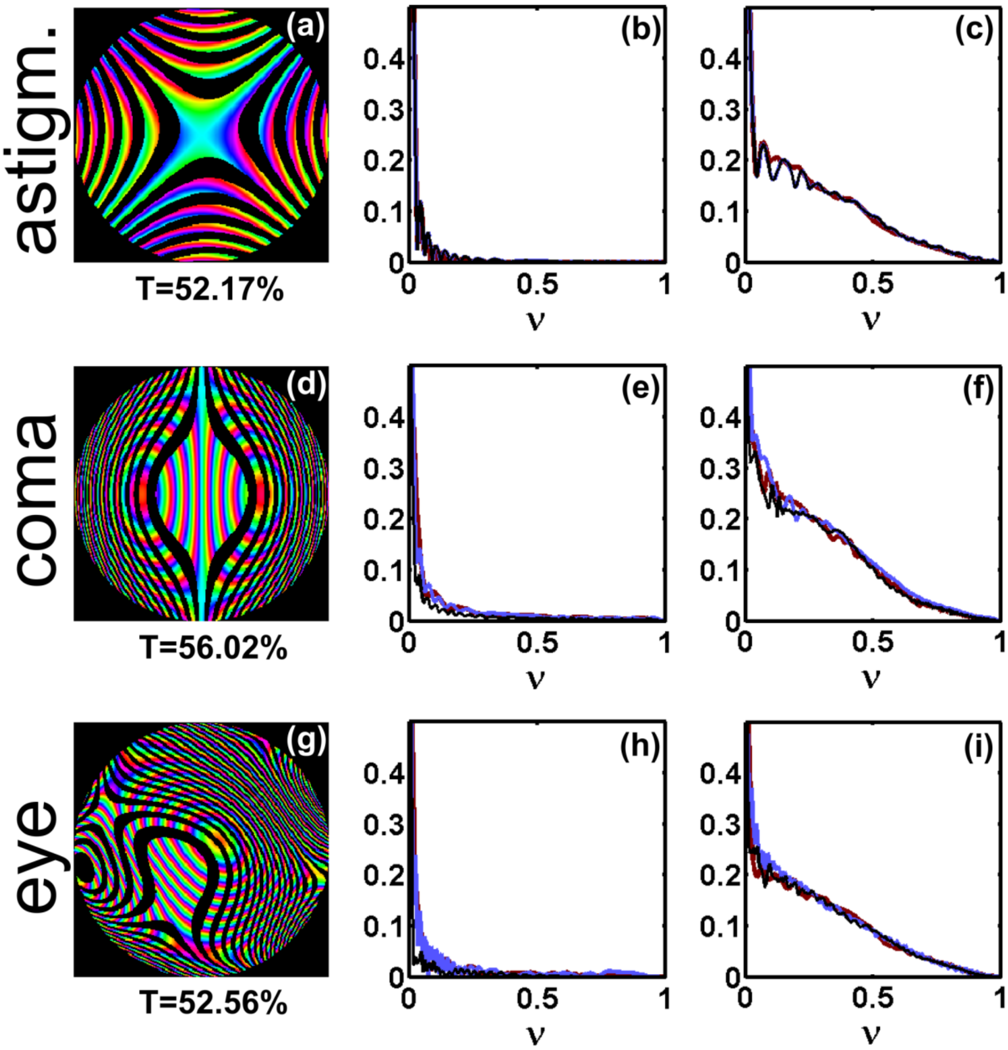

Figure 1. Aberration compensation of astigmatism (a-c) coma (d-f) with a root-mean-square optical-path difference of 2λ and a typical eye aberration. The aberration phases and contour masks with the relative transmission noted below are shown in (a,d,g). The tangential (black), sagittal (blue), and diagonal (red) MTF without mask in (b,e,h), and with mask in (c,f,i).

3.2 Multiple discretized phase masks

The optimization of multiple discretized masks described in section 2.2 is applicable to an arbitrary number of masks, but we restrict our attention here to an example of two 16×16 pixel binary-amplitude masks optimized to correct aberrations introduced by an ellipsoidal dome as shown in Fig. 2. We restrict our consideration to monochromatic light. The imaging system employs a reconfigurable shutter array at the aperture stop, which can change the binary-amplitude pattern depending on the field of view of interest, as in foveal imaging. Ray-tracing shows that the magnitude of the aberrations introduced by the dome increases gradually with field of view (FoV) up to a maximum of 4 waves at ±45°.

Two masks for each position were optimized using a differential evolution algorithm. Example coding masks, optimized to mitigate aberrations at (0,45°) are shown in Fig. 3 together with the aberrating functions they mitigate. The transmissions of each masks is about 60%. Optimisation was achieved using a single wavelength of 4μm, but PSFs were calculated for a rectangular pass-band between 3 and 5μm. The PSFs without coding, with coding using masks 1 and 2 and the recovered PSFs are shown in Fig 4.

The tangential and sagittal MTFs of the system at (0^{o}, 45^{o}) with and without masks are shown in Fig.5. Clearly, in spite of the 40% reduction in transmission of the amplitude masks the magnitude of coded MTFs is significantly higher than the MTF of the conventional aberrated system and, more importantly, the coded masks have eradicated the nulls across most of the spatial frequency space by reducing destructive interferences in the optical transfer function. It is notable that the MTFs are again approximately 20% of the diffraction-limited MTF scaled by the mask transmissivity – this is in agreement with equation (2.2)

Restored images using regularized multi-frame recovery are displayed in Fig. 5 for both conventional and binary-amplitude mask systems. Both systems use a sequence of two masks (fully open in the conventional system) and the images are acquired with the same integration time, so they have equal additive Gaussian noise levels but different SNR at the time of detection (the ratio of peak signal to root-mean-square noise of the conventional and amplitude-mask images are 37dB and 30dB respectively). The images recovered from the conventional system without aperture coding are shown on the left and the images recovered from the aperture-coded system are shown on the right. The subjective improvements in image quality with aperture coding are apparent. In each case the root-mean square error is approximately halved by the use of aperture coding: from 35% to 15% for the spoke target and from 27% to 13% for the USAF target.

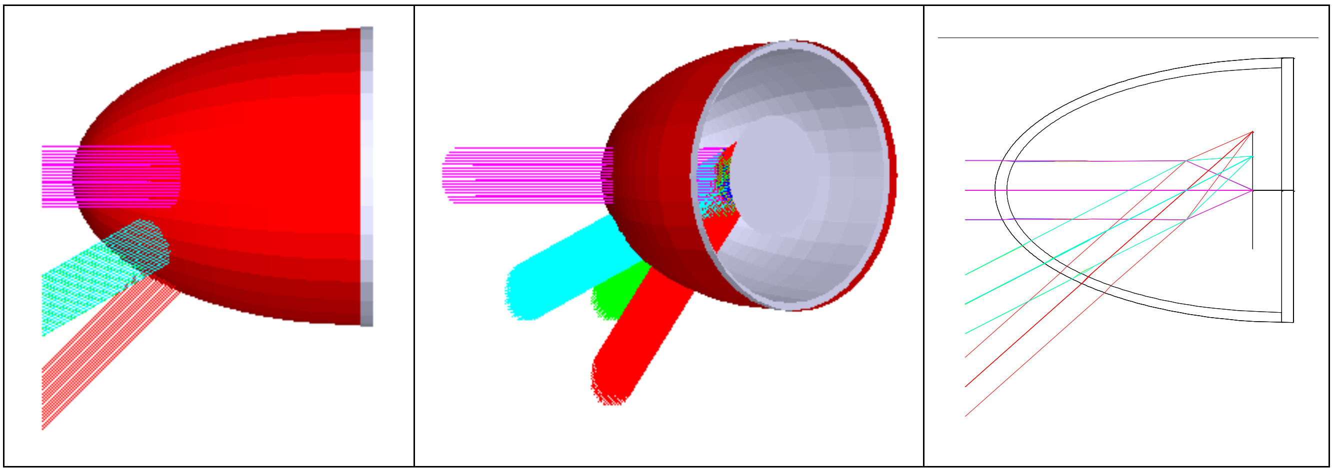

Figure 2. Three views of the MgF2 missile dome optical design from Zemax.

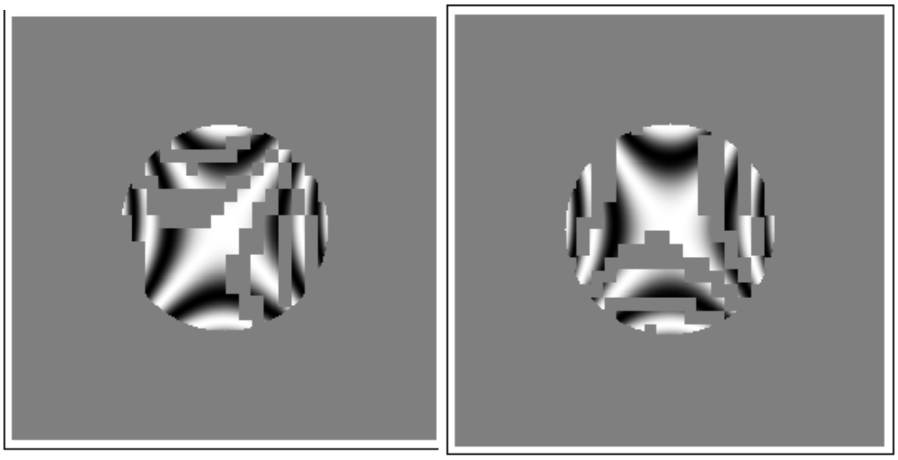

Figure 3. Two coded masks optimised to improve performance at FoV (0^o, 45^o).

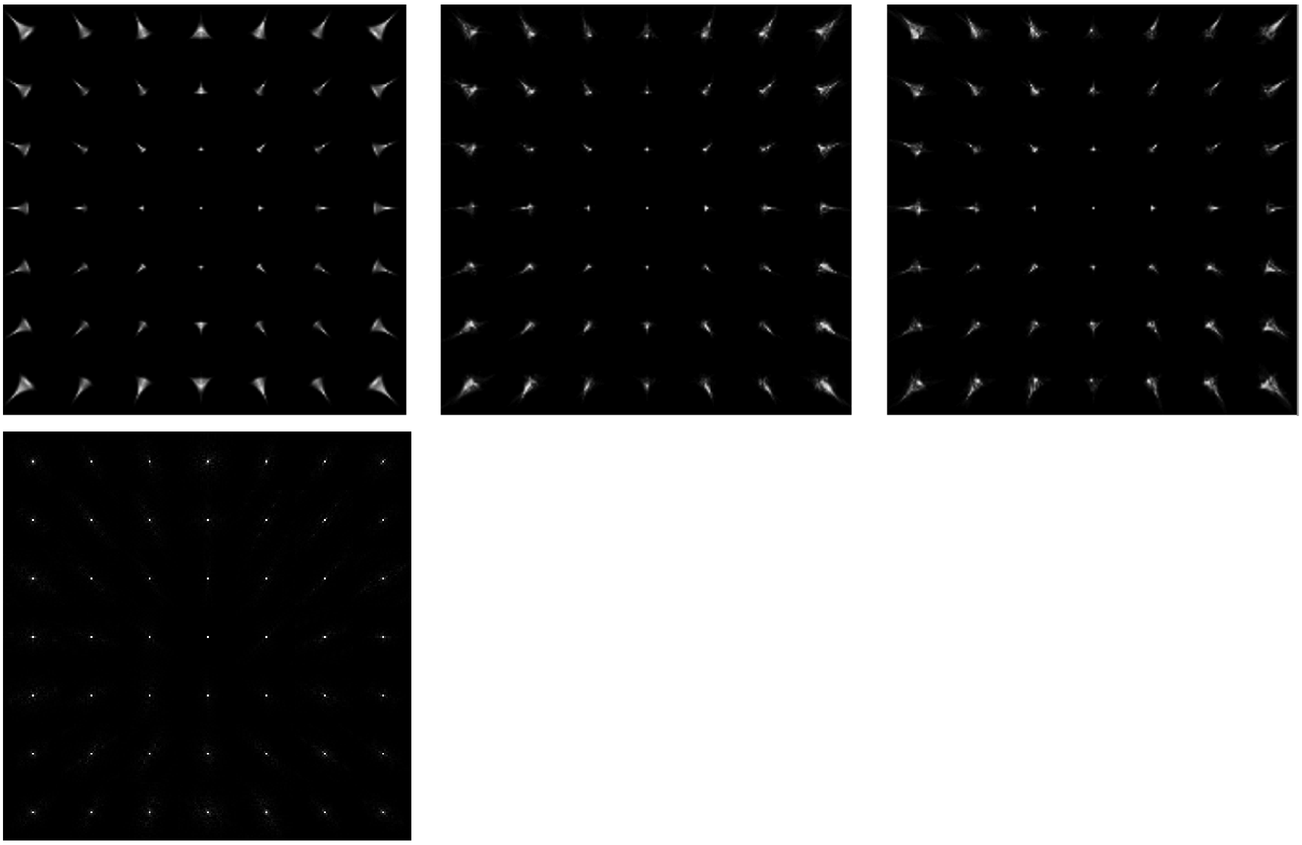

Figure 4 Calculated PSFs across the {±45 °,±45 °} field of view for no mask (top left), masks 1 and 2 (top middle and top right) and recovered PSFs (bottom row).

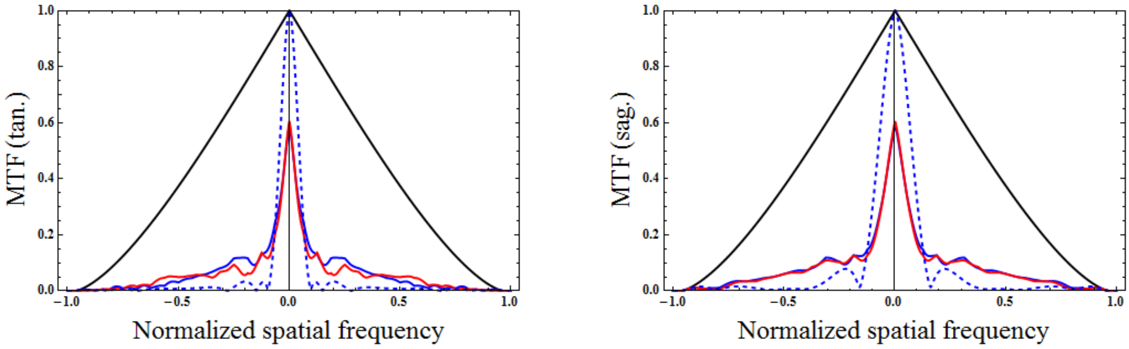

Figure 5. Tangential (left) and sagittal (right) unrestored MTFs of the diffraction limited (solid black), aberrated (dashed), coded mask 1 (solid blue) and coded mask 2 (solid red) systems. MTFs scaled according to transmission. The MTFs have been scaled according to transmission of the aperture.

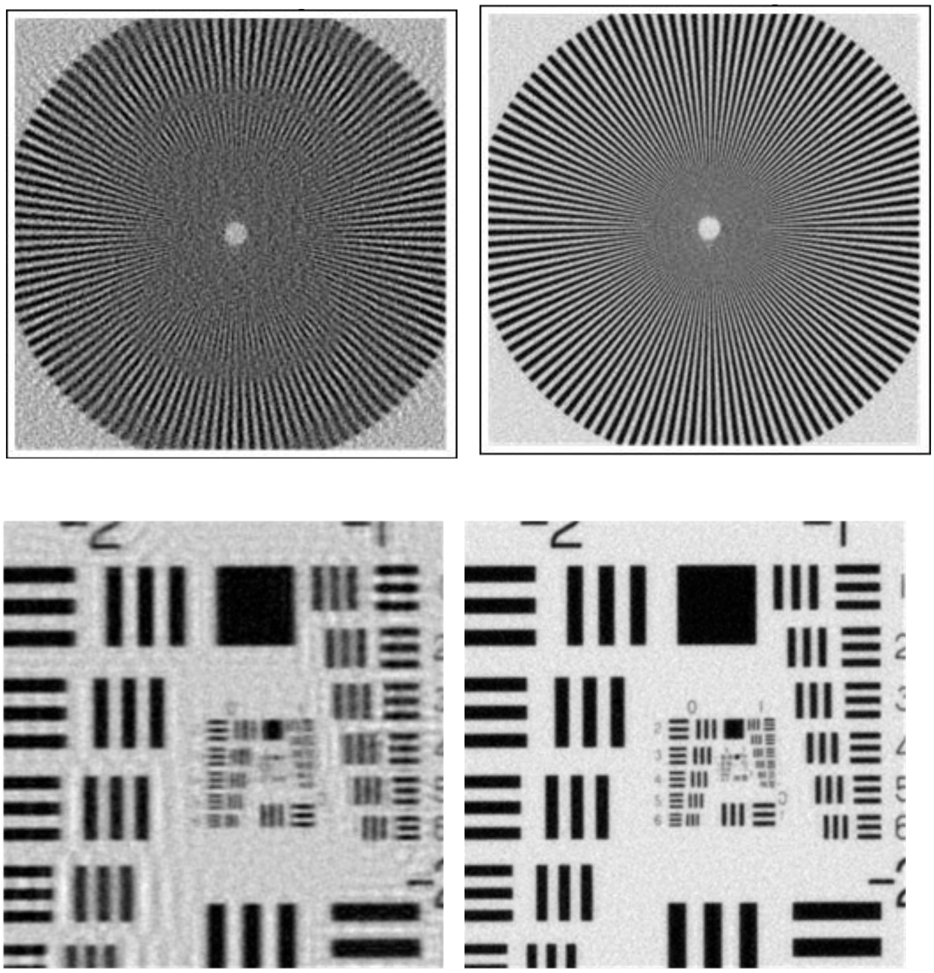

Figure 6. Restored images from the conventional system (left) and the amplitude-mask system (right) for FoV at (0^{o},45^{o}).

4. Conclusions

We have shown that binary-amplitude masks can be designed to mitigate optical phase aberrations to yield, in general, an improved MTF (approximately 20% of the magnitude of the diffraction-limited MTF for contour masks) and image quality. The absence of nulls and the relatively modest reduction in the MTF allows for digital recovery of a high quality image. Although the benefits of the masks are most pronounced for monochromatic imaging, it might be reasonable to think that two or more masks could give better results for broadband imaging.

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